Finite volume effects in the scalar meson sector and generalization of Luescher approach

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Chiral Unitary approach for finite volume
Generalization of Luescher approach to relativistic energies.
Generalization of Luescher approach to two channels
Approximations to be avoided. Critical view on “signals of resonances”
Results for the scalar mesons f0(980) and a0(980)

We simulate the QCD lattice results using the chiral unitary approach in the scalar meson sector

$$T = [1 - VG]^{-1} V$$

where V is a 2x2 matrix accounting for the s-wave $\pi\pi \to \pi\pi$, $K\bar{K} \to K\bar{K}$ and $\pi\pi \to K\bar{K}$ potentials

$$G_j = \int^{|\vec{q}|<q_{\max}} \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{2\omega_1(\vec{q})\,\omega_2(\vec{q})} \frac{\omega_1(\vec{q}) + \omega_2(\vec{q})}{(P^0)^2 - (\omega_1(\vec{q}) + \omega_2(\vec{q}))^2 + i\epsilon}, \quad \omega_{1,2}(\vec{q}) = \sqrt{m_{1,2}^2 + \vec{q}^{\,2}}$$

Next we study the system in a finite box of dimension L^3, demanding periodic boundary conditions

$$\tilde{G}_j = \frac{1}{L^3} \sum_{\vec{q}_i}^{|\vec{q}|<q_{\max}} \frac{1}{2\omega_1(\vec{q}_i)\,\omega_2(\vec{q}_i)} \frac{\omega_1(\vec{q}_i) + \omega_2(\vec{q}_i)}{(P^0)^2 - (\omega_1(\vec{q}_i) + \omega_2(\vec{q}_i))^2}, \quad \vec{q}_i = \frac{2\pi}{L}\vec{n}_i, \quad \vec{n}_i \in \mathbb{Z}^3$$

Then we look for poles of T in the finite volume: If we had only one channel

$$V^{-1}(E) - \tilde{G}(E) = 0 \quad \rightarrow \quad V^{-1}(E) = \tilde{G}(E)$$

For one channel and the energies E, eigenenergies of the box

In the continuum:

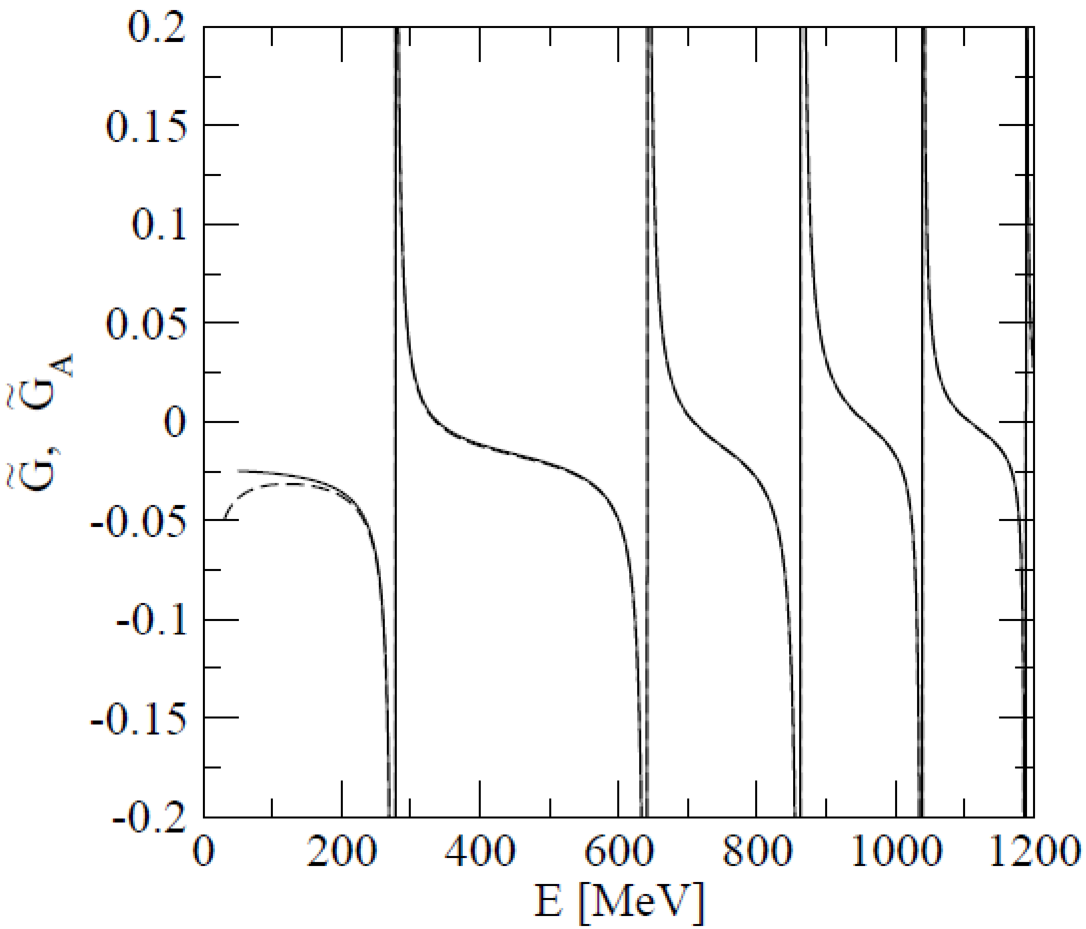
$$T(E) = \left(V^{-1}(E) - G(E)\right)^{-1} = \left(\tilde{G}(E) - G(E)\right)^{-1}$$

END OF FORMALISM IN ONE CHANNEL

This difference is cut off independent

π π channel   We fix a certain value of L

Determine E via the equation $V^{-1}(E) - \tilde{G}(E) = 0$



Connection to Luescher approach

$$\beta T(E) = f(E) = \frac{1}{p\cot\delta(p) - i\,p}$$

$$\mathrm{Im}\, G = -\frac{1}{8\pi}\,\frac{p}{\sqrt{s}}$$

$$p\cot\delta(p) = -8\pi\sqrt{s}\left(\tilde{G}(E) - \mathrm{Re}\, G(E)\right)$$

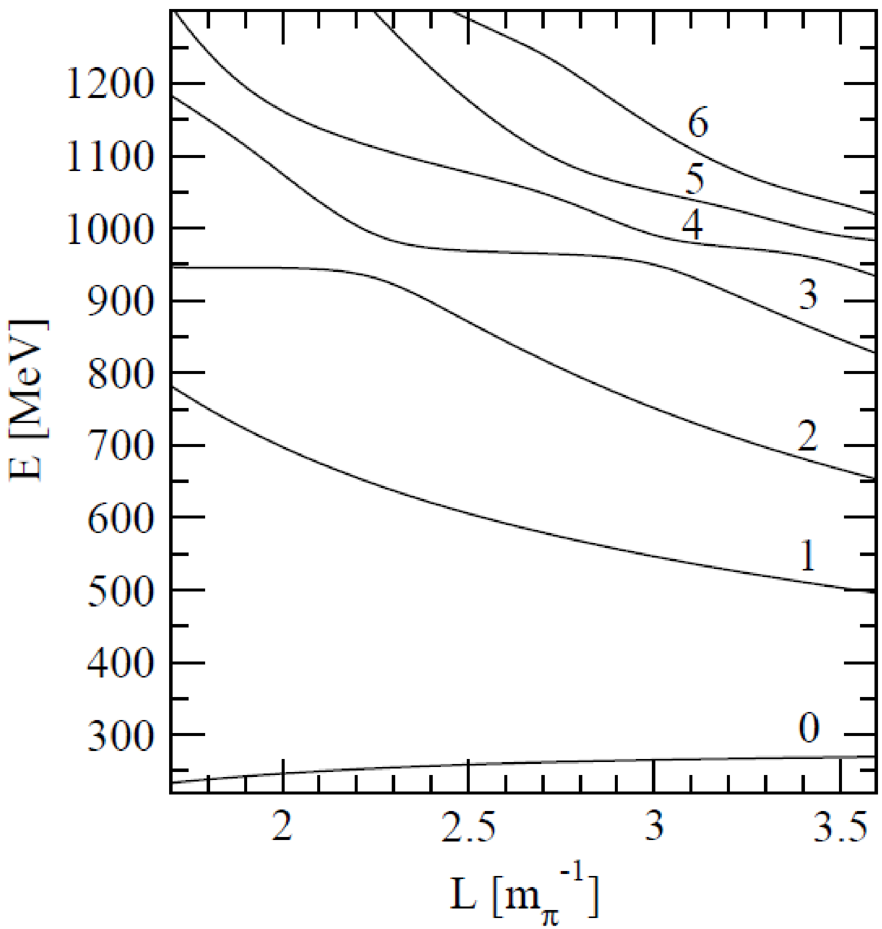
In Luescher approach one obtains:

$$p\cot\delta(p) = \frac{1}{\pi L}\sum_{\vec{n}_i}^{\Lambda}\frac{1}{|\vec{n}_i|^2 - [Lp/(2\pi)]^2} - \frac{4\,\Lambda}{L} = \frac{2\pi}{L}\,\pi^{-3/2}\,\mathcal{Z}_{00}(1;\tilde{p}^2)$$

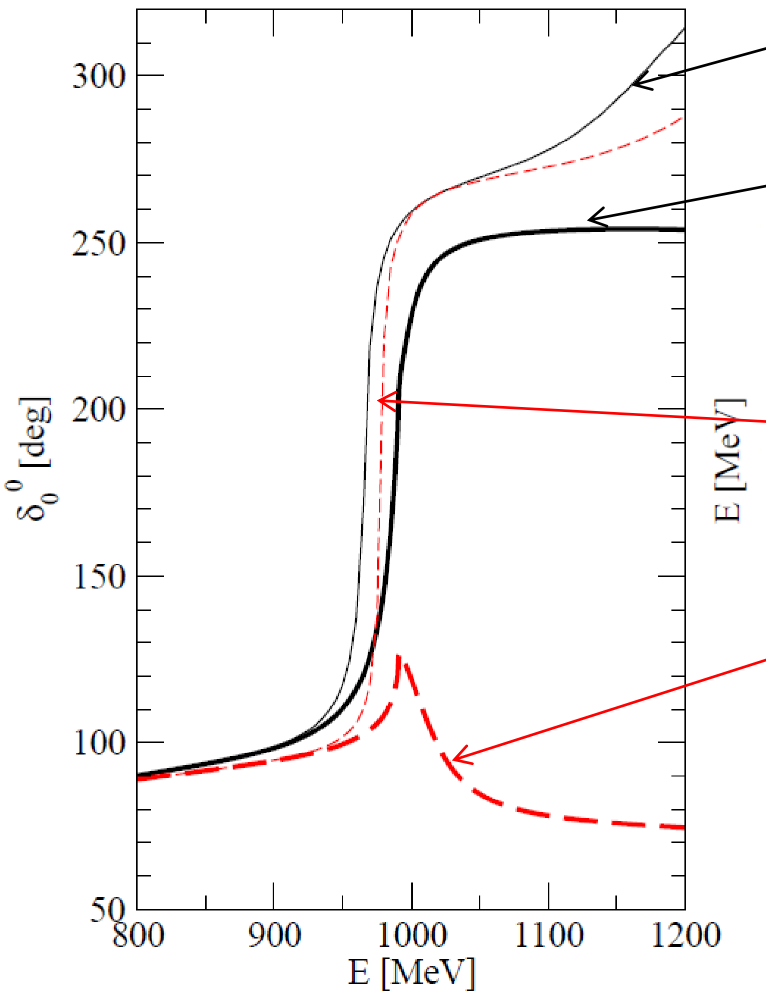
As a side effect of our approach, we can evaluate the Z function making our G functions non relativistic (practical and efficient method), but they are not needed in our approach.

Determination of the eigenenergies in the box with two channels

$$\det\left(1 - V\tilde{G}\right) = \begin{vmatrix} 1 - V_{11}\tilde{G}_1 & -V_{12}\tilde{G}_2 \\ -V_{21}\tilde{G}_1 & 1 - V_{22}\tilde{G}_2 \end{vmatrix} = 1 - V_{11}\tilde{G}_1 - V_{22}\tilde{G}_2 + \left(V_{11}V_{22} - V_{12}^2\right)\tilde{G}_1\tilde{G}_2 \equiv 0$$



Analysis of two channel results with finite volume using our approach with just the ππ channel = Relativistic Luescher approach



Luescher one channel analysis

Exact result: the rise of δ indicates the appearance of the $f_0(980)$

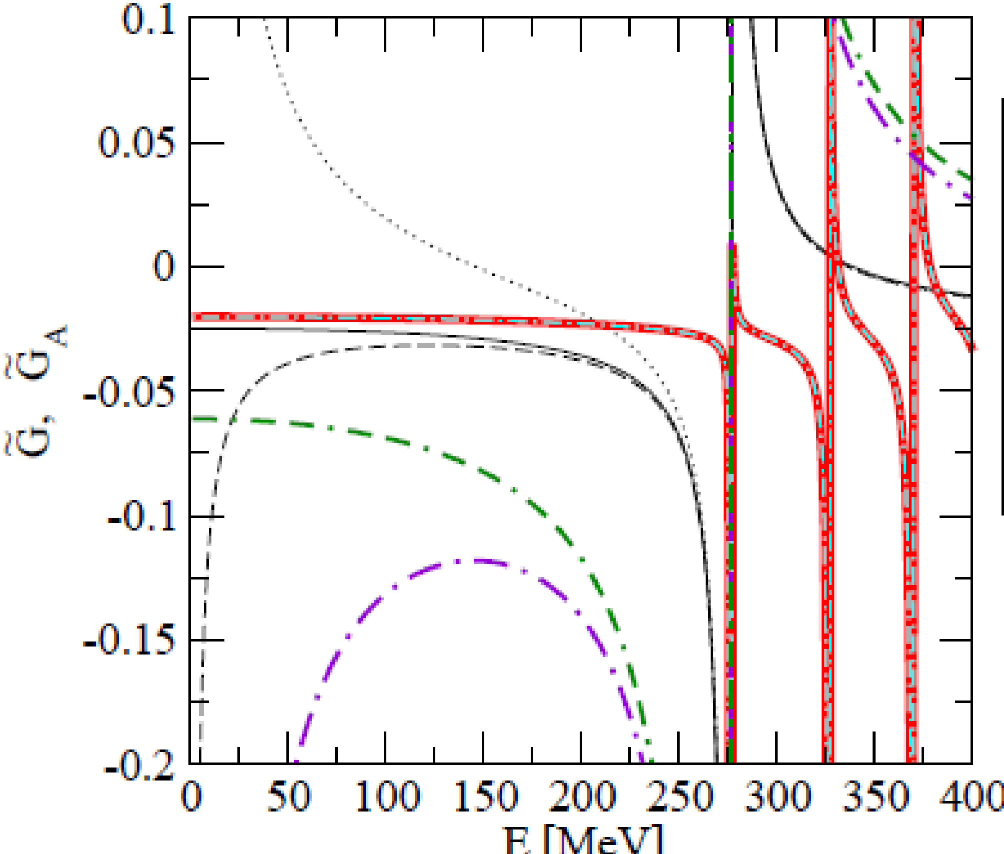
Luescher one channel analysis

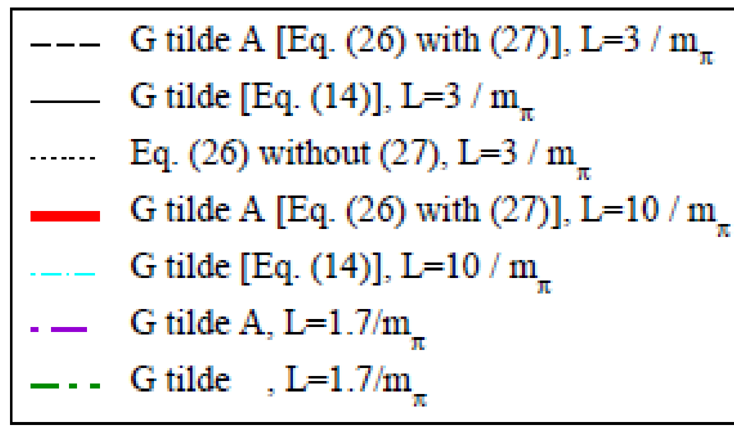
Exact result

The one channel gives a rise of δ, as if there was a resonance, even when it has disappeared.   WARNING for Lattice

Relativistic approximation done in Bernard, Lage, Rusetsky, Meissner **JHEP (2011).** Keep only first term to be able to use Luescher function.

$$\frac{1}{2\,\omega_1\,\omega_2}\,\frac{\omega_1+\omega_2}{(P^0)^2-(\omega_1+\omega_2)^2} = -\frac{1}{2P^0}\,\frac{1}{q^2-p^2} - \frac{2}{2\,\omega_1\,2\,\omega_2}\,\frac{1}{\omega_1+\omega_2+P^0}$$
$$-\frac{1}{2\,\omega_1\,2\,\omega_2}\,\frac{1}{\omega_1-\omega_2-P^0} - \frac{1}{2\,\omega_1\,2\,\omega_2}\,\frac{1}{\omega_2-\omega_1-P^0}$$





Approximation fails below threshold. Better avoid it in coupled channels.

We use exact result now.

Strategy to get phase shifts in two channel analysis: Take three trajectories (E versus L) and determine three L's for the same energy

$$V_{11}\tilde{G}_1^{(i)} + V_{22}\tilde{G}_2^{(i)} - \left(V_{11}V_{22} - V_{12}^2\right)\tilde{G}_1^{(i)}\tilde{G}_2^{(i)} = 1, \quad i = 1,2,3$$

$$\begin{pmatrix} V_{11} \\ V_{22} \\ V_{12}^2 - V_{11}V_{22} \end{pmatrix} = \tilde{G}_L^{-1}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad G_L = \begin{pmatrix} \tilde{G}_1^{(1)} & \tilde{G}_2^{(1)} & \tilde{G}_1^{(1)}\tilde{G}_2^{(1)} \\ \tilde{G}_1^{(2)} & \tilde{G}_2^{(2)} & \tilde{G}_1^{(2)}\tilde{G}_2^{(2)} \\ \tilde{G}_1^{(3)} & \tilde{G}_2^{(3)} & \tilde{G}_1^{(3)}\tilde{G}_2^{(3)} \end{pmatrix}$$

These equations determine the 3 V's for each E. With them we use the Bethe Salpeter equations with a cut off for G the same as the one chosen for $\tilde{G}(E)$, the results are cut off independent

$$T = [1 - VG]^{-1}V$$

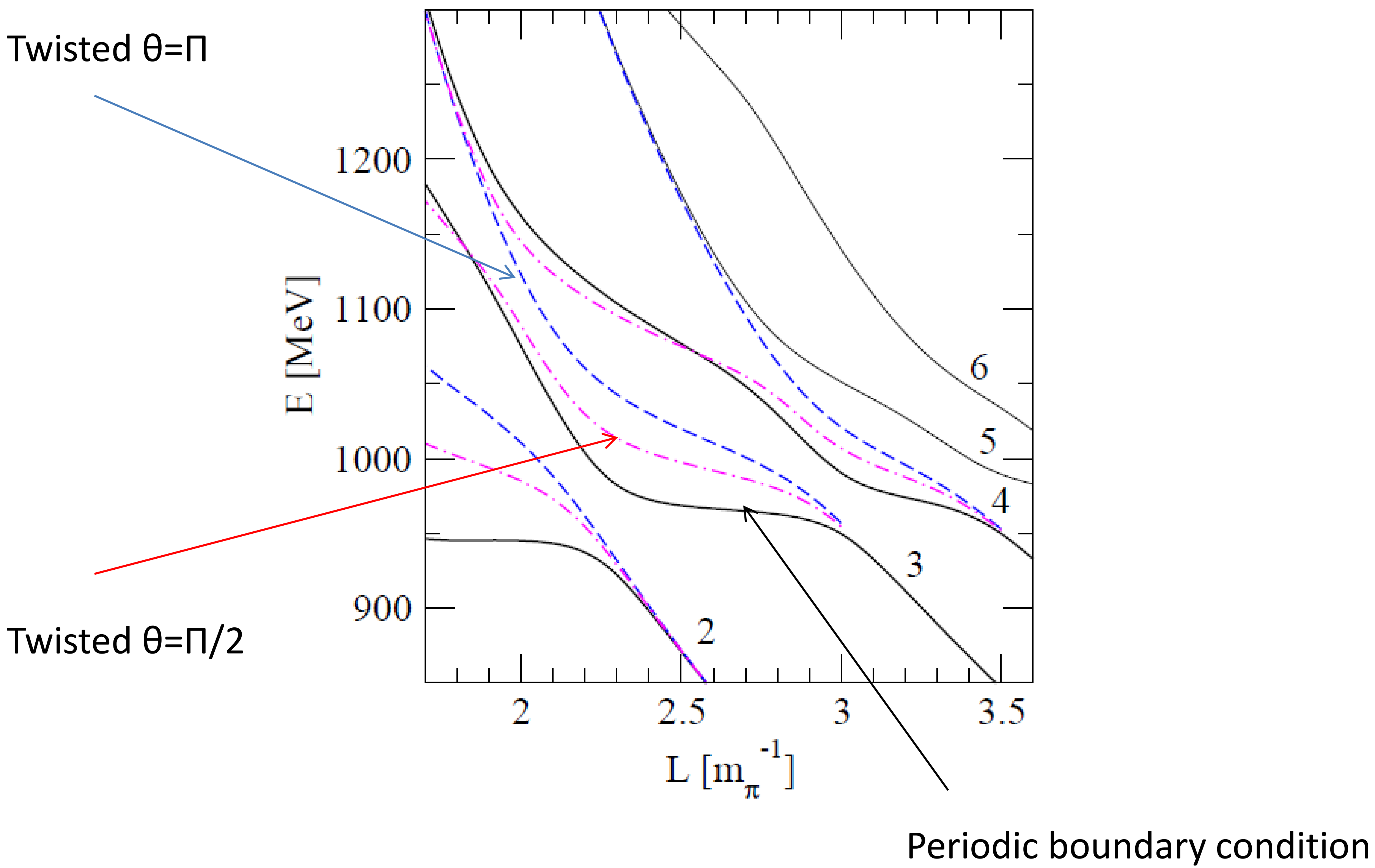
Three methods used:

1) Use three different trajectories using standard boundary conditions
2) Use asymmetric boxes, L_x, L_y, L_z different
3) Use twisted boundary conditions

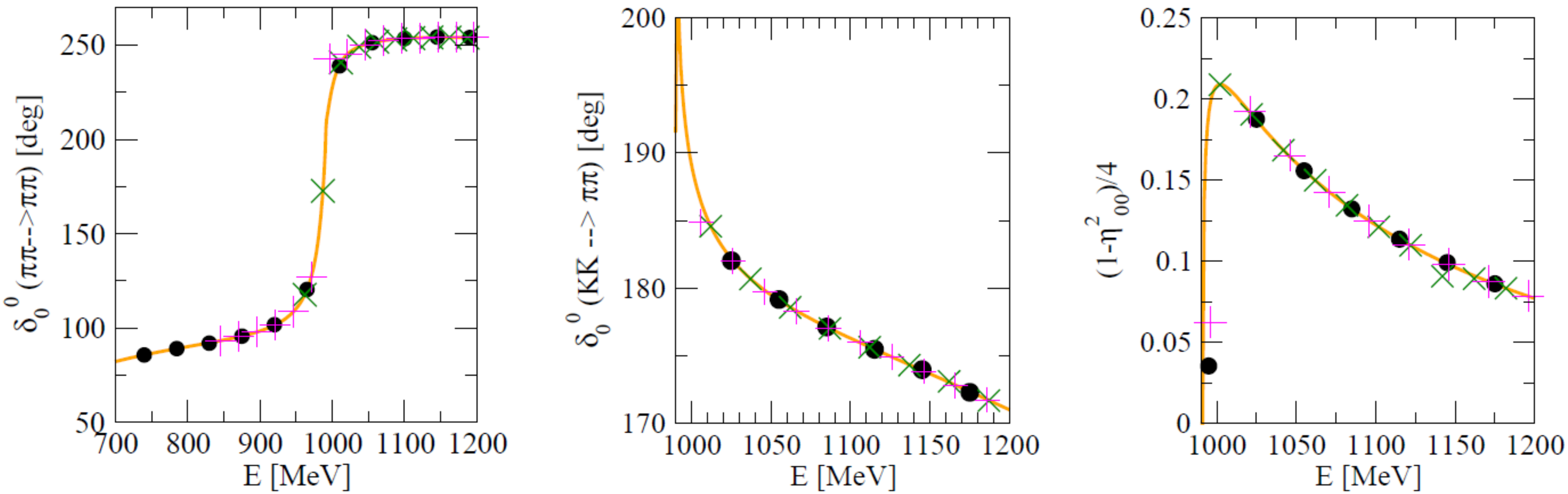
$$\Psi(\vec{r} + L\vec{e}_i) = e^{i\theta_i}\Psi(\vec{r})$$

$$\vec{p}_j \to \vec{p}_j + \frac{\vec{\theta}}{L}, \quad \vec{p}_j = \frac{2\pi}{L}\vec{n}_j, \quad \vec{n}_j \in \mathbb{Z}^3$$

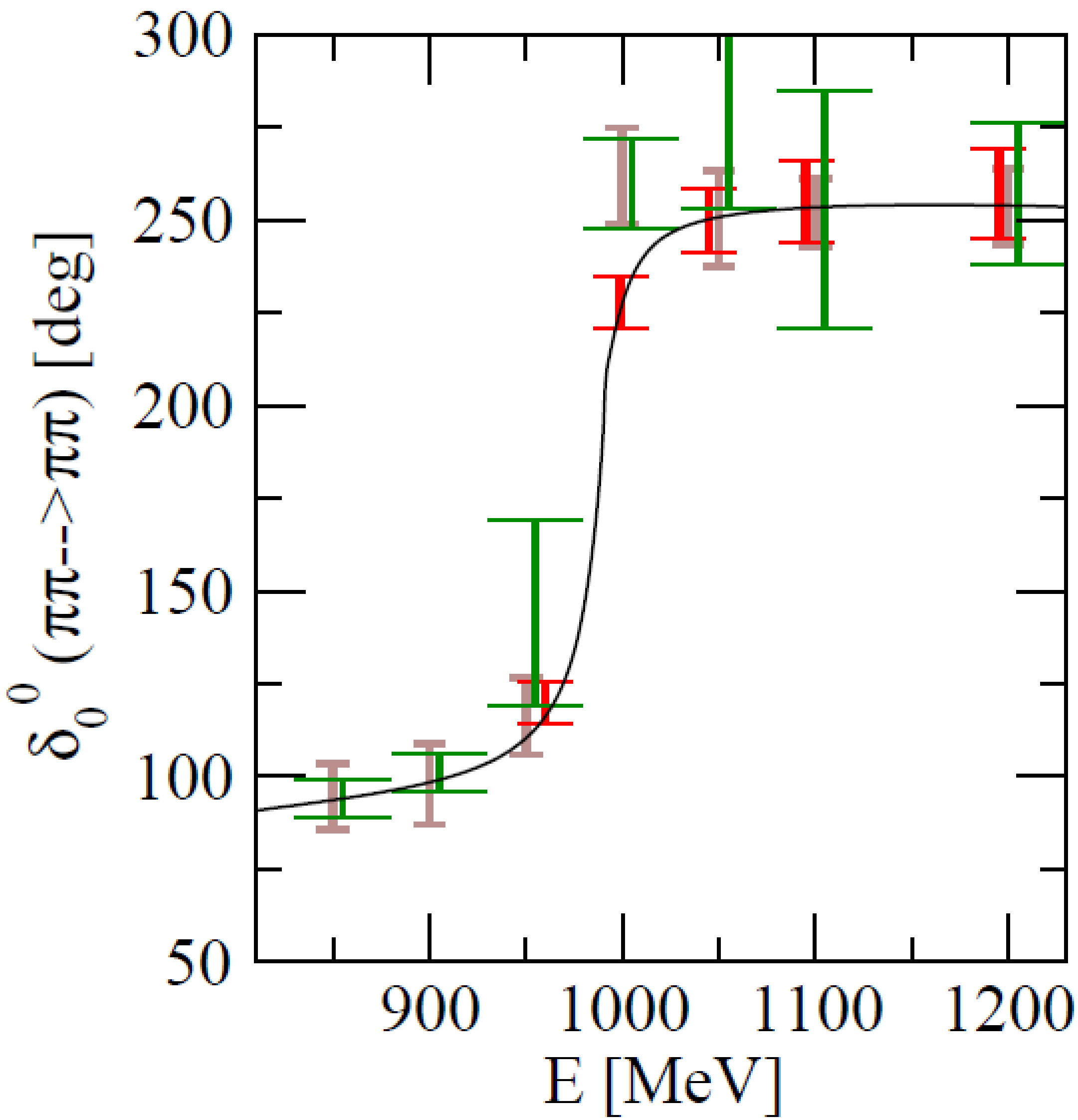
The three methods work


Twisted θ=Π
Twisted θ=Π/2
Periodic boundary condition
E [MeV]
L [m_π^-1]
1200
1100
1000
900
2
2.5
3
3.5
2
3
4
5
6

These results are not tautology: the initial chiral unitary approach required a certain cut off . The results obtained from our analysis are cut off independent



Analysis of errors: $\Delta L = 0.02\, m_\pi^{-1}$



Red: twisted boundary conditions

Green: asymmetric boxes

Brown: three different levels

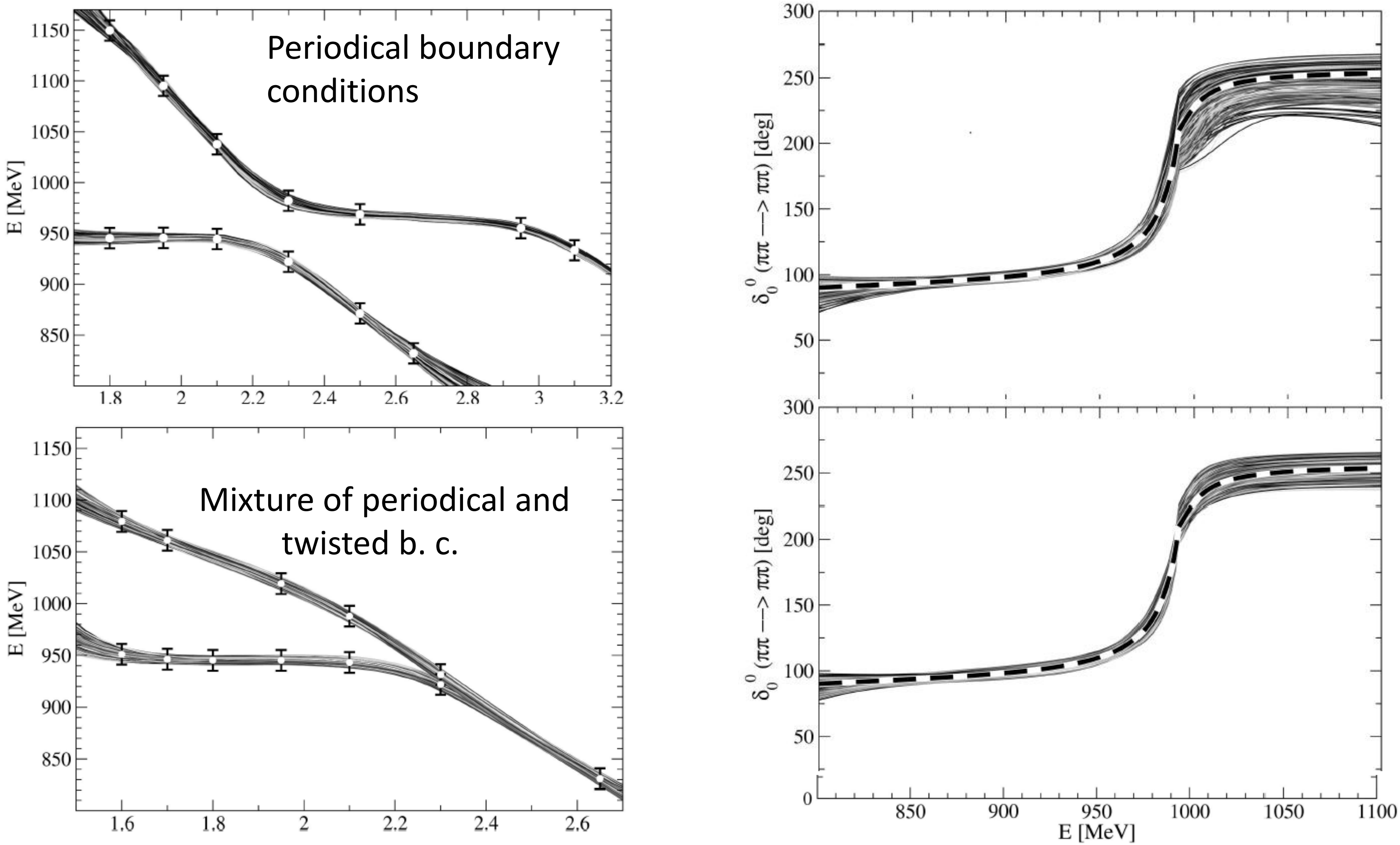
Different strategy: aproximate method, model dependent. Assume, as it occurs with chiral potentials at lowest order, that

$$V(i \to j) = a_{ij} + b_{ij}[s - (2m_K)^2]$$

Take about 15 eigenenergies of the box with errors of 10 MeV

Make a fit to the data to determine the 6 parameters, a_{ij}, b_{ij}

To determine errors of induced phase shift, choose random a_{ij}, b_{ij} such that Xisq= $Xisq_{min}$+1



Method works, but if original potential has a different s dependence than the linear one assumed in the analysis, the erros become larger

Conclusions

Even if the levels are obtained in two channels, the one channel ππ analysis works well till close to the K Kbar threshold: BUT RELATIVISTIC FORMALISM IS NECESSARY

The raise of δ close to the K Kbar threshold in one channel IS NOT indicative of the coming f_0 resonance, but a threshold effect

BEWARE OF SUCH SIGNALS IN QCD LATTICE RESULTS

We provide an exact relativistic treatment, which generalizes Luescher nonrelatistic approach. No need of the ordinary Luescher function Z

The formalism is far simpler than the standard Luescher approach.

An extension is done to two channels with relativistic kinematics

We prove that the method works and it is possible to obtain resonances that couple to two channels (most of them) from future lattice results.