Structure of scalar mesons and the Higgs sector of strong interaction

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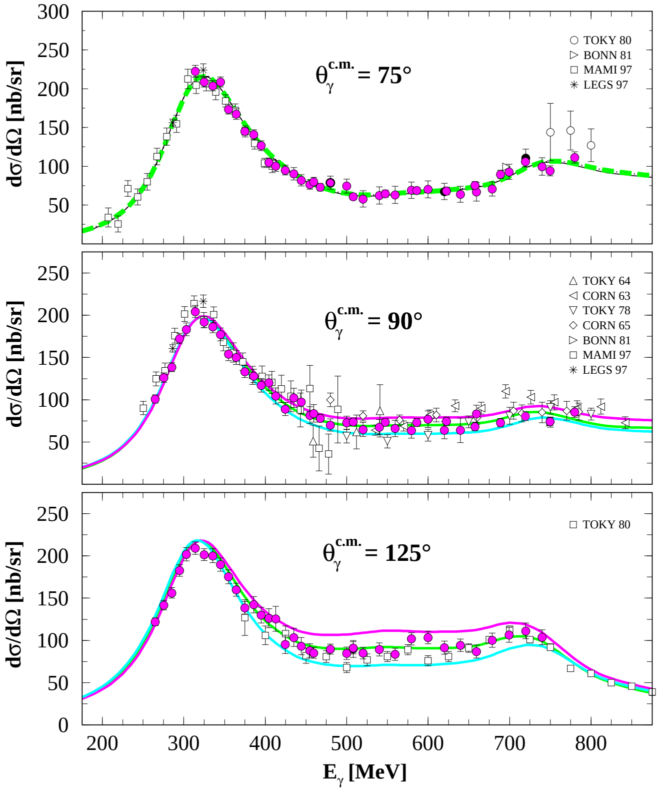
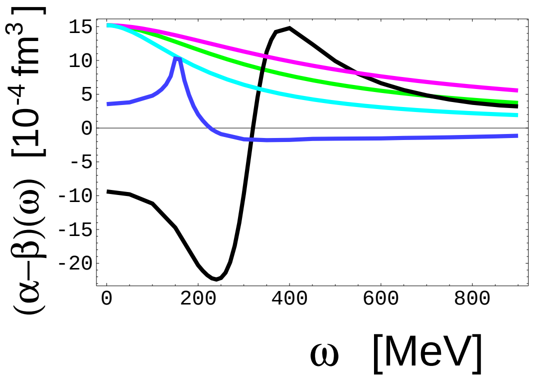
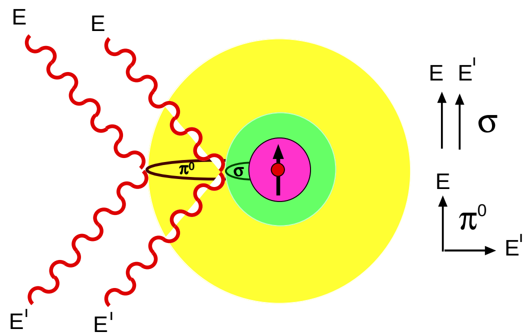
Topics:

1. Motivation: Compton scattering and polarizabilities
2. The tetraquark model of scalar mesons
3. The doorway model of scalar mesons
4. Mass generation via spontaneous and explicit symmetry breaking

Observation of the σ on the constituent quark







Direct observation of the sigma mesons as part of the constituent quark inside the nucleon via Compton scattering by the nucleon.

The π^0 and σ meson pole contributions

$$|\pi^0\rangle = \tfrac{1}{\sqrt{2}}(-|u\bar{u} + |d\bar{d}\rangle) \quad \mathcal{M}(\pi^0 \to \gamma\gamma) = \tfrac{\alpha_{em} N_c}{\pi f_\pi}\left[-\left(\tfrac{2}{3}\right)^2 + \left(\tfrac{-1}{3}\right)^2\right]$$

$$\gamma^t_{\pi(p,n)} = \frac{g_{\pi^0 NN}\mathcal{M}(\sigma \to \gamma\gamma)}{2\pi m^2_{\pi^0} m}\tau_3 = -46.7\,\tau_3 \;\; 10^{-4}\mathrm{fm}^4$$

$$|\sigma\rangle = \tfrac{1}{\sqrt{2}}(|u\bar{u} + |d\bar{d}\rangle) \quad \mathcal{M}(\sigma \to \gamma\gamma) = \tfrac{\alpha_{em} N_c}{\pi f_\pi}\left[\left(\tfrac{2}{3}\right)^2 + \left(\tfrac{-1}{3}\right)^2\right]$$

$$(\alpha - \beta)^t_{p,n} = \frac{g_{\sigma NN} M(\sigma \to \gamma\gamma)}{2\pi m^2_\sigma} = 15.2 \;\; 10^{-4}\mathrm{fm}^3, \quad (\alpha + \beta)^t_{p,n} = 0,$$

$$\alpha^t_{p,n} = +7.6 \;\; 10^{-4}\mathrm{fm}^3, \quad \beta^t_{p,n} = -7.6 \;\; 10^{-4}\mathrm{fm}^3$$

s-channel and t-channel polarisabilities

	α_p	β_p	α_n	β_n
σ pole	$+7.6$	-7.6	$+7.6$	-7.6
f_0 pole	$+0.3$	-0.3	$+0.3$	-0.3
a_0 pole	-0.4	$+0.4$	$+0.4$	-0.4
const. quark	$+7.5$	-7.5	$+8.3$	-8.3
nucleon	$+4.5$	$+9.4$	$+5.1$	$+10.1$
total pred.	$+12.0$	$+1.9$	$+13.4$	$+1.8$
exp. result	$+(12.0 \pm 0.6)$	$+(1.9 \mp 0.6)$	$+(12.5 \pm 1.7)$	$+(2.7 \mp 1.8)$
	unit 10^{-4} fm^3			

The nucleon structure component is calculated from photo-meson data.

s-channel - and t-channel spin polarisabilities

spin polarizabilities	$\gamma_\pi^{(p)}$	$\gamma_\pi^{(n)}$
π^0 pole	-46.7	+46.7
η pole	+1.2	+1.2
η' pole	+0.4	+0.4
const. quark structure	-45.1	$+48.3$
nucleon structure	+8.5	+10.4
total predicted	-36.6	$+58.3$
exp. result	$-(36.4 \pm 1.5)$	$+(58.6 \pm 4.0)$
	unit 10^{-4} fm^4	

The nucleon structure component is calculated from photo-meson data.

The tetra quark model of scalar mesons

Scalar mesons in the $(q\bar{q})^2$ representation

Y/I_3	-1	$-1/2$	0	$+1/2$	$+1$		f_s
$+1$		$d\bar{s}u\bar{u}$		$u\bar{s}d\bar{d}$		$\kappa(800)$	$1/4$
0			$u\bar{d}d\bar{u}$			$\sigma(600)$	0
0	$d\bar{u}s\bar{s}$		$s\bar{s}(u\bar{u}-d\bar{d})/\sqrt{2}$		$u\bar{d}s\bar{s}$	$a_0(980)$	$1/2$
0			$s\bar{s}(u\bar{u}+d\bar{d})/\sqrt{2}$			$f_0(980)$	$1/2$
-1		$s\bar{u}d\bar{d}$		$s\bar{d}u\bar{u}$		$\bar{\kappa}(800)$	$1/4$

The equal strange quark fractions in the tetraquark structures lead to equal masses for the $a_0(980)$ and $f_0(980)$ mesons. In flavour $SU(3)$ the neutral $a_0(980)$ has the structure $1/\sqrt{2}(-u\bar{u}+d\bar{d})$ and the $f_0(980)$ the structure $s\bar{s}$ and, therefore, they have very different masses.

Doorway model of neutral scalar mesons

The $(q\bar{q})^2$ tetraquark structure is partly dissociated into a $q\bar{q}$ diquark structure and into those meson pairs which show up in the decay channel.

$$\sigma = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \leftrightarrow u\bar{u}d\bar{d} \leftrightarrow \pi\pi$$

$$f_0 \approx \frac{1}{\sqrt{2}}\left(\frac{u\bar{u} + d\bar{d}}{\sqrt{2}} - s\bar{s}\right) \leftrightarrow \frac{s\bar{s}(u\bar{u} + d\bar{d})}{\sqrt{2}} \leftrightarrow \pi\pi, K\bar{K}$$

$$a_0 \approx \frac{1}{\sqrt{2}}\left(\frac{-u\bar{u} + d\bar{d}}{\sqrt{2}} + s\bar{s}\right) \leftrightarrow s\bar{s}\frac{(u\bar{u} - d\bar{d})}{\sqrt{2}} \leftrightarrow \eta\pi, K\bar{K}$$

In two-photon fusion reactions the transition proceeds first into the $q\bar{q}$ structure component serving as a doorway state. Then by rearrangement the $(q\bar{q})^2$ structure is formed.

Two-photon widths and doorway-structure

The direct two-photon fusion into the tetraquark structure can be neglegted. The two-photon width, therefore, is given by the $q\bar{q}$ structure of the doorway state. The transition matrix element is given by

$$|q\bar{q}\rangle = a\,|u\bar{u}\rangle + b\,|d\bar{d}\rangle + c\,|s\bar{s}\rangle, \quad a^2 + b^2 + c^2 = 1, \quad m_s/\hat{m} \simeq 1.44$$

$$\mathcal{M}(M \to \gamma\gamma) = \frac{\alpha_e}{\pi f_\pi} N_c \sqrt{2} \langle e_q^2 \rangle, \quad \langle e_q^2 \rangle = a\, e_u^2 + b\, e_d^2 + c\, \hat{m}/m_s) e_s^2,$$

$$\Gamma(M \to \gamma\gamma) = \frac{m_M^3}{64\pi} |\mathcal{M}(M \to \gamma\gamma)|^2$$

The quantity m_M is the bare mass. For the σ meson we have $m_\sigma = 666$ MeV. $m_s/\hat{m}$ is the strange-quark light-quark mass ratio.

Structure of pseudoscalar and scalar mesons

Structures of pseudoscalar and scalar mesons fitted to the two-photon widths.

$$
\begin{aligned}
|\pi^0\rangle &= |IV\rangle = \tfrac{1}{\sqrt{2}}(-|u\bar{u}\rangle + |d\bar{d}\rangle && {}^1S_0\\
|\eta\rangle &= \tfrac{1}{\sqrt{2}}(1.04\,|IS\rangle - 0.96\,|s\bar{s}\rangle) && {}^1S_0\\
|\eta'\rangle &= \tfrac{1}{\sqrt{2}}(0.83\,|IS\rangle + 1.15\,|s\bar{s}\rangle) && {}^1S_0\\
\\
|\sigma(666)\rangle &= |IS\rangle = \tfrac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle && {}^3P_0\\
|f_0(980)\rangle &= \tfrac{1}{\sqrt{2}}(0.52\,|IS\rangle - 1.31\,|s\bar{s}\rangle) && {}^3P_0\\
|a_0(985)\rangle &= \tfrac{1}{\sqrt{2}}(0.83\,|IV\rangle + 1.15\,|s\bar{s}\rangle) && {}^3P_0
\end{aligned}
$$

Pseudoscalar and scalar mesons differ by the angular momentum structures, being ${}^1S_0$ for pseudoscalar mesons and ${}^3P_0$ for scalar mesons.

Decay amplitudes and meson-quark couplings

	$\mathcal{M}(M \to \gamma\gamma)$ [10^{-2} GeV^{-1}]	g_{MNN}	$\Gamma_{M\gamma\gamma}$ [keV]
π^0	-2.513 ± 0.007	13.169 ± 0.057	$(7.74 \pm 0.55) \times 10^{-3}$
η	$+2.50 \pm 0.06$	5.79 ± 0.15	0.510 ± 0.026
η'	$+3.13 \pm 0.05$	4.63 ± 0.08	4.29 ± 0.15
σ	$+4.19 \pm 0.21$	13.169 ± 0.057	2.58 ± 0.26
f_0	$+0.79 \pm 0.11$	5.8 ± 0.8	$0.29^{+0.07}_{-0.09}$
a_0	-0.79 ± 0.13	7.7 ± 1.2	0.30 ± 0.10

$\mathcal{M}(M \to \gamma\gamma)$ is the meson decay amplitude, g_{MNN} the meson-nucleon coupling constant and $\Gamma_{M\gamma\gamma}$ the experimental meson decay width.

Bare mass and pole on the second Riemann sheet

$$P(s) = \frac{1}{m_\sigma^2 + \Pi(s) - s} = \frac{1}{m^2(s) - s - i\, m_{\rm BW} \Gamma_{\rm tot}(s)}, \tag{1}$$

Compton scattering

$$\gamma\gamma \to \sigma \to N\bar{N} \quad \Longrightarrow \quad \Pi(s) \equiv 0 \tag{2}$$

Bare mass of the σ meson $m_\sigma = 666$ MeV

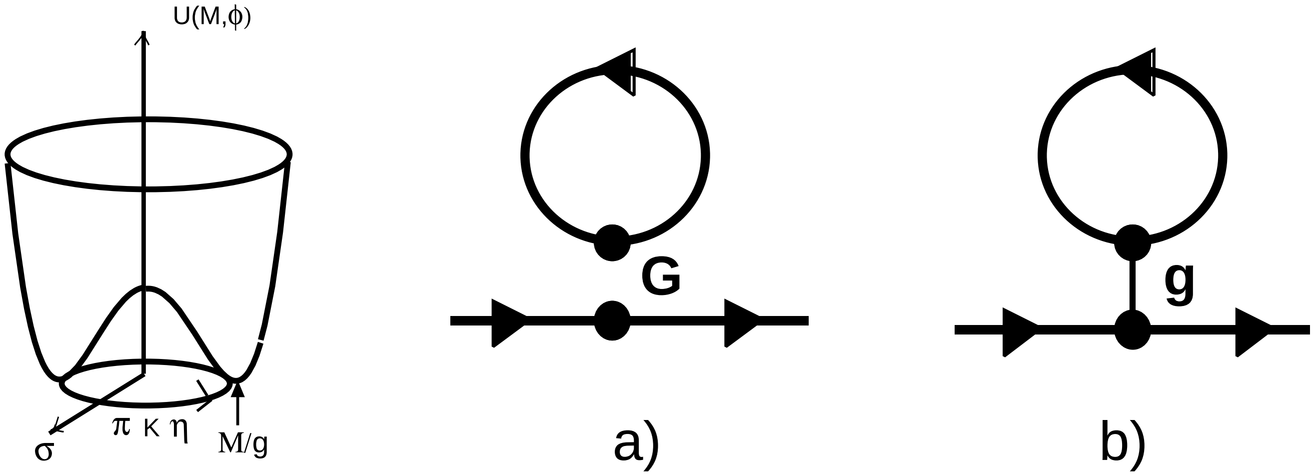
Two-pion decay $$\gamma\gamma \to \sigma \to \pi\pi \tag{3}$$

Pole on the second Riemann sheet at

$$m_\sigma^2 + \Pi(s_\sigma) - s_\sigma = 0 \tag{4}$$

$$\sqrt{s_\sigma} = M_\sigma - i\, \Gamma_\sigma/2, \quad M_\sigma = 441^{+16}_{-8} \text{ MeV}, \; \Gamma_\sigma = 554^{+18}_{-25} \text{ MeV} \tag{5}$$

Spontaneous and explicit symmetry breaking 1



Left panel: Mexican-hat potential for scalar and pseudo goldstone bosons. Right panel: Nambu–Jona-lasinio model: a) four-fermion version, b) bosonized version. In the chiral limit the Goldstone bosons π, K, and η have zero mass. The scalar mesons σ, κ, $f_0(980)$ and $a_0(980)$ have the mass $m_\sigma^{\rm cl} = 652$ MeV.

Spontaneous and explicit symmetry breaking 2

Linerar σ model:

$$\mathcal{L}_\sigma = \tfrac{1}{2}\partial_\mu \pi \cdot \partial^\mu \pi + \tfrac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \tfrac{\mu^2}{2}(\sigma^2 + \pi^2) - \tfrac{\lambda}{4}(\sigma^2 + \pi^2)^2 + f_\pi m_\pi^2 \sigma$$

Nambu–Jona-Lasinio model:

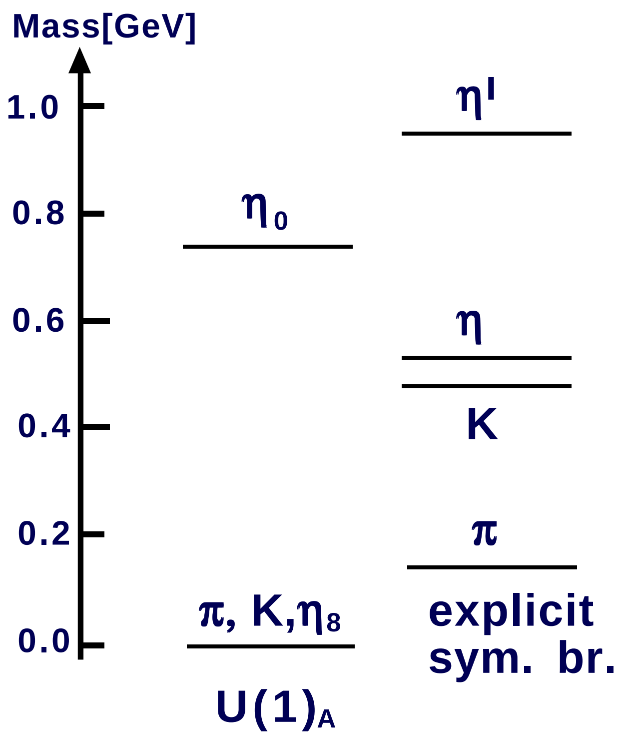
$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\partial\!\!\!/ - m_0)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \tau \psi)^2],$$

$$\mathcal{L}'_{\rm NJL} = \bar{\psi} i\partial\!\!\!/ \psi - g\bar{\psi}(\sigma + i\gamma_5 \tau \cdot \pi)\psi - \frac{1}{2}\delta\mu^2(\sigma^2 + \pi^2) + \frac{g m_0}{G}\sigma,$$

$$G = g^2/\delta\mu^2, \quad \delta\mu^2 = (m_\sigma^{\rm cl})^2, \quad G = \lambda/(\sqrt{2} m_\sigma^{\rm cl})^2, \quad g = \sqrt{\lambda/2}$$

Predictions: $m_\sigma^{\rm cl} = 652$ MeV, $m_\sigma = (\frac{16\pi^2}{3} f_\pi^2 + m_\pi^2)^{1/2} = 685$ MeV, $\lambda = \frac{8\pi^2}{3} = 26.3$, $\mu = \sqrt{\frac{2}{3}} 2\pi f_0 = 461$ MeV, $f_0 = 89.8$ MeV, $f_\pi = 92.42$ MeV.

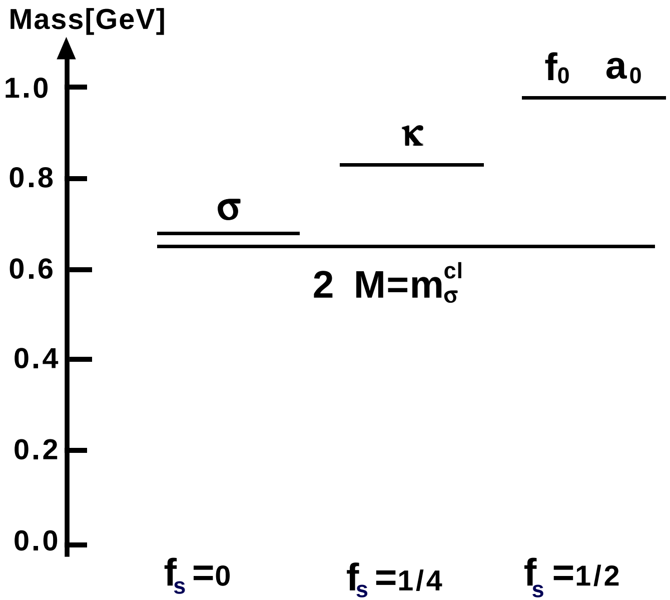
Explicit symmetry breaking for pseudoscalar mesons



In the chiral limit the pseudoscalar mesons π, K and η_8 have zero mass. Due to the $U(1)_A$ anomaly η_0 has a nonzero mass and, therefore, is not a Goldstone boson. For non-zero current-quark masses the Goldstone bosons acquire mass according to the Gell-Mann–Oakes–Renner relation:

$$m^2_\pi f^2_\pi = -\tfrac{1}{2}(m^0_u + m^0_d)\langle \bar{u}u + \bar{d}d\rangle + \cdots$$
$$m^2_{K^+} f^2_{K^+} = -\tfrac{1}{2}(m^0_u + m^0_s)\langle \bar{u}u + \bar{s}s\rangle + \cdots$$
$$m^2_\eta f^2_\eta = -\tfrac{1}{4}(m^0_u + m^0_d + 2m^0_s)\langle \bar{u}u + \bar{s}s\rangle + \cdots$$

Spont. and explicit symmetry breaking for scalar mesons



In the chiral limit the scalar mesons $\sigma(600)$, $\kappa(800)$, $f_0(980)$ and $a_0(980)$ have the same mass $2M = m_\sigma^{\rm cl} = 652$ MeV where $M = \frac{2\pi}{\sqrt{3}} f_0$ with $f_0 = 98.8$ MeV. Explicit symmetry breaking leads to

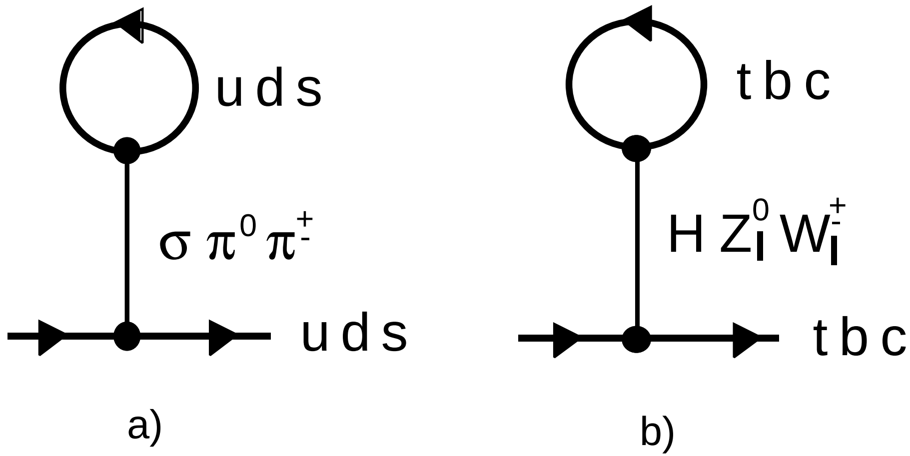
$$m^2_\sigma = \frac{16\pi^2}{3} f^2_\pi + m^2_\pi$$
$$m^2_\kappa = \frac{16\pi^2}{3}\frac{1}{2}(f^2_\pi + f^2_K) + \frac{1}{2}(m^2_\pi + m^2_K)$$
$$m^2_{a_0,f_0} = \frac{16\pi^2}{3} f^2_K + m^2_\eta$$

With $f_\pi = 92.42 \pm 0.26$ MeV and $f_\eta \approx f_K = 113.0 \pm 1.0$ MeV we arrive at $m_\sigma = 685$ MeV, $m_\kappa = 834$ MeV and $m_{a_0,f_0} = 986$ MeV.

Summary

- A complete description of the Higgs sector of strong interaction is presented
- The σ meson is directly observed as part of the constituent quark via Compton scattering by the nucleon
- The elecromagnetic polarizabilities definitely prove that the $q\bar{q}$ structures of the scalar mesons correctly describe the intermediate state of nucleon Compton scattering
- The on-shell scalar mesons have a tetraquark structure with the $q\bar{q}$ substructures serving as doorway states in two-photon fusion reactions
- The masses of scalar mesons can quantitatively be predicted in terms dynamical/spontaneous and explicit symmetry breaking

Outlook on electroweak symmetry breaking



Strong symmetry breaking: $m_\sigma^2 = 4M^{*2} + m_\pi^2 = (685\ \mathrm{MeV})^2$

Electroweak symmetry breaking: $m_H^2 = 4m_t^2 - m_Z^2 - 2m_W^2 = (317\ \mathrm{GeV})^2$

M.D. Scadron et al. J. Phys. G 32 (2006) 735 and references therein