Review on results by the FLAG working group

Hadron 2011

Künstlerhaus, München

13.-17.06.2011

Andreas Jüttner for FlaviA net Lattice WG

CERN **Theory Division**

Strong claims were recently made based on lattice QCD results:

- “We find a (2-3)σ tension in the unitarity triangle”
  Laiho, Lunghi, Van de Water, PRD 81 (2010) 034503
- “. . . confirming CKM unitarity at the permille level”
  FLAG arXiv:1011.4408
- “. . . we find evidence of new physics in both B_d and B_s systems . . . ”
  CKMfitter Group PRD 83 (2011) 036004
- “Possible evidence for the breakdown of the CKM-paradigm of CP-violation”
  Lunghi, Soni, PLB 697, 323-328 (2011)

Strong claims were recently made based on lattice QCD results:

- "We find a (2-3)σ tension in the unitarity triangle"
  Laiho, Lunghi, Van de Water, PRD 81 (2010) 034503
- ". . . confirming CKM unitarity at the permille level"
  FLAG arXiv:1011.4408
- ". . . we find evidence of new physics in both B_d and B_s systems . . . "
  CKMfitter Group PRD 83 (2011) 036004
- "Possible evidence for the breakdown of the CKM-paradigm of CP-violation"
  Lunghi, Soni, PLB 697, 323-328 (2011)

such statements require a precise screening of lattice results and therefore a good understanding of lattice QCD

FLAG

→ Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulness of lattice results

People: G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig

FLAG

→ Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulness of lattice results

People: G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig

→ quantities we consider: m_u, m_d, m_s, $f_+^{K\pi}(0)$, f_K/f_π, B_K, NLO LEC's, potentially more in the future

→ What is the current lattice value? Is the quoted uncertainty reliable?

→ provides:

- relevant formulae and notation
- detailed quality assessment
- average/recommended range in those cases where quality of lattice results considered very high
- lattice dictionary for non-experts
- details of every single lattice simulation (appendix)

→ planned periodic updates of *arXiv:1011.4408, http://itpwiki.unibe.ch/flag*

FLAG

→ Flavia Net Lattice Averaging Group (**FLAG**) was founded to allow also to an outsider to judge the quality and 'state-of-the-art'-fulness of lattice results

People: G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig

→ quantities we consider: m_u, m_d, m_s, $f_+^{K\pi}(0)$, f_K/f_π, B_K, NLO LEC's, potentially more in the future

→ What is the current lattice value? Is the quoted uncertainty reliable?

→ provides:

- relevant formulae and notation
- detailed quality assessment
- average/recommended range in those cases where quality of lattice results considered very high
- lattice dictionary for non-experts
- details of every single lattice simulation (appendix)

→ planned periodic updates of *arXiv:1011.4408, http://itpwiki.unibe.ch/flag*

Other efforts: Laiho, Lunghi, Van de Water:
$N_f = 2+1$ light and heavy-light meson observables for CKM-triangle analysis

Lattice QCD inputs to the CKM unitarity triangle analysis, Phys.Rev. D81 (2010) 034503 http://www.latticeaverages.org

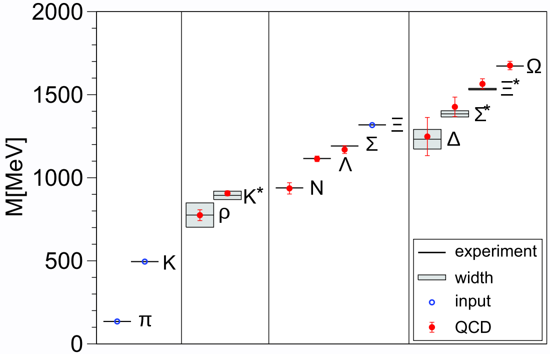
Why lattice QCD?

- perturbation theory works well for weak coupling
- bound state observables like the proton mass or the *B*-decay constant for example cannot be predicted by perturbation theory

Why lattice QCD?

- perturbation theory works well for weak coupling
- bound state observables like the proton mass or the *B*-decay constant for example cannot be predicted by perturbation theory

but simulations of lattice QCD can do this:



BMW Collaboration, Science 322 (2008) 1224-1227

NOTE: only three input-parameters!!!

What is . . . ?

	QCD
N_c	3
N_f, fundamental	1+1+1+1+1+1
$SU(2)$ iso-spin brk.	✓
m_π	135MeV
V	∞
a	0

What is . . . ?

	QCD	Lattice QCD
N_c	3	3
N_f, fundamental	1+1+1+1+1+1	0, 2, 2+1, 2+1+1
$SU(2)$ iso-spin brk.	✔	✗
m_π	135MeV	$\lesssim m_\pi^{\rm sim}$
V	∞	2-3fm
a	0	0.05-0.1fm

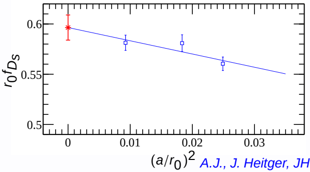
Systematics

- most results systematics-dominated
- extrapolation of lattice data to the physical point very often tricky

Systematics

- most results systematics-dominated
- extrapolation of lattice data to the physical point very often tricky

$a \to 0$ Symanzik eff. th.

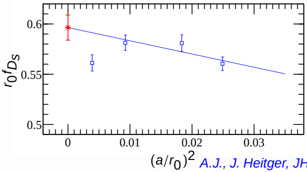


A.J., J. Heitger, JHEP 0905 (2009) 101

Systematics

- most results systematics-dominated
- extrapolation of lattice data to the physical point very often tricky

$a \to 0$ Symanzik eff. th.

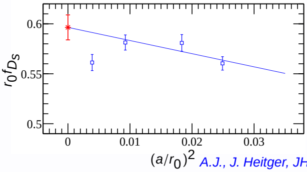


A.J., J. Heitger, JHEP 0905 (2009) 101

Systematics

- most results systematics-dominated
- extrapolation of lattice data to the physical point very often tricky

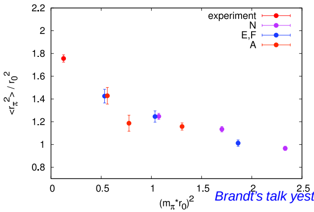
$a \to 0$ Symanzik eff. th.



A.J., J. Heitger, JHEP 0905 (2009) 101

$m_q \to m_q^{\rm phys}$ chiral eff. th.

also talks by Brandt, Zanotti



Brandt's talk yesterday

Systematics continued

- renormalisation, e.g. quark masses and B_K
  although techniques for full non-perturbative renormalisation and running
  Martinelli et al., Nucl. Phys. B445 (1995) 81-108, Lüscher et al., Nulc. Phys. B384 (1992) 168-228
  are standard by now some collaborations still employ perturbation theory

Systematics continued

- renormalisation, e.g. quark masses and B_K
  although techniques for full non-perturbative renormalisation and running
  Martinelli et al., Nucl. Phys. B445 (1995) 81-108, Lüscher et al., Nulc. Phys. B384 (1992) 168-228
  are standard by now some collaborations still employ perturbation theory
- scale setting
- finite size errors
- chosen discretisation
- . . .

FLAG criteria

- chiral extrapolation
- continuum extrapolation
- finite volume errors
- renormalisation
- renormalisation scale running
- publication status

FLAG criteria

- chiral extrapolation
- continuum extrapolation
- finite volume errors
- renormalisation
- renormalisation scale running
- publication status

FLAG's colour coding:

★ when the systematic error has been estimated in a satisfactory manner and convincingly shown to be under control

● when a reasonable attempt at estimating the systematic error has been made, although this could be improved;

■ when no or a clearly unsatisfactory attempt at

Current color coding

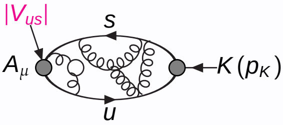
- Chiral extrapolation:
  - ★ $M_{\pi,\mathrm{min}} < 250$ MeV
  - ● 250 MeV $\leq M_{\pi,\mathrm{min}} \leq$ 400 MeV
  - ■ $M_{\pi,\mathrm{min}} > 400$ MeV
- Continuum extrapolation:
  - ★ 3 or more lattice spacings, at least 2 points below 0.1 fm
  - ● 2 or more lattice spacings, at least 1 point below 0.1 fm
  - ■ otherwise
- Finite-volume effects:
  - ★ $M_{\pi,\mathrm{min}}L > 4$ or at least 3 volumes
  - ● $M_{\pi,\mathrm{min}}L > 3$ and at least 2 volumes
  - ■ otherwise

Current colour coding

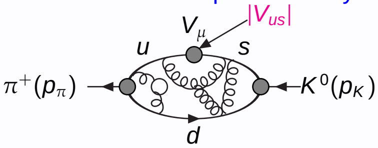
- Renormalisation (where applicable):
  - ★ non-perturbative
  - ● 2-loop perturbation theory
  - ■ otherwise
- Publication status:
  - A published or plain update of published results
  - P preprint
  - C conference contribution
- Only published results enter averages (where applicable)
- results with different number of dynamical quark flavours considered separately

A FLAG example - the kaon sector

K leptonic decay



$K \rightarrow \pi$ semi-leptonic decay

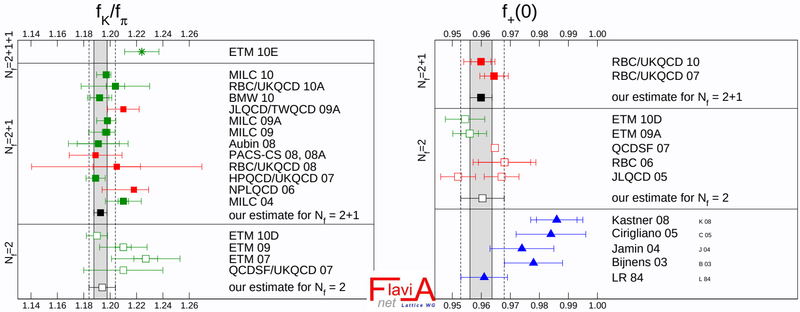


$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{m_K(1 - m_\mu^2/m_K^2)}{m_\pi(1 - m_\mu^2/m_\pi^2)} 0.9930(35) \times \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2$$

(Marciano, Phys.Rev.Lett. 2004)

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{\rm EW}[1 + \Delta_{SU(2)} + \Delta_{\rm EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$

A FLAG example - the kaon sector



- agreement amongst independent lattice determinations
- current level of precision does not reveal dependence on flavour content of simulation

A FLAG example - the kaon sector

Collaboration	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	f_K/f_π
ETM 10E	2+1+1	C	●	●	●	$1.224(13)_{stat}$
MILC 10	2+1	C	●	★	★	$1.197(2)(^{+3}_{-7})$
RBC/UKQCD 10A	2+1	P	●	●	★	1.204(7)(25)
BMW 10	2+1	A	★	★	★	1.192(7)(6)
JLQCD/TWQCD 09A	2+1	C	●	■	■	$1.210(12)_{\mathrm{stat}}$
MILC 09A	2+1	C	●	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09	2+1	A	●	★	★	$1.197(3)(^{+6}_{-13})$
Aubin 08	2+1	C	●	●	●	1.191(16)(17)
PACS-CS 08, 08A	2+1	A	★	■	■	1.189(20)
RBC/UKQCD 08	2+1	A	●	■	★	1.205(18)(62)
HPQCD/UKQCD 07	2+1	A	●	★	●	1.189(2)(7)
NPLQCD 06	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 10D	2	C	●	★	●	$1.190(8)_{stat}$
ETM 09	2	A	●	★	●	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	●	●	★	1.21(3)

FLAG arXiv:1011.4408

A FLAG example - the kaon sector

For both $N_f = 2 + 1$ and $N + f = 2$ FLAG identified high quality lattice results and provides averages:

FLAG averages *FLAG arXiv:1011.4408*:

N_f	$f_+^{K\pi}(0)$	f_K/f_π
2+1	0.9599(34)(41)	1.193(5)
2	0.9560(57)(62)	1.210(6)(17)

A FLAG example - the kaon sector

For both $N_f = 2 + 1$ and $N + f = 2$ FLAG identified high quality lattice results and provides averages:

FLAG averages *FLAG arXiv:1011.4408*:

N_f	$f_+^{K\pi}(0)$	f_K/f_π
2+1	0.9599(34)(41)	1.193(5)
2	0.9560(57)(62)	1.210(6)(17)

Together with experimental input

$\lvert V_{us} f_+^{K\pi}(0) \rvert$	=	0.2163(5)
$\left\lvert \frac{f_K V_{us}}{f_\pi V_{ud}} \right\rvert$	=	0.2758(5)
$\lvert V_{ud} \rvert$	=	0.97425(22)

FLAVIA KAON WG Eur.Phys.J. C69 (2010) 399-424
Hardy, Towner, Phys. Rev., C79, 2009,05550

A FLAG example - the kaon sector

For both $N_f = 2 + 1$ and $N + f = 2$ FLAG identified high quality lattice results and provides averages:

FLAG averages *FLAG arXiv:1011.4408*:

N_f	$f_+^{K\pi}(0)$	f_K/f_π
2+1	0.9599(34)(41)	1.193(5)
2	0.9560(57)(62)	1.210(6)(17)

Together with experimental input

$\lvert V_{us} f_+^{K\pi}(0)\rvert$	=	0.2163(5)
$\left\lvert \frac{f_K V_{us}}{f_\pi V_{ud}} \right\rvert$	=	0.2758(5)
$\lvert V_{ud}\rvert$	=	0.97425(22)

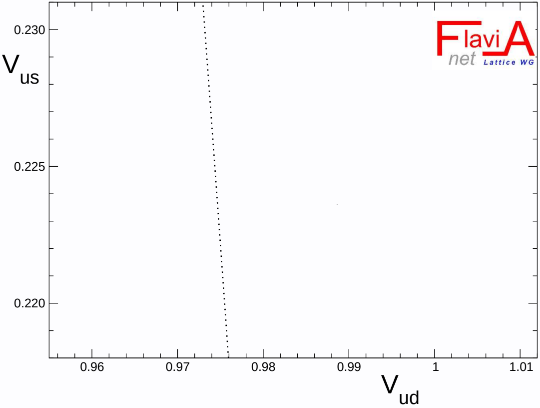
FLAVIA KAON WG Eur.Phys.J. C69 (2010) 399-424
Hardy, Towner, Phys. Rev., C79, 2009,05550

FLAG did two kind of analysis:

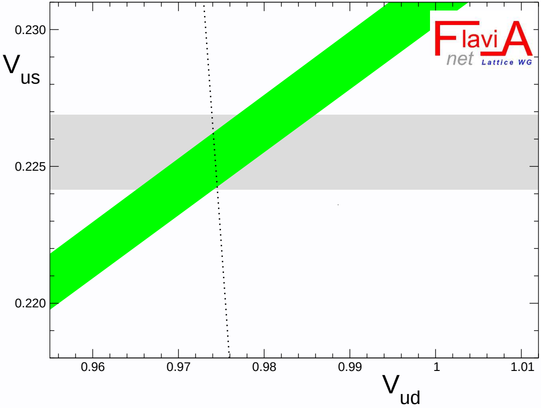
a) test the SM: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$

b) using SM-correlations

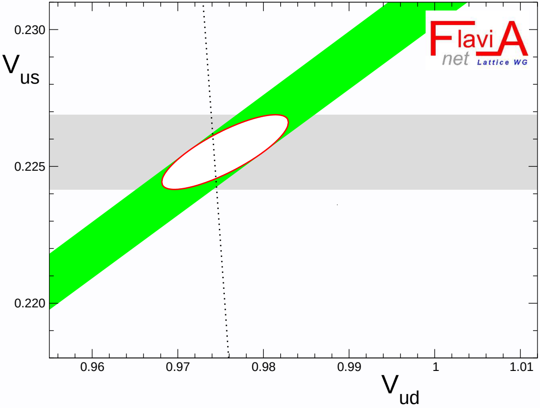
FLAG tests the SM: $|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$



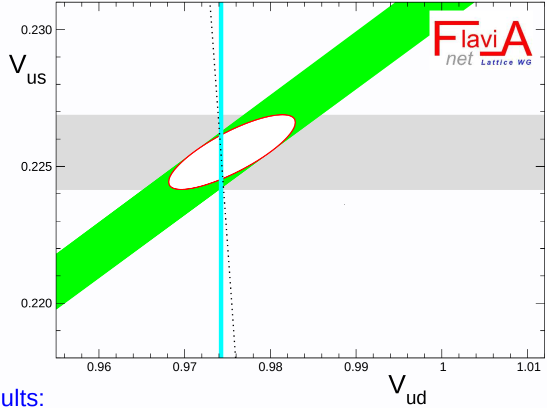
FLAG tests the SM: $|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$



FLAG tests the SM: $|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$



FLAG tests the SM: $|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

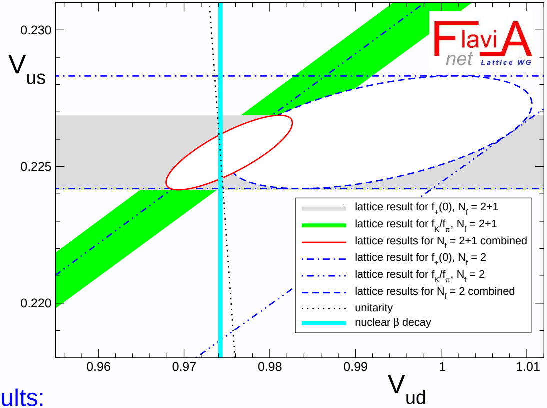


Results:

N_f	$\lvert V_u\rvert^2\rvert_{no\,\lvert V_{ud}\rvert}$	$\lvert V_u\rvert^2\rvert_{\lvert V_{ud}\rvert}$ from $f_+^{K\pi}(0)$	$\lvert V_u\rvert^2\rvert_{\lvert V_{ud}\rvert}$ from f_K/f_π
2+1	1.002(15)	1.0000(7)	0.9999(6)
2	1.037(36)	1.0004(10)	0.9985(16)

CKM-unitarity confirmed at the per-mil level

FLAG tests the SM: $|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

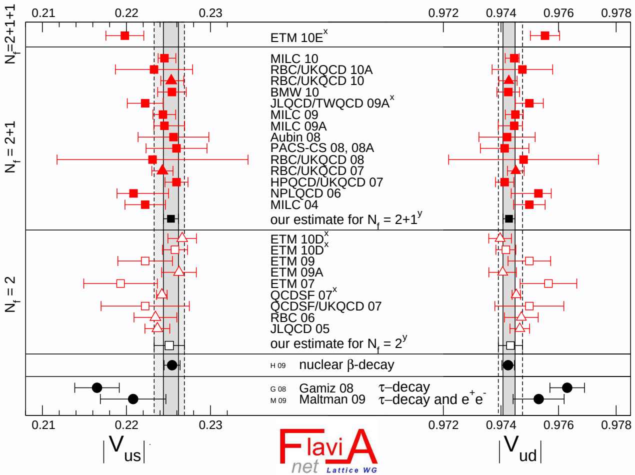


Results:

N_f	$\lvert V_u\rvert^2\rvert_{no\lvert V_{ud}\rvert}$	$\lvert V_u\rvert^2\rvert_{\lvert V_{ud}\rvert}$ from $f_+^{K\pi}(0)$	$\lvert V_u\rvert^2\rvert_{\lvert V_{ud}\rvert}$ from f_K/f_π
2+1	1.002(15)	1.0000(7)	0.9999(6)
2	1.037(36)	1.0004(10)	0.9985(16)

CKM-unitarity confirmed at the per-mil level

If FLAG assumes first row unitarity



Nice result: Assuming first row unitarity lattice QCD makes prediction for $|V_{ud}|$ with same precision as super-allowed nuclear beta decays and fully compatible with it

B_K - neutral kaon mixing

- $\epsilon_K \propto B_K$ - hyperbola in the $\bar{\eta} - \bar{\rho}$-plane

$$\epsilon_K \;=\; \exp(i\phi_\epsilon)\;\sin(\phi_\epsilon)\left[\frac{\Im[\langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle]}{\Delta M_K} \;+\; \frac{\Im(A_0)}{\Re(A_0)}\right]$$

B_K - neutral kaon mixing

- $\epsilon_K \propto B_K$ - hyperbola in the $\bar{\eta} - \bar{\rho}$-plane

$$\epsilon_K = \exp(i\phi_\epsilon)\ \sin(\phi_\epsilon) \left[\frac{\Im[\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle]}{\Delta M_K} + \frac{\Im(A_0)}{\Re(A_0)} \right]$$

- tremendous progress in the last 5 years or so:

  cheaper lattice actions break chiral symmetry $\rightarrow$ lattice 4-fermion operator mixing under renormalisation becomes complicated task
  use of chiral symmetry respecting fermion formulations have allowed for breakthrough in predictions for B_K

B_K - neutral kaon mixing

- $\epsilon_K \propto B_K$ - hyperbola in the $\bar{\eta} - \bar{\rho}$-plane

$$\epsilon_K = \exp(i\phi_\epsilon)\ \sin(\phi_\epsilon) \left[\frac{\Im[\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle]}{\Delta M_K} + \frac{\Im(A_0)}{\Re(A_0)} \right]$$

- tremendous progress in the last 5 years or so:

  cheaper lattice actions break chiral symmetry $\rightarrow$ lattice 4-fermion operator mixing under renormalisation becomes complicated task
  use of chiral symmetry respecting fermion formulations have allowed for breakthrough in predictions for B_K

- B_K naively in lattice scheme, matching to $\overline{MS}$ done in perturbation theory

  alternative: cite Renormalisation Group Invariant B_K: $\hat{B}_K$

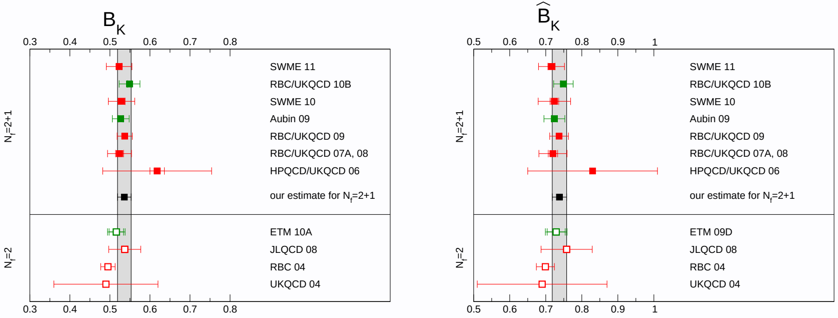
$$\hat{B}_K = \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \exp\left\{ \int_0^{\bar{g}(\mu)} dg \left(\frac{\gamma(g)}{\beta(g)} + \frac{\gamma_0}{\beta_0 g} \right) \right\} B_K(\mu)$$

  lattice can in principle provide $\hat{B}_K$ in a fully non-perturbative way

B_K - neutral kaon mixing

Collaboration	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalisation	running	B_K	$\hat{B}_K$
SWME 11	2+1	P	★	●	●	■	●	0.523(7)(26)	0.716(10)(35)
RBC/UKQCD 10B	2+1	P	●	●	★	★	●	0.549(5)(26)	0.749(7)(26)
SWME 10	2+1	A	★	●	●	■	●	0.529(9)(32)	0.724(12)(43)
Aubin 09	2+1	A	●	★□	●	★	●	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 07A, 08	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 10A	2	A	★	●	●	★	●	0.516(18)(12)	0.729(25)(17)
JLQCD 08	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	2	A	■	■	■†	■	●	0.49(13)	0.69(18)

B_K - neutral kaon mixing



Results:

$$N_f = 2 \qquad B_K^{\overline{MS}}(2\mathrm{GeV}) = 0.516(18)(12) \qquad \hat{B}_K = 0.729(25)(17)$$

Aubin 09
RBC/UKQCD 10B

$$N_f = 2+1 \qquad B_K^{\overline{MS}}(2\mathrm{GeV}) = 0.536(17) \qquad \hat{B}_K = 0.738(20)$$

ETM 09D

Light quark masses m_u, m_d, m_s

- quark masses cannot be measured directly in experiment
- procedure:
  - compute three experimentally measurable quantities to tune bare parameters $m_{ud} = \frac{1}{2}(m_u + m_d)$, m_s and Λ_{QCD}
  - renormalisation of bare parameters
- lattice QCD calculations in the iso-spin limit $m_u = m_d$

Light quark masses m_u, m_d, m_s

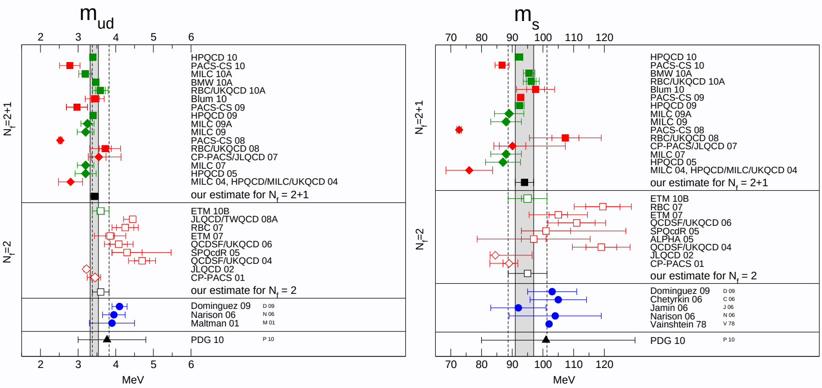
- quark masses cannot be measured directly in experiment
- procedure:
  - compute three experimentally measurable quantities to tune bare parameters $m_{ud} = \frac{1}{2}(m_u + m_d)$, m_s and Λ_{QCD}
  - renormalisation of bare parameters
- lattice QCD calculations in the iso-spin limit $m_u = m_d$
- electro-magnetic corrections have to be taken into account

FLAG summary for quark masses

Collaboration	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalisation	running	m_{ud}	m_s
PACS-CS 10	P	★	■	■	★	*a*	2.78(27)	86.7(2.3)
MILC 10A	C	●	★	★	●	–	3.19(4)(5)(16)	–
HPQCD 10	A	●	★	★	★	–	$3.39(6)^*$	92.2(1.3)
BMW 10A, $10B^+$	P	★	★	★	★	*b*	3.469(47)(48)	95.5(1.1)(1.5)
RBC/UKQCD 10A	P	●	●	★	★	*c*	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum $10^\dagger$	P	●	■	●	★	–	3.44(12)(22)	97.6(2.9)(5.5)
PACS-CS 09	A	★	■	■	★	*a*	2.97(28)(3)	92.75(58)(95)
HPQCD 09	A	●	★	★	★	–	3.40(7)	92.4(1.5)
MILC 09A	C	●	★	★	●	–	3.25 (1)(7)(16)(0)	89.0(0.2)(1.6)(4.5)(0.1)
MILC 09	A	●	★	★	●	–	3.2(0)(1)(2)(0)	88(0)(3)(4)(0)
PACS-CS 08	A	★	■	■	■	–	2.527(47)	72.72(78)
RBC/UKQCD 08	A	●	■	★	★	–	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	A	■	★	★	■	–	$3.55(19)(^{+56}_{-20})$	$90.1(4.3)(^{+16.7}_{-4.3})$
HPQCD 05	A	●	●	●	●	–	$3.2(0)(2)(2)(0)^\ddagger$	$87(0)(4)(4)(0)^\ddagger$
MILC 04, HPQCD/ MILC/UKQCD 04	A	●	●	●	■	–	2.8(0)(1)(3)(0)	76(0)(3)(7)(0)

table shows $N_f = 2+1$ flavour results only

FLAG summary for quark masses



FLAG summary for quark masses

FLAG Results:

$N_f = 2$	$m_{ud}(2\mathrm{GeV}) = 3.6(1)(2)\mathrm{MeV}$	$m_s(2\mathrm{GeV}) = 95(2)(6)\mathrm{MeV}$	*ETM 10N*
$N_f = 2+1$	$m_{ud}(2\mathrm{GeV}) = 3.43(11)\mathrm{MeV}$	$m_s(2\mathrm{GeV}) = 94(3)\mathrm{MeV}$	*MILC 09A, RBC/UKQCD 10A, HPQCD 10*

FLAG summary for quark masses

FLAG Results:

$N_f = 2$ $m_{ud}(2\mathrm{GeV}) = 3.6(1)(2)\mathrm{MeV}$ $m_s(2\mathrm{GeV}) = 95(2)(6)\mathrm{MeV}$ *ETM 10N*

$N_f = 2+1$ $m_{ud}(2\mathrm{GeV}) = 3.43(11)\mathrm{MeV}$ $m_s(2\mathrm{GeV}) = 94(3)\mathrm{MeV}$ *MILC 09A, RBC/UKQCD 10A, HPQCD 10*

PDG Result:

$m_s(2\mathrm{GeV}) = 101(^{+29}_{-21})\mathrm{MeV}$

FLAG summary for quark masses

FLAG Results:

$N_f = 2$ $m_{ud}(2\mathrm{GeV}) = 3.6(1)(2)\mathrm{MeV}$ $m_s(2\mathrm{GeV}) = 95(2)(6)\mathrm{MeV}$ *ETM 10N*

$N_f = 2+1$ $m_{ud}(2\mathrm{GeV}) = 3.43(11)\mathrm{MeV}$ $m_s(2\mathrm{GeV}) = 94(3)\mathrm{MeV}$ *MILC 09A, RBC/UKQCD 10A, HPQCD 10*

PDG Result:

$$m_s(2\mathrm{GeV}) = 101(^{+29}_{-21})\mathrm{MeV}$$

Note: $N_f = 2+1$ naive average would yield

$$\begin{aligned} m_{ud} &= 3.38(6)\ \mathrm{MeV} \quad \text{and} \\ m_s &= 92.8(1.3)\mathrm{MeV} \end{aligned}$$

our estimate leaves room for potential charm quark contributions

FLAG: is the up-quark massless?

FLAG Results:

$N_f = 2+1$ $m_u(2\mathrm{GeV}) = 2.19(15)\mathrm{MeV}$ $m_d(2\mathrm{GeV}) = 4.67(20)\mathrm{MeV}$ *MILC + FLAG*

FLAG: is the up-quark massless?

FLAG Results:

$N_f = 2+1$ $m_u(2\mathrm{GeV}) = 2.19(15)\mathrm{MeV}$ $m_d(2\mathrm{GeV}) = 4.67(20)\mathrm{MeV}$ *MILC + FLAG*

- determined from $K^+ - K^0$-splitting (in QCD, i.e. elm. effects subtracted)
- m_u different from zero by 15 standard deviations
- error dominated by QED corrections
- QED contributions need to be taken into account in future lattice simulations

FLAG: is the up-quark massless?

FLAG Results:

$N_f = 2+1$ $m_u(2\mathrm{GeV}) = 2.19(15)\mathrm{MeV}$ $m_d(2\mathrm{GeV}) = 4.67(20)\mathrm{MeV}$ *MILC + FLAG*

- determined from $K^+ - K^0$-splitting (in QCD, i.e. elm. effects subtracted)
- m_u different from zero by 15 standard deviations
- error dominated by QED corrections
- QED contributions need to be taken into account in future lattice simulations

PDG Result:

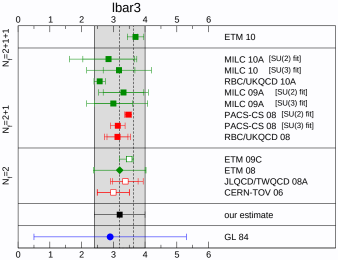
$$m_u(2\mathrm{GeV}) = 1.7 - 3.3\mathrm{MeV} \quad m_d(2\mathrm{GeV}) = 4.1 - 5.8\mathrm{MeV}$$

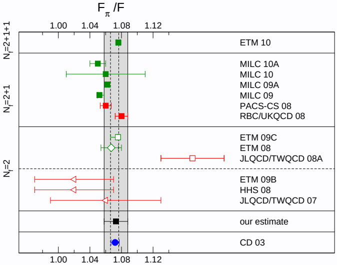
Low energy constants

- NLO $SU(2)$ constants: Σ, F, $\bar{l}_{3,4}$, $\bar{l}_6$, c_V, $\bar{l}_1 - \bar{l}_2$
  NLO $SU(3)$ constants: Σ_0, F_0, $\bar{L}_{4,5,6,8,9,10}$,
- results not always coherent yet

Low energy constants

- NLO $SU(2)$ constants: Σ, F, $\bar{l}_{3,4}$, $\bar{l}_6$, c_V, $\bar{l}_1 - \bar{l}_2$
  NLO $SU(3)$ constants: Σ_0, F_0, $\bar{L}_{4,5,6,8,9,10}$,
- results not always coherent yet
- example: $\bar{l}_3 = 3.2 \pm 0.8$, $F_\pi / F = 1.073 \pm 0.015$





Outlook

- in most of the considered quantities the results are mutually compatible
- per cent level precision (and below) possible
- lattice QCD is a continously evolving, so will FLAG's criteria

Outlook

- in most of the considered quantities the results are mutually compatible
- per cent level precision (and below) possible
- lattice QCD is a continously evolving, so will FLAG's criteria
- the FLAG document provides a thorough analysis of state-of-the art lattice results and averages or recommended intervals where appropriate
- FLAG's appendix: detailed information on the characteristics of each simulation (lattice discretisation, simulation parameters, fitting model, . . .)

Outlook

- in most of the considered quantities the results are mutually compatible
- per cent level precision (and below) possible
- lattice QCD is a continously evolving, so will FLAG's criteria
- the FLAG document provides a thorough analysis of state-of-the art lattice results and averages or recommended intervals where appropriate
- FLAG's appendix: detailed information on the characteristics of each simulation (lattice discretisation, simulation parameters, fitting model, . . .)
- periodic updates are planned
- extension to larger set of quantities
- FLAG is a service for the larger community; the hard work has been done by lattice collaborations and we kindly ask users of FLAG-results to quote also the original work along with FLAG-averages