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Mixing properties of $a_1(1260)$ meson consisting of
hadronic composite and quark composite



Hideko NAGAIRO (Nara Women's University)

Collaborators :

K. Nawa (RCNP), S. Ozaki (U. Tokyo)
Daisuke Jido (YITP, Kyoto U.)
Atsushi Hosaka (RCNP, Osaka U.)

Introduction

Exotic hadrons : *not* simple $q\bar{q}$ (or qqq) state

- › σ , $f_0(980)$, $a_0(980)$, ... , $a_1(1260)$, $K_1(1270)$, ..., $N^*(1535)$, $\Lambda(1405)$, ... etc...
- molecule, $qq\bar{q}\bar{q}$ state, or ***mixed states among them ?***

Nature of axial vector meson $a_1(1260)$: $m = 1230 \pm 40$ MeV, $\Gamma=260$ to 600 MeV [PDG]

» *elementary* field (or $q\bar{q}$) : chiral partner of ρ meson

[Hidden local sym.] Bando-Kugo-Yamawaki; PR164(88)217; Kaiser-Meissner, NPA519(90)671, ...

[$q\bar{q}$ -NJL] M. Wakamatsu *et al.*, ZPA311(88)173, A. Hosaka, PLB244(90)363-367, ...

[Lattice QCD] M. Wingate *et al.*, PRL74(95)4596, ...

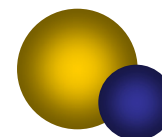
[Holographic QCD] T. Sakai, S. Sugimoto, PTP113 (05) 843; *ibid.*114(05)1083,



» dynamically generated resonance [$\pi\rho$ composite]

[coupled-channel BS] Lutz-Kolomeitsev, NPA730(04)392, ...

[Chiral Unitary model] Roca-Oset-Singh, PRD72(05)014002, ...



» applications

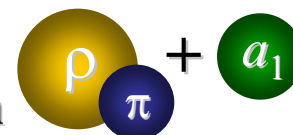
[τ -decay spectrum] M. Wagner and S. Leupold, PRD78(08)053001, ...

[radiative decay width] H. Nagahiro, L. Roca, A. Hosaka, E. Oset, PRD79(09)014015, ...

Aim of this study

We study the ***“mixing properties”*** of exotic hadrons

as a quark-core + hadronic composite system



A model for a_1 , π and ρ mesons

hidden local sym. or holographic model

Bando-Kugo-Yamawaki, PR164(88)217

Sakai-Sugimoto, PTP113(05)843

$$\mathcal{L}_{\text{WT}} = -\frac{g_4}{4f_\pi^2} \text{tr}([\rho^\mu, \partial^\nu \rho_\mu][\pi, \partial_\nu \pi]) \implies \begin{array}{cc} \pi & \text{---} & \pi \\ & \diagdown & / \\ & \rho & \rho \end{array} \implies \text{composite } a_1$$

$$\mathcal{L}_{a_1\pi\rho} = -g_{a_1\pi\rho} \frac{i\sqrt{2}}{f_\pi} \left\{ \text{tr}[(\partial_\mu a_{1\nu} - \partial_\nu a_{1\mu})[\partial^\mu \pi, \rho^\nu]] + \text{tr}[(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)[\partial^\mu \pi, a_1^\nu]] \right\} \implies \begin{array}{c} \text{elementary } a_1 \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \pi \\ \text{---} \\ \rho \end{array}$$

A good model for **composite a_1** and **elementary a_1**

mass of elementary a_1 meson : in holographic model

$$m_{a_1} = 1189 \text{ MeV}$$

Sakai-Sugimoto, PTP113(05)843; PTP114(05)1083]

$$f_\pi = 92.4 \text{ MeV}, \quad m_\rho = 776 \text{ MeV}$$

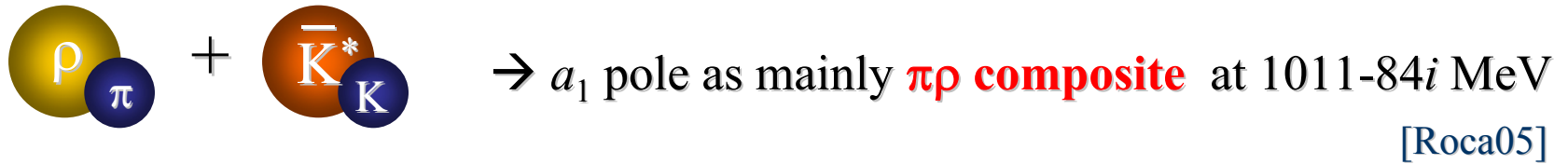
input parameters

- ✓ this elementary a_1 meson does **not** have molecular component
[hQCD is constructed in the large N_c limit]

Dynamically generated resonances : **composite a_1 meson**

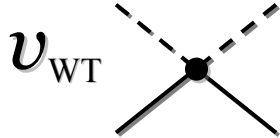
a_1 meson as a $\pi\rho$ -composite : chiral unitary model

L.Roca, E.Oset and J.Singh, PRD72(05)014002



$\pi\rho$ scattering amplitude with coupled-channel calculation

$$T_{\pi\rho \rightarrow \pi\rho} = \frac{v_{WT}}{1 - v_{WT}G} \vec{\epsilon} \cdot \vec{\epsilon}'$$

v_{WT} 

$$= \text{[Series of diagrams: a vertex, a vertex with a loop, a vertex with two loops, etc.] } = g_R(s) \frac{1}{s - s_p} g_R(s)$$

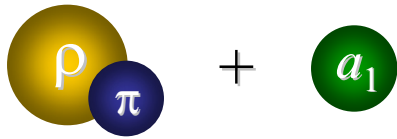
regularization constant

$a(\mu) = -1.85$ ($\mu=900\text{MeV}$) [Roca *et al.*] $\rightarrow a(\mu) = -0.2$ (natural)
 to avoid the double counting.

[T. Hyodo, D.Jido, A.Hosaka, PRC78(08)025203]₄

Formalism : elementary a_1 field through additional interaction

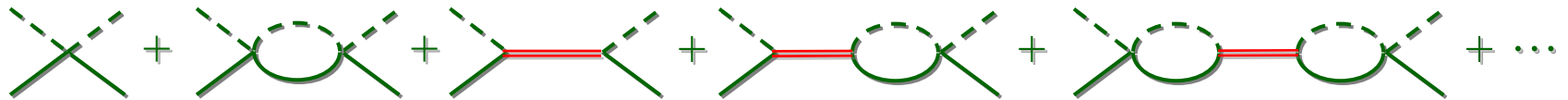
mixed states of $\pi\rho$ composite a_1 and elementary a_1 mesons



elementary a_1 meson contributes to $T_{\pi\rho}$ through the additional interaction v_{a1}

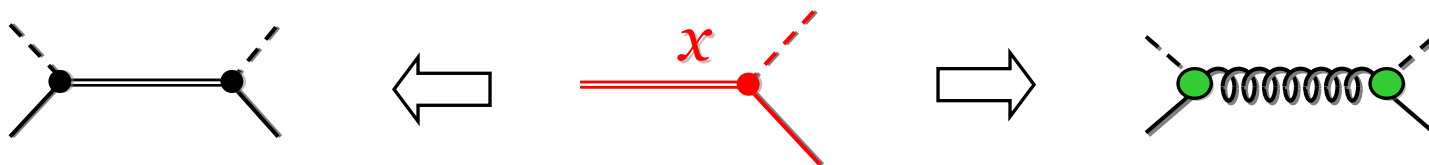
full scattering amplitude

$$T_{\pi\rho \rightarrow \pi\rho} = \frac{v_{WT} + v_{a_1}}{1 - (v_{WT} + v_{a_1})G} \vec{\epsilon} \cdot \vec{\epsilon}'$$

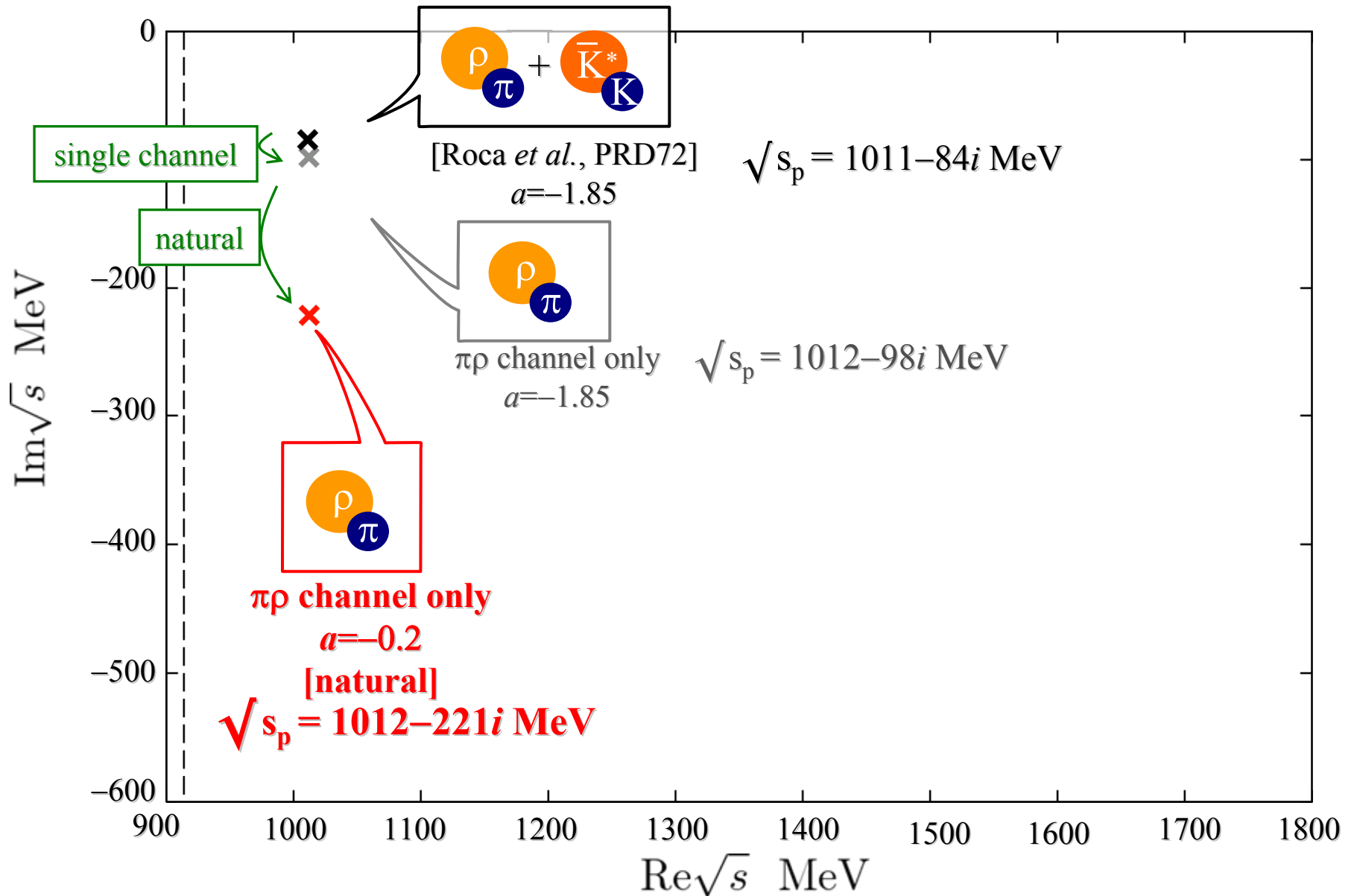


= physical resonant a_1 states

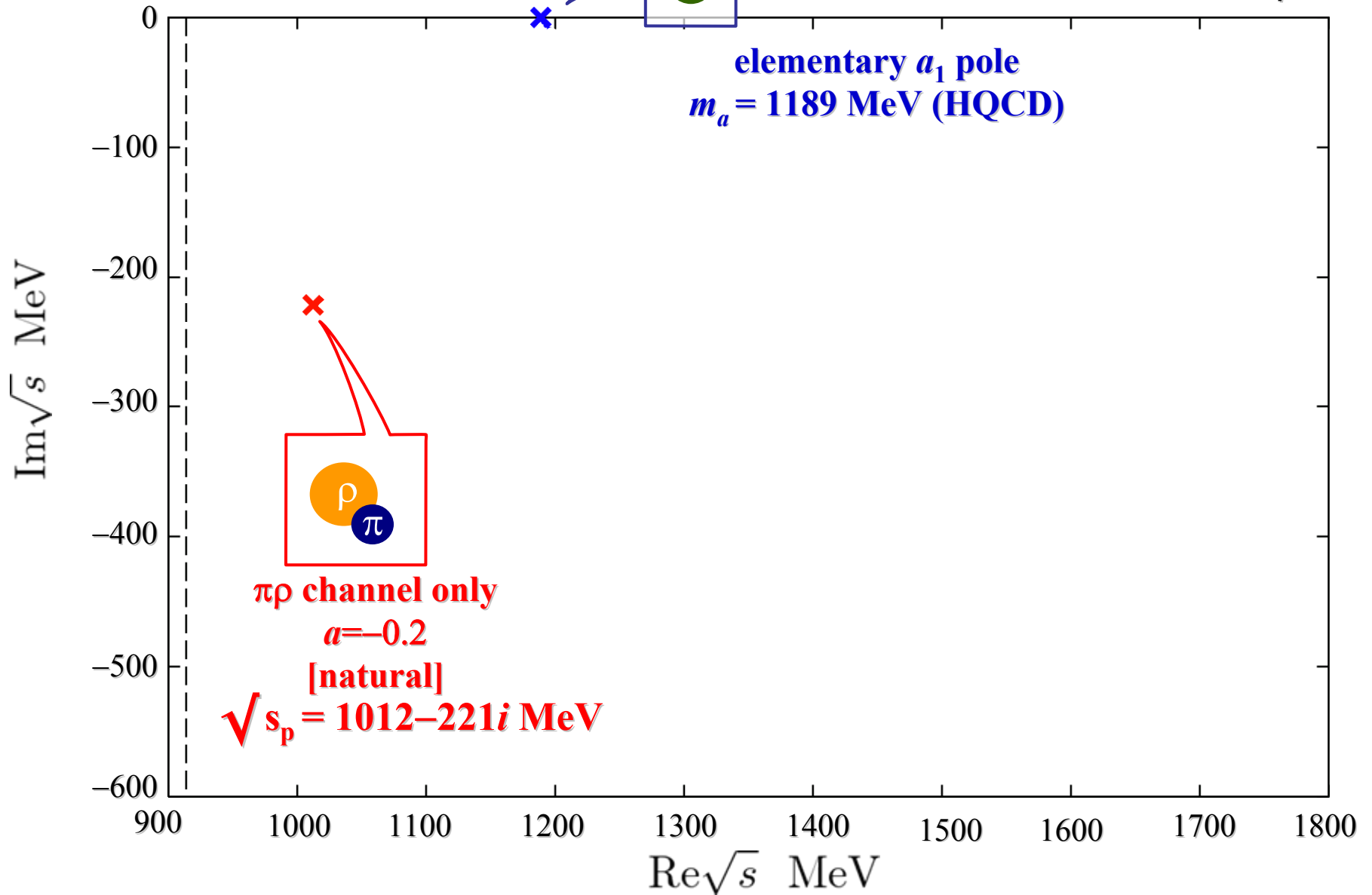
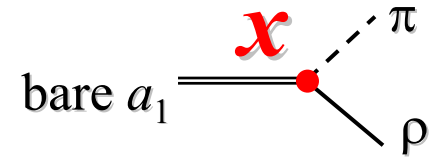
causes the mixing



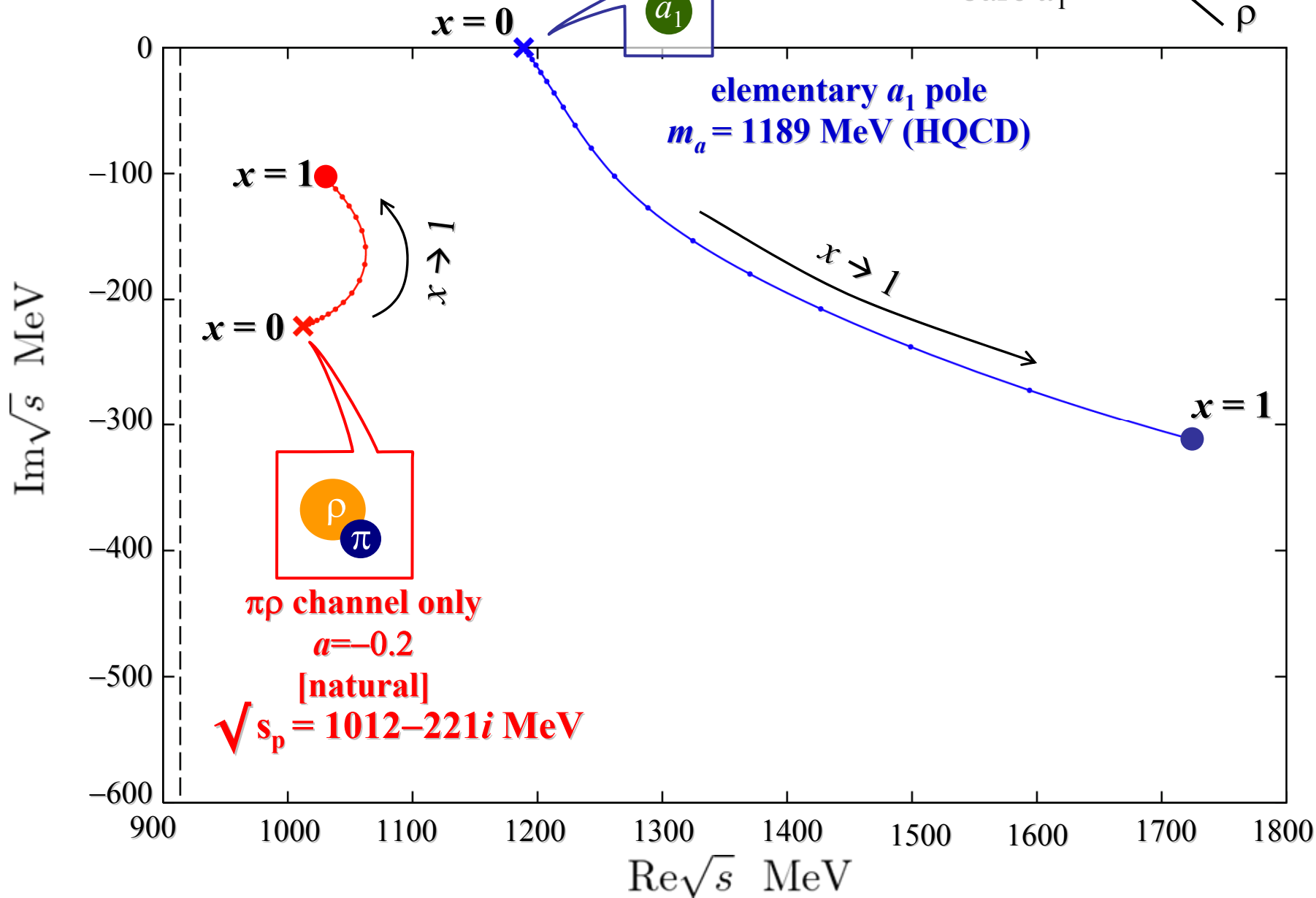
Numerical result 1 : pole-flow of T-matrix



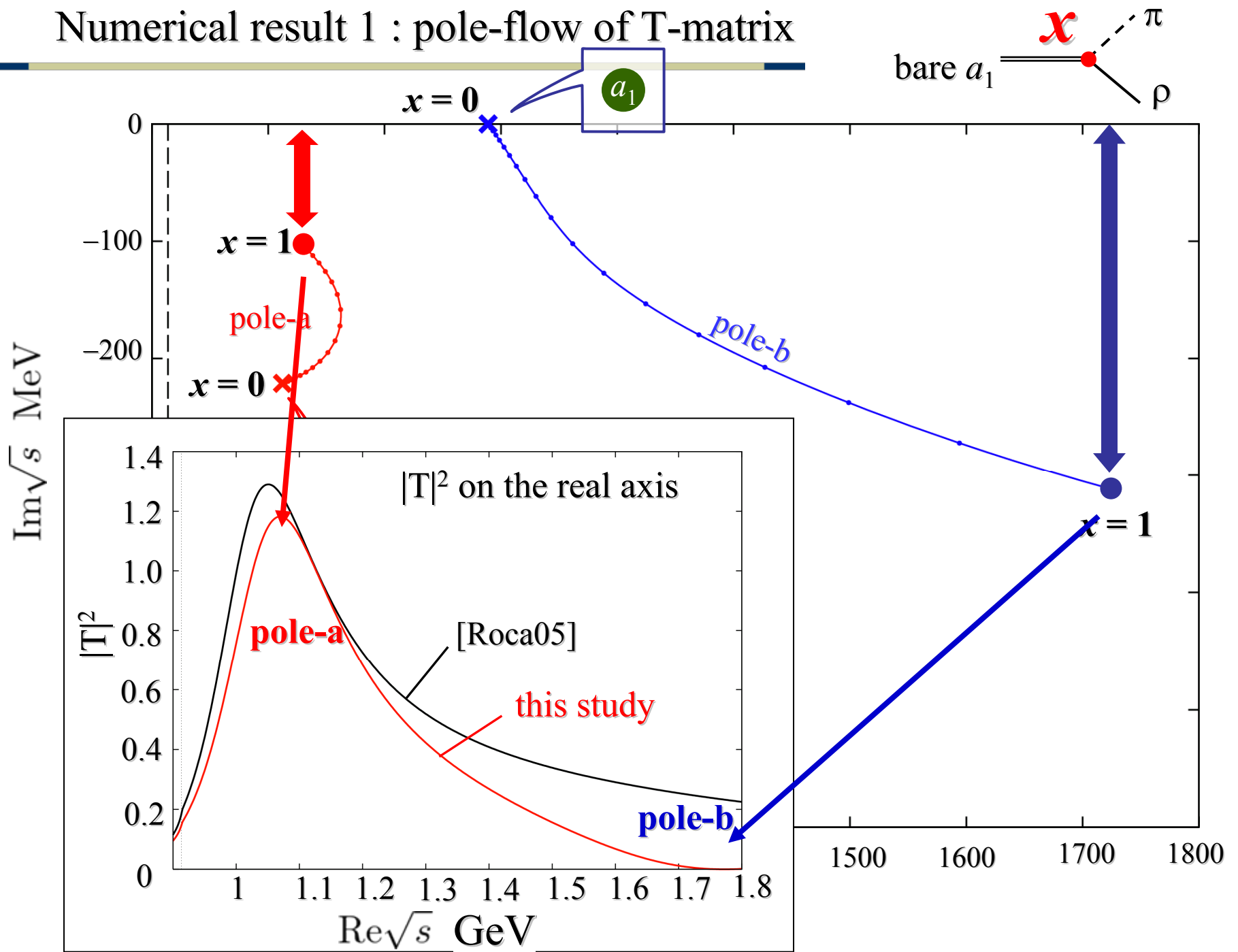
Numerical result 1 : pole-flow of T-matrix



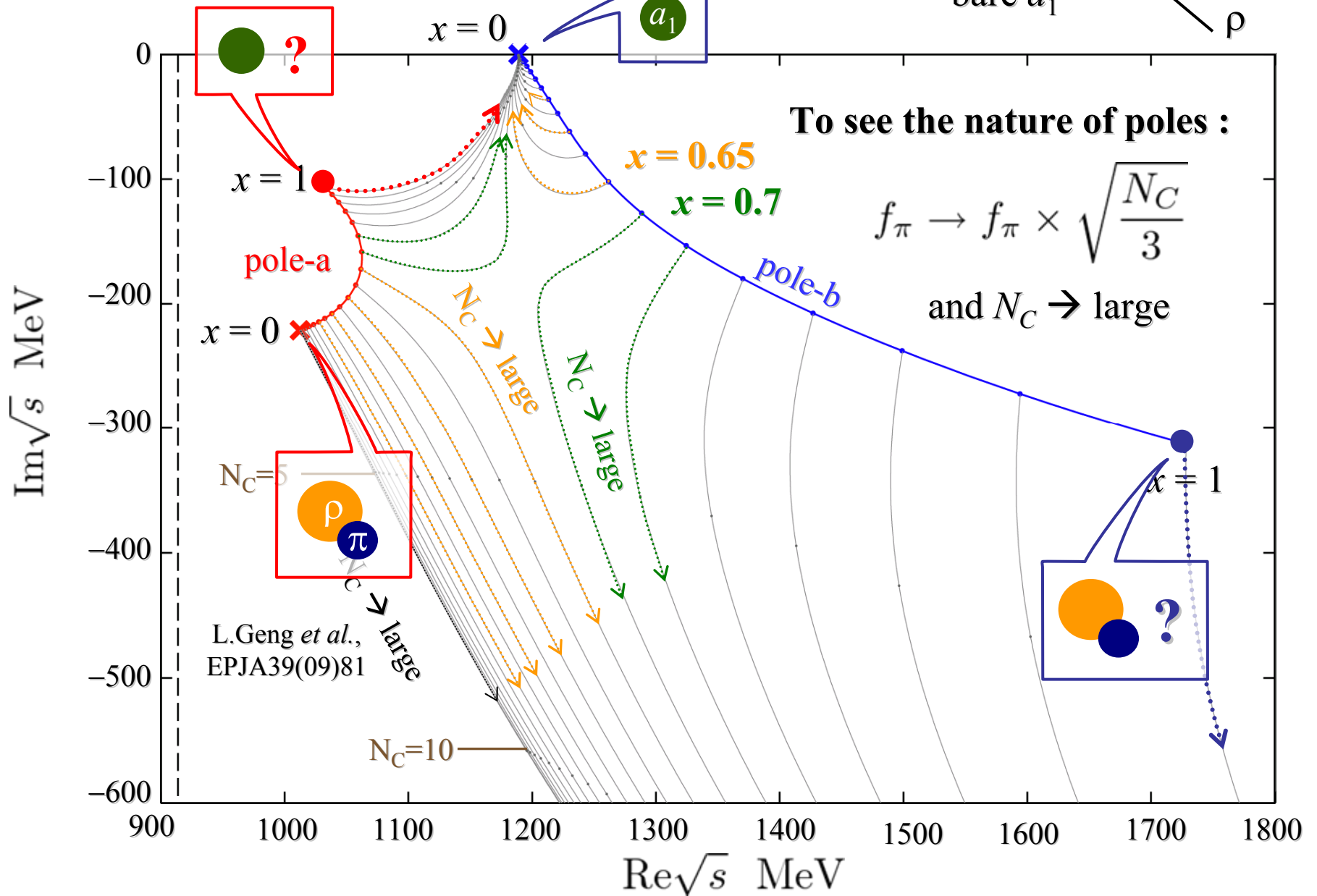
Numerical result 1 : pole-flow of T-matrix



Numerical result 1 : pole-flow of T-matrix



Numerical result 1 : large N_C flow



Alternative expression for the *full* $\pi\rho$ scattering amplitude T

$$T = \frac{v_{WT} + v_{a_1}}{1 - (v_{WT} + v_{a_1})G} = (g_R, g) \left\{ \begin{pmatrix} s-s_p & \\ & s-m_{a_1}^2 \end{pmatrix} - \begin{pmatrix} g_R G g & \\ g G g_R & g G g \end{pmatrix} \right\}^{-1} \begin{pmatrix} g_R \\ g \end{pmatrix}$$

$$= \left(\begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} \right) \left\{ \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix}^{-1} - \begin{pmatrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{pmatrix} \right\}^{-1} \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix}$$

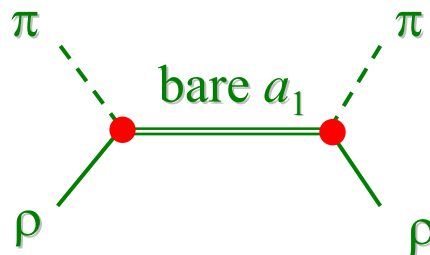
$\pi\rho$ -composite a_1 pole

$$T_{WT} = \frac{v_{WT}}{1 - v_{WT}G}$$

$$\text{---} + \text{---} + \text{---} + \dots \equiv \text{---} = g_R(s) \frac{1}{s - s_p} g_R(s)$$

a_1 pole term

$$V_{a_1} = g(s) \frac{\vec{\epsilon} \cdot \vec{\epsilon}'}{s - m_a^2} g(s)$$



Alternative expression for the *full* $\pi\rho$ scattering amplitude T

$$T = \frac{v_{WT} + v_{a_1}}{1 - (v_{WT} + v_{a_1})G} = (g_R, g) \left\{ \begin{pmatrix} s-s_p & \\ & s-m_{a_1}^2 \end{pmatrix} - \begin{pmatrix} g_R G g & \\ g G g_R & g G g \end{pmatrix} \right\}^{-1} \begin{pmatrix} g_R \\ g \end{pmatrix}$$

\hat{D}

$$= \left(\begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} \right) \left\{ \begin{pmatrix} \text{---} & \\ & \text{---} \end{pmatrix}^{-1} - \begin{pmatrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{pmatrix} \right\}^{-1} \begin{pmatrix} \text{---} \\ \text{---} \end{pmatrix}$$

In this form, we can analyze the mixing nature of the physical a_1 in terms of the **original two bases**: and

composite a_1

elementary a_1



$$= \boxed{\text{---} \text{---} \text{---}}_{D^{11}} + \boxed{\text{---} \text{---} \text{---}}_{D^{21}} + \boxed{\text{---} \text{---} \text{---}}_{D^{12}} + \boxed{\text{---} \text{---} \text{---}}_{D^{22}}$$

Features of the full propagators

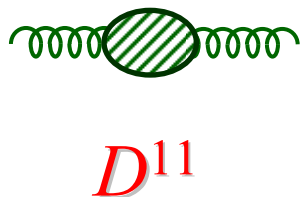
- ✓ The propagators have two poles exactly at the same position as T_{full}
- ✓ the residues z_a^{ii} have the information on the mixing rate

$$z_a^{11} = |\langle 1|a\rangle|^2 = |\langle \text{pole-a} \rangle|^2$$

→ *Probability of finding the original composite a_1 in pole-a*

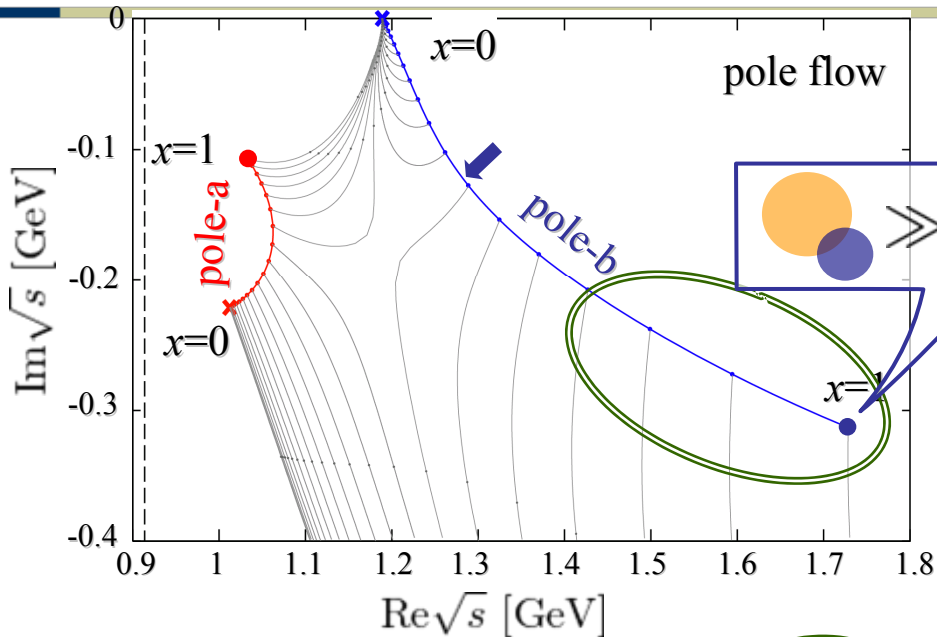
In this form, we can analyze the mixing nature of the physical a_1 in terms of the **original two bases**:  and 

composite a_1 elementary a_1



$$D^{11} = \frac{z_a^{11}}{s - M_a^2} + \frac{z_b^{11}}{s - M_b^2} + \text{regular term}$$

Residues : probabilities of finding two a_1 's in pole-a and -b

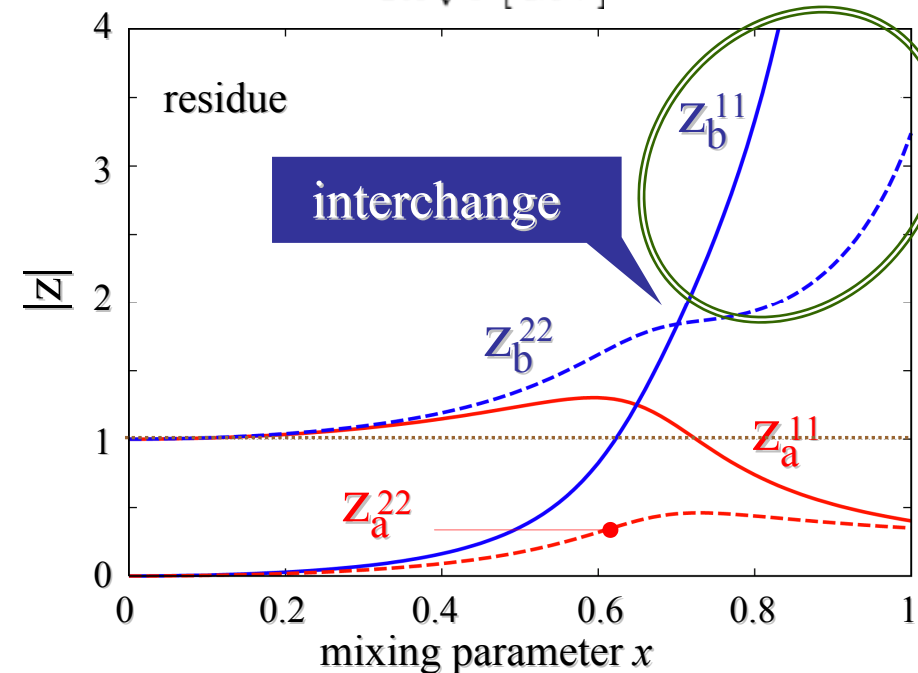


$$[\hat{D}_{\text{full}}]^{11} = \frac{z_a^{11}}{E - E_a} + \frac{z_b^{11}}{E - E_b} + (\text{regular})$$

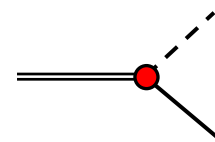
$$[\hat{D}_{\text{full}}]^{22} = \frac{z_a^{22}}{E - E_a} + \frac{z_b^{22}}{E - E_b} + (\text{regular})$$

$$|a\rangle = \sqrt{z_a^{11}} |\text{orange, blue}\rangle + \sqrt{z_a^{22}} |\text{green}\rangle$$

$$|b\rangle = \sqrt{z_b^{11}} |\text{orange, blue}\rangle + \sqrt{z_b^{22}} |\text{green}\rangle$$

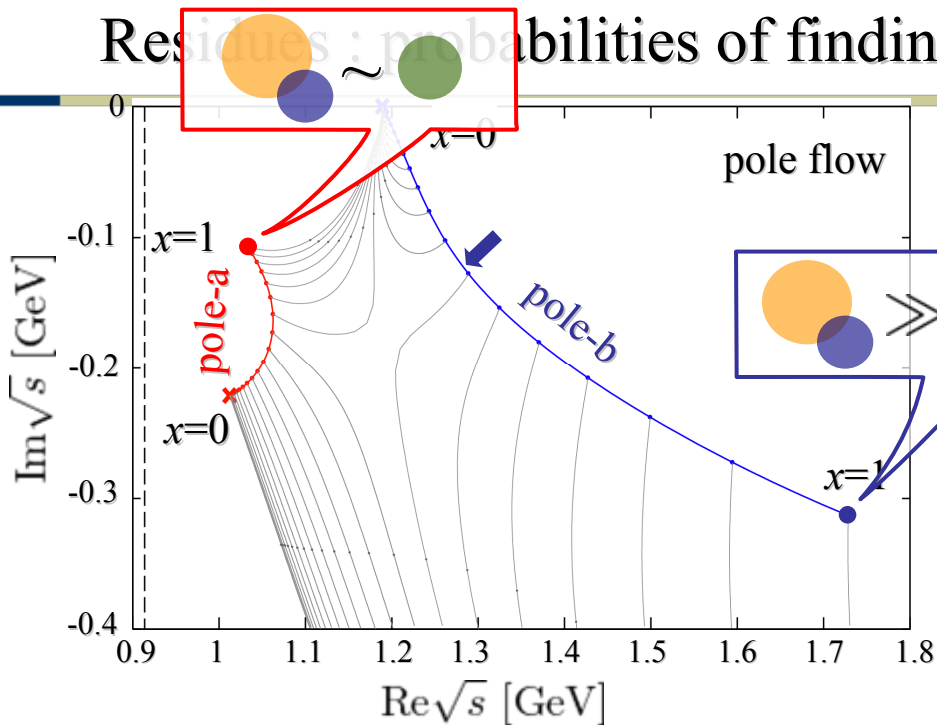


Large residue
→ due to the ene-dep.



$$g(s) = \frac{2\sqrt{2}}{f_\pi} g_{a_1\pi\rho} (s - M_\rho^2)$$

Residues: probabilities of finding two a_1 's in pole-a and -b

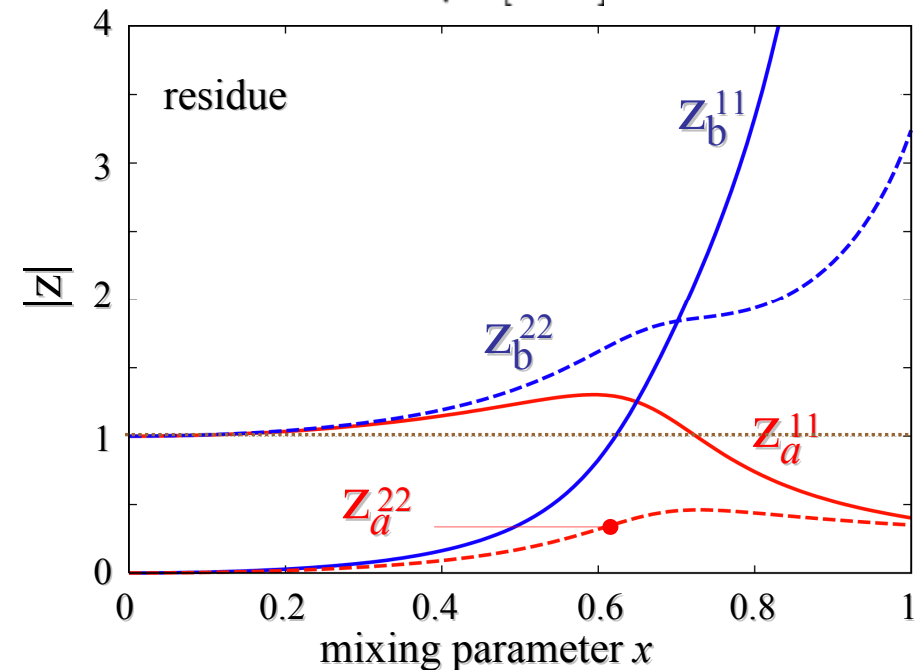


$$[\hat{D}_{\text{full}}]^{11} = \frac{z_a^{11}}{E - E_a} + \frac{z_b^{11}}{E - E_b} + (\text{regular})$$

$$[\hat{D}_{\text{full}}]^{22} = \frac{z_a^{22}}{E - E_a} + \frac{z_b^{22}}{E - E_b} + (\text{regular})$$

$$|a\rangle = \sqrt{z_a^{11}} |\text{pole-a}\rangle + \sqrt{z_a^{22}} |\text{pole-b}\rangle$$

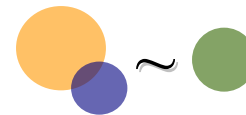
$$|b\rangle = \sqrt{z_b^{11}} |\text{pole-a}\rangle + \sqrt{z_b^{22}} |\text{pole-b}\rangle$$



at physical point ($x=1$)

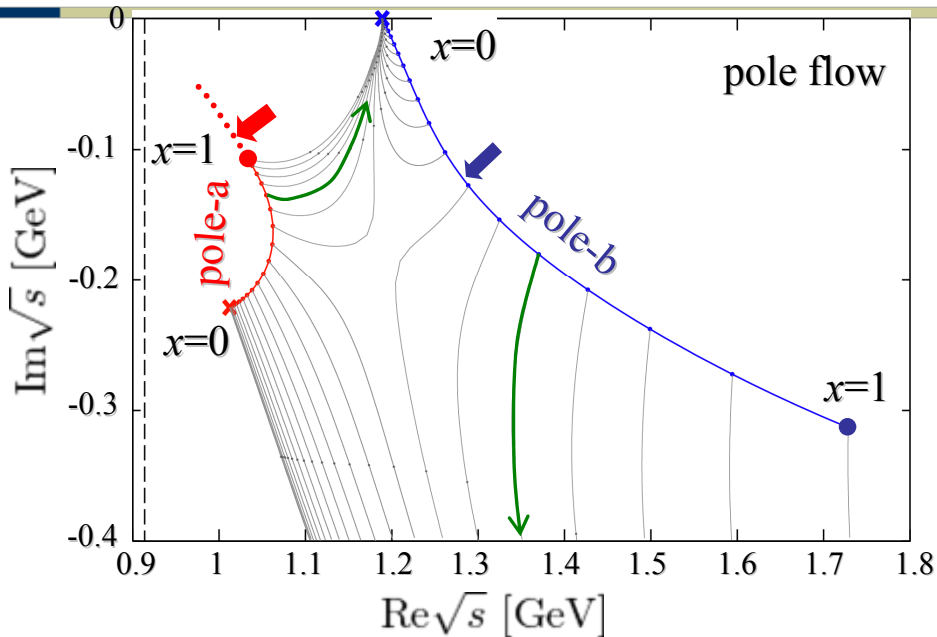
- **pole-a** has a component of the elementary a_1 meson *comparable to* that of composite a_1 .

(pole-a at $x=1$ is possibly observed one)



non-zero comp. of

Last question : large N_C limit vs. nature of poles

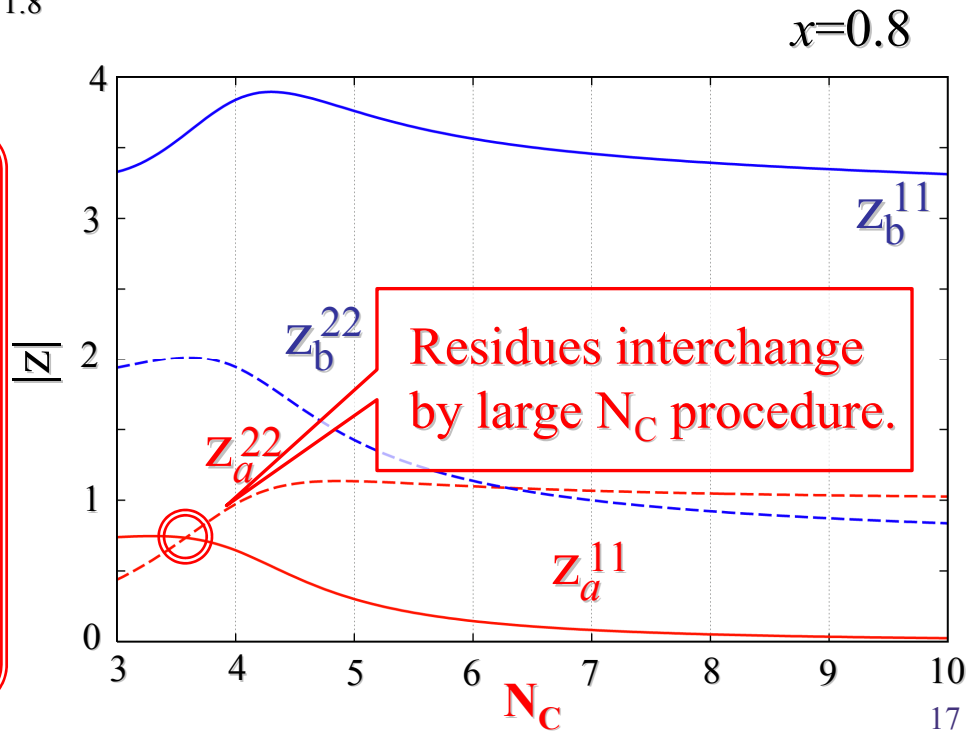


- for pole-b, large N_C flip point \sim residue-interchange
- for pole-a, large N_C flip point \ll residue-interchange

$\propto 1/N_C$ $\propto 1/\sqrt{N_C}$

mixing nature *changes* as N_C is increased

Large N_C limit doesn't always reflect the world at $N_C=3$.



Conclusions

- » We discussed the **mixing properties** of $a_1(1260)$ meson as the superposition of the hadronic $\pi\rho$ composite and elementary a_1 based on the holographic QCD Lagrangian.
 - › bare a_1 ... doesn't have molecule nature
 - › $\pi\rho$ molecule ... “natural” regularization
- ← Important to avoid the double-counting
- » **We analyzed the pole nature by residues**
 - ✓ the pole expected to be observed is *pole-a*: having finite ● comp.
 - ✓ Non-trivial N_C dependence pole-nature ← ? → large N_C limit

Future works

phenomenological interests

- » τ -decay spectrum with our model parameter

[Wagner and Leupold, PRD78(08)053001, ...]

- » radiative decay width

[H. Nagahiro, L. Roca, A. Hosaka, E. Oset, PRD79(09)014015, ...]

- » etc...

to see **how the nature of poles affects *observables***