Mixing properties of $a_1(1260)$ meson consisting of hadronic composite and quark composite

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H. Nagahiro et al., PRD83(11)111504(R), arXiv:1101.3623 [hep-ph].
Introduction

Exotic hadrons: *not* simple $q\bar{q}$ (or $qqq$) state

- $\sigma, f_0(980), a_0(980), \ldots, a_1(1260), K_1(1270), \ldots, N^*(1535), \Lambda(1405), \ldots$ etc…

→ molecule, $qq\bar{q}\bar{q}$ state, or *mixed states among them?*

Nature of axial vector meson $a_1(1260): m = 1230 \pm 40$ MeV, $\Gamma = 260$ to 600 MeV [PDG]

- *elementary* field (or $q\bar{q}$): chiral partner of $\rho$ meson
  - [Hidden local sym.] Bando-Kugo-Yamawaki; PR164(88)217; Kaiser-Meissner, NPA519(90)671, …
  - [$q\bar{q}$-NJL] M. Wakamatsu *et al.,* ZPA311(88)173, A. Hosaka, PLB244(90)363-367, …
  - [Lattice QCD] M. Wingate *et al.,* PRL74(95)4596, …
  - [Holographic QCD] T. Sakai, S. Sugimoto, PTP113 (05) 843; *ibid.*114(05)1083,

- *dynamically generated resonance* [$\pi\rho$ *composite*]
  - [coupled-channel BS] Lutz-Kolomeitsev, NPA730(04)392, …
  - [Chiral Unitary model] Roca-Oset-Singh, PRD72(05)014002, …

- *applications*
  - [$\tau$-decay spectrum] M. Wagner and S. Leupold, PRD78(08)053001, …

Aim of this study

*We study the “mixing properties” of exotic hadrons as a quark-core + hadronic composite system*
A model for $a_1$, $\pi$ and $\rho$ mesons

hidden local sym. or holographic model

Bando-Kugo-Yamawaki, PR164(88)217
Sakai-Sugimoto, PTP113(05)843

\[
\mathcal{L}_{WT} = -\frac{g^4}{4f^2_\pi} \text{tr} \left( [\rho^\mu, \partial^\nu \rho_\mu][\pi, \partial_\nu \pi] \right)
\]

\[
\mathcal{L}_{a_1 \pi \rho} = -g_{a_1 \pi \rho} \frac{i\sqrt{2}}{f_\pi} \left\{ \text{tr} \left( [\partial_\mu a_1, \partial_\nu a_1][\partial^\mu \pi, \rho^\nu] \right) \\
+ \text{tr} \left( [\partial_\mu \rho, \partial_\nu \rho][\partial^\mu \pi, a_1^\nu] \right) \right\}
\]

A good model for composite $a_1$ and elementary $a_1$

mass of elementary $a_1$ meson: in holographic model

\[
m_{a_1} = 1189 \text{ MeV}
\]

Sakai-Sugimoto, PTP113(05)843; PTP114(05)1083

\[
f_\pi = 92.4\text{MeV}, \quad m_\rho = 776\text{MeV}
\]

input parameters

✓ this elementary $a_1$ meson does not have molecular component

[hQCD is constructed in the large Nc limit]
Dynamically generated resonances: composite $a_1$ meson

$a_1$ meson as a $\pi\rho$-composite: chiral unitary model

L. Roca, E. Oset and J. Singh, PRD72(05)014002

$\rho_\pi + \bar{K}^* K \rightarrow a_1$ pole as mainly $\pi\rho$ composite at 1011-84i MeV

$\pi\rho$ scattering amplitude with coupled-channel calculation

$$T_{\pi\rho \rightarrow \pi\rho} = \frac{\mathcal{U}_{WT}}{1 - \mathcal{U}_{WT}G} \vec{\epsilon} \cdot \vec{\epsilon}'$$

$$= \times + \times \times + \times \times \times + \ldots = \times \times \times \times \times \times \times$$

regularization constant

$a(\mu) = -1.85$ ($\mu=900$ MeV) [Roca et al.] $\Rightarrow a(\mu) = -0.2$ (natural)

to avoid the double counting.

[T. Hyodo, D. Jido, A. Hosaka, PRC78(08)025203]
Formalism: **elementary $a_1$ field through additional interaction**

mixed states of $\pi\rho$ composite $a_1$ and elementary $a_1$ mesons

full scattering amplitude

$$T_{\pi\rho \rightarrow \pi\rho} = \frac{\nu_{WT} + \nu_{a_1}}{1 - (\nu_{WT} + \nu_{a_1})G} \bar{\epsilon} \cdot \bar{\epsilon}' + \cdots$$

= physical resonant $a_1$ states

causes the mixing
Numerical result 1: pole-flow of T-matrix

[Roca et al., PRD72]  
\[ a = -1.85 \]  
\[ \sqrt{s_p} = 1011 - 84i \text{ MeV} \]

\[ a = -0.2 \]  
\[ \sqrt{s_p} = 1012 - 221i \text{ MeV} \]
Numerical result 1: pole-flow of T-matrix

elementary $a_1$ pole

$m_a = 1189$ MeV (HQCD)

$\sqrt{s_p} = 1012 - 221i$ MeV

$\rho$ channel only

$a = -0.2$

[natural]
Numerical result 1: pole-flow of T-matrix

\[ \text{elementary } a_1 \text{ pole} \]
\[ m_a = 1189 \text{ MeV (HQCD)} \]

\[ \pi\rho \text{ channel only} \]
\[ a = -0.2 \]
\[ [\text{natural}] \]
\[ \sqrt{s_p} = 1012 - 221i \text{ MeV} \]
Numerical result 1: pole-flow of T-matrix

\[ |T|^2 \text{ on the real axis} \]

- pole-a
- pole-b

\[ x = 0 \]
\[ x = 1 \]

\[ \text{Im} \sqrt{s} \text{ MeV} \]

\[ \text{Re} \sqrt{s} \text{ GeV} \]
Numerical result 1: large $N_C$ flow

To see the nature of poles:

$$f_{\pi} \to f_{\pi} \times \sqrt{\frac{N_C}{3}}$$

and $N_C \to \text{large}$

L. Geng et al., EPJA39(09)81
Alternative expression for the full $\pi\rho$ scattering amplitude $T$

\[
T = \frac{v_{WT} + v_{a_1}}{1 - (v_{WT} + v_{a_1})G} = (g_R, g) \left\{ \left( s - s_p - m_{a_1}^2 \right) - \left( g_{Gg_R} g_{Gg} \right) \right\}^{-1} \left( g_R \right)
\]

\[
= (\pi, \rho) \left\{ \left( \pi \right) \right\}^{-1} - \left\{ \left( \rho \right) \right\}^{-1}
\]

\[
\pi\rho\text{-composite } a_1 \text{ pole}
\]

\[
T_{WT} = \frac{v_{WT}}{1 - v_{WT}G}
\]

\[
\pi\rho + \pi\rho + \pi\rho + \cdots = g_R(s) \frac{1}{s - s_p} g_R(s)
\]

\[
a_1 \text{ pole term}
\]

\[
V_{a_1} = g(s) \frac{\vec{\epsilon} \cdot \vec{\epsilon}'}{s - m_{a_1}^2} g(s)
\]
Alternative expression for the full $\pi p$ scattering amplitude $T$

$$T = \frac{\nu_{WT} + \nu_{a_1}}{1 - (\nu_{WT} + \nu_{a_1})G} = (g_R, g) \left\{ \begin{array}{c} s-s_p \\ s-m^2_{a_1} \end{array} \right\}^{-1} - \left( \begin{array}{cc} g_R G g \\ g G g_R \end{array} \right) \right\}^{-1} \left( \begin{array}{c} g_R \\ g \end{array} \right)$$

$$= (\text{wire}, \text{wire}) \left\{ \begin{array}{c} \text{wire} \\ \text{wire} \end{array} \right\}^{-1} - \left( \begin{array}{c} \text{wire} \\ \text{wire} \end{array} \right) \right\}^{-1} \left( \begin{array}{c} \text{wire} \end{array} \right)$$ 

In this form, we can analyze the mixing nature of the physical $a_1$ in terms of the original two bases: \text{wire} and composite $a_1$ and elementary $a_1$.
Features of the full propagators

- The propagators have two poles exactly at the same position as $T_{\text{full}}$
- The residues $z_a^{ii}$ have the information on the mixing rate

$$z_a^{11} = |\langle 1 | a \rangle|^2 = |\langle \text{pole-a} | a \rangle|^2$$

→ Probability of finding the original composite $a_1$ in pole-$a$

In this form, we can analyze the mixing nature of the physical $a_1$ in terms of the original two bases: $\ldots$ and $\ldots$ composite $a_1$ and elementary $a_1$

$$D^{11} = \frac{z_a^{11}}{s - M_a^2} + \frac{z_b^{11}}{s - M_b^2} + \text{regular term}$$
Residues: probabilities of finding two $a_1$'s in pole-a and -b

\[
[D_{\text{full}}]^{11} = \frac{z_a^{11}}{E - E_a} + \frac{z_b^{11}}{E - E_b} + \text{(regular)}
\]
\[
[D_{\text{full}}]^{22} = \frac{z_a^{22}}{E - E_a} + \frac{z_b^{22}}{E - E_b} + \text{(regular)}
\]
\[
|a\rangle = \sqrt{z_a^{11}} \left| \begin{array}{c} 1 \\ 1 \end{array} \right> + \sqrt{z_a^{22}} \left| \begin{array}{c} 2 \\ 2 \end{array} \right>
\]
\[
|b\rangle = \sqrt{z_b^{11}} \left| \begin{array}{c} 1 \\ 1 \end{array} \right> + \sqrt{z_b^{22}} \left| \begin{array}{c} 2 \\ 2 \end{array} \right>
\]

Large residue $\rightarrow$ due to the ene-dep.

\[
g(s) = \frac{2\sqrt{2}}{f_\pi} g_{\alpha_1 \pi \rho} (s - M_\rho^2)
\]
Residues: probabilities of finding two $a_1$'s in pole-a and -b

\[
[D_{\text{full}}]^{11} = \frac{z_a^{11}}{E - E_a} + \frac{z_b^{11}}{E - E_b} + \text{(regular)}
\]

\[
[D_{\text{full}}]^{22} = \frac{z_a^{22}}{E - E_a} + \frac{z_b^{22}}{E - E_b} + \text{(regular)}
\]

\[
|a\rangle = \sqrt{z_a^{11}} |\text{a} \rangle + \sqrt{z_a^{22}} |\text{b} \rangle
\]

\[
|b\rangle = \sqrt{z_b^{11}} |\text{a} \rangle + \sqrt{z_b^{22}} |\text{b} \rangle
\]

at physical point ($x=1$)

- pole-a has a component of the elementary $a_1$ meson comparable to that of composite $a_1$.

(pole-a at $x=1$ is possibly observed one)

non-zero comp. of
Last question: large $N_C$ limit vs. nature of poles

- for pole-b, large $N_C$ flip point $\sim$ residue-interchange
- for pole-a, large $N_C$ flip point $\ll$ residue-interchange

Residues interchange by large $N_C$ procedure.

Large $N_C$ limit doesn’t always reflect the world at $N_C=3$. 
Conclusions

We discussed the mixing properties of $a_1(1260)$ meson as the superposition of the hadronic $\pi\rho$ composite and elementary $a_1$ based on the holographic QCD Lagrangian.

- bare $a_1$ … doesn’t have molecule nature
- $\pi\rho$ molecule … “natural” regularization

Important to avoid the double-counting

We analyzed the pole nature by residues

- the pole expected to be observed is pole-a: having finite comp.
- Non-trivial $N_C$ dependence pole-nature $\leftrightarrow$ ? $\rightarrow$ large $N_C$ limit

Future works

phenomenological interests

- $\tau$-decay spectrum with our model parameter
  [Wagner and Leupold, PRD78(08)053001, …]
- radiative decay width
  [H. Nagahiro, L. Roca, A. Hosaka, E. Oset, PRD79(09)014015, …]
- etc…

to see how the nature of poles affects observables