How the small hyperfine splitting of P-wave mesons evades large loop corrections

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Spin-dependence in quark models

Mass formula in perturbation theory,

$$M_{SLJ} = M + \Delta_s \langle \tfrac{1}{2} \tfrac{1}{2} \rangle_S + \Delta_t \langle \mathbf{T} \rangle_{SLJ} + \Delta_o \langle \mathbf{L} \cdot \mathbf{S} \rangle_{SLJ},$$

for mesons with spin S, orbital L and total J angular momenta.

- $\langle \ldots \rangle$ are model independent.
- M and the Δ's are model dependent, but common.

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P-wave mesons: theory

Four equations, and four unknowns:

$$M_{^1\mathrm{P}_1} = M - \frac{3}{4}\Delta_s$$

$$M_{^3\mathrm{P}_0} = M + \frac{1}{4}\Delta_s + 2\Delta_t - 2\Delta_o$$

$$M_{^3\mathrm{P}_1} = M + \frac{1}{4}\Delta_s - \Delta_t - \Delta_o$$

$$M_{^3\mathrm{P}_2} = M + \frac{1}{4}\Delta_s + \frac{1}{5}\Delta_t + \Delta_o$$

Hyperfine splitting:

$$\frac{1}{9}\left(M_{^3\mathrm{P}_0} + 3M_{^3\mathrm{P}_1} + 5M_{^3\mathrm{P}_2}\right) - M_{^1\mathrm{P}_1} = \Delta_s \approx 0$$

P-wave mesons: experiment

Charmonia:

$$\overline{M}_{\chi_c(1\mathrm{P})} - M_{h_c(1\mathrm{P})} = -0.05 \pm 0.19 \pm 0.16 \text{ MeV}$$

Bottomonia:

$$\overline{M}_{\chi_b(1\mathrm{P})} - M_{h_b(1\mathrm{P})} = +2 \pm 4 \pm 1 \text{ MeV} \qquad \text{(BaBar)}$$

$$\overline{M}_{\chi_b(1\mathrm{P})} - M_{h_b(1\mathrm{P})} = +1.62 \pm 1.52 \text{ MeV} \qquad \text{(Belle)}$$

$$\overline{M}_{\chi_b(2\mathrm{P})} - M_{h_b(2\mathrm{P})} = +0.48^{+1.57}_{-1.22} \text{ MeV} \qquad \text{(Belle)}$$

Mass shifts due to channel coupling

Coupling to open flavour pairs

$$(Q\overline{Q}) \leftrightarrow (Q\overline{q})(q\overline{Q})$$

- unquenching causes mass shifts
- χ_0, χ_1, χ_2 and h couple to different channels and with different strengths, so their mass shifts differ
- expect violations to the mass formula

$$\frac{1}{9}\left(M_{^3\mathrm{P}_0} + 3M_{^3\mathrm{P}_1} + 5M_{^3\mathrm{P}_2}\right) - M_{^1\mathrm{P}_1} = 0$$

Mass shifts due to channel coupling

Charmonia

Mass shifts of

- χ_{c0}, χ_{c1}, χ_{c2} and h_c,

due to couplings

- $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$, and
- $D_s\bar{D}_s$, $D_s\bar{D}_s^*$, $D_s^*\bar{D}_s^*$

Bottomonia

Mass shifts of

- χ_{b0}, χ_{b1}, χ_{b2} and h_b,

due to couplings

- $B\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$, and
- $B_s\bar{B}_s$, $B_s\bar{B}_s^*$, $B_s^*\bar{B}_s^*$

Literature

Barnes & Swanson (BT)
Kalashnikova (K)
Li, Meng & Chao (LMC)
Yang, Li, Chen & Deng (YLCD)
Ono & Törnqvist (OT)
Liu & Ding (LD)

Mass shifts due to channel coupling

		$\Delta M_{^3P_0}$	$\Delta M_{^3P_1}$	$\Delta M_{^3P_2}$	$\Delta M_{^1P_1}$	Induced Δ_s
BS	(1P,$c\bar{c}$)	459	496	521	504	
K	(1P,$c\bar{c}$)	198	215	228	219	
LMC	(1P,$c\bar{c}$)	35	38	63	52	
YLCD	(1P,$c\bar{c}$)	131	152	175	162	
OT	(1P,$c\bar{c}$)	173	180	185	182	
OT	(1P,$b\bar{b}$)	43	44	45	44	
OT	(2P,$b\bar{b}$)	55	56	58	57	
LD	(1P,$b\bar{b}$)	80.777	84.823	87.388	85.785	
LD	(2P,$b\bar{b}$)	73.578	77.608	80.146	78.522	

- ΔM_{SLJ} can be very large
- $\Delta M_{S'L'J'} - \Delta M_{SLJ}$ is smaller
- $-\frac{1}{9}\left(\Delta M_{^3P_0} + 3\Delta M_{^3P_1} + 5\Delta M_{^3P_2}\right) + \Delta M_{^1P_1}$ is smaller still

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K	(1P,$c\bar{c}$)	198	215	228	219	– 1.3
LMC	(1P,$c\bar{c}$)	35	38	63	52	– 2.9
YLCD	(1P,$c\bar{c}$)	131	152	175	162	– 0.4
OT	(1P,$c\bar{c}$)	173	180	185	182	– 0.0
OT	(1P,$b\bar{b}$)	43	44	45	44	– 0.4
OT	(2P,$b\bar{b}$)	55	56	58	57	– 0.0
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Mass shifts due to channel coupling

The models differ in many ways:

- perturbation theory *vs*. coupled channel equations
- harmonic oscillator *vs*. coulomb + linear wavefunctions
- universal *vs*. flavour-dependent wavefunctions
- exact SU(3) *vs*. broken SU(3) in pair creation

But have important common features:

- coupling $(Q\overline{Q}) \rightarrow (Q\overline{q})(q\overline{Q})$ has $q\overline{q}$ in spin triplet
- spin and spatial degrees of freedom factorise
- spin is conserved

Computing the mass shifts

The mass shift

- of a state with S, L, J quantum numbers
- due to coupling with mesons spins s_1 and s_2 in partial wave l

$$\Delta M_{SLJ}^{s_1 s_2 l} = C_{SLJ}^{s_1 s_2 l} \int dp \frac{p^2 |A_l(p)|^2}{\epsilon_{SLJ}^{s_1 s_2} + p^2/2\mu_{s_1 s_2}}$$

- $\epsilon_{SLJ}^{s_1 s_2}$ and $\mu_{s_1 s_2}$ are binding energy and reduced mass
- $C_{SLJ}^{s_1 s_2 l}$ depends only on the angular momenta
- $A_l(p)$ depends only on the spatial degrees of freedom
- $A_l(p)$ is common to all channels if the radial wavefunctions $\chi_0 = \chi_1 = \chi_2 = h$ and $D = D^*$

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Computing the mass shifts

Total mass shifts

$$\Delta M_{SLJ} = \sum_{s_1 s_2 l} C_{SLJ}^{s_1 s_2 l} \int dp \frac{p^2 |A_l(p)|^2}{\epsilon_{SLJ}^{s_1 s_2} + p^2/2\mu_{s_1 s_2}}$$

Continuum probability

$$P_{SLJ} = \sum_{s_1 s_2 l} C_{SLJ}^{s_1 s_2 l} \int dp \frac{p^2 |A_l(p)|^2}{\left(\epsilon_{SLJ}^{s_1 s_2} + p^2/2\mu_{s_1 s_2}\right)^2}$$

The equal mass limit

In the equal mass limit ($\chi_0 = \chi_1 = \chi_2 = h$ and $D = D^*$)

- $\epsilon_{SLJ}^{s_1 s_2} = \epsilon$
- $\mu_{s_1 s_2} = \mu$

The integrals are common to all channels

$$\Delta M^l = \int dp \frac{p^2 |A_l(p)|^2}{\epsilon + p^2/2\mu}$$

$$P^l = \int dp \frac{p^2 |A_l(p)|^2}{(\epsilon + p^2/2\mu)^2}$$

The equal mass limit

Total mass shifts

$$\Delta M_{SLJ} = \sum_{s_1 s_2 l} C_{SLJ}^{s_1 s_2 l} \Delta M^l$$
$$= \sum_{l} \Delta M^l \sum_{s_1 s_2} C_{SLJ}^{s_1 s_2 l}$$

Continuum probabilities

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$$\Delta M_{SLJ} = \sum_{s_1 s_2 l} C^{s_1 s_2 l}_{SLJ} \Delta M^l$$
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Continuum probabilities

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The equal mass limit

The coefficients $C_{SLJ}^{s_1 s_2 l}$:

	l	$^3P_0$	$^3P_1$	$^3P_2$	$^1P_1$
$D\overline{D}$	S	$3/4$	0	0	0
$D^*\overline{D}$	S	0	1	0	$1/2$
$D^*\overline{D^*}$	S	$1/4$	0	1	$1/2$
$D\overline{D}$	D	0	0	$3/20$	0
$D^*\overline{D}$	D	0	$1/4$	$9/20$	$1/2$
$D^*\overline{D}^*$	D	1	$3/4$	$2/5$	$1/2$

The equal mass limit

Mass shift and probability are independent of S and J:

$$\Delta M_{SLJ} = \sum_l \Delta M^l$$
$$P_{SLJ} = \sum_l P^l$$

Mass formula after shifts

$$M'_{SLJ} = M' + \Delta_s \langle \tfrac{1}{2}\tfrac{1}{2} \rangle_S + \Delta_t \langle \mathbf{T} \rangle_{SLJ} + \Delta_o \langle \mathbf{L} \cdot \mathbf{S} \rangle_{SLJ},$$

- with a renormalisation of $M' = M - \sum_l \Delta M^l$
- loop theorem (Barnes and Swanson)

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With physical masses

Expanding around $\mu_{s_1 s_2}\epsilon^{s_1 s_2}_{SLJ} = \mu\epsilon(1 + X^{s_1 s_2}_{SLJ})$:

$$\Delta M^{s_1 s_2 l}_{SLJ} = C^{s_1 s_2 l}_{SLJ} \int dp \frac{p^2 |A_l(p)|^2}{\epsilon^{s_1 s_2}_{SLJ} + p^2/2\mu_{s_1 s_2}}$$

$$= C^{s_1 s_2 l}_{SLJ} \frac{\mu_{s_1 s_2}}{\mu} \frac{1}{\epsilon} \sum_{n=0}^{\infty} (-X^{s_1 s_2}_{SLJ})^n \int dp \frac{p^2 |A_l(p)|^2}{(1 + p^2/2\mu\epsilon)^{n+1}}.$$

- Integrals are common to all channels, and
- the first two are ΔM^l and P^l

Model-independent formula for the mass shift

$$\Delta M^{s_1 s_2 l}_{SLJ} \approx C^{s_1 s_2 l}_{SLJ} \frac{\mu_{s_1 s_2}}{\mu} \left(\Delta M^l - X^{s_1 s_2}_{SLJ} \epsilon P^l\right)$$

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With physical masses

The total mass shift

$$\Delta M_{SLJ} = \sum_{s_1 s_2 l} C_{SLJ}^{s_1 s_2 l} \frac{\mu_{s_1 s_2}}{\mu} \left(\Delta M^l - X_{SLJ}^{s_1 s_2} \epsilon P^l \right)$$

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- everything is expressed in terms of ΔM^l and P^l

Mass formula after shifts

$$M'_{SLJ} = M' + \Delta'_s \langle \tfrac{1}{2} \tfrac{1}{2} \rangle_S + \Delta'_t \langle \mathbf{T} \rangle_{SLJ} + \Delta'_o \langle \mathbf{L} \cdot \mathbf{S} \rangle_{SLJ}$$

- With renormalised M', Δ'_s, Δ'_t and Δ'_o

With physical masses

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- With renormalised M', Δ'_s, Δ'_t and Δ'_o

With physical masses

Renormalisation:

$$M' = M - \sum_l \Delta M^l$$

$$\Delta_s' = \Delta_s \left(1 - \sum_l P^l\right)$$

$$\Delta_t' = \Delta_t \left(1 - \sum_l P^l\right)$$

$$\Delta_o' = \Delta_o \left(1 - \sum_l P^l\right) - \sum_l \xi_l \delta \left(\frac{\Delta M^l}{2m} - \left(\frac{\epsilon}{2m} + 1\right) P^l\right)$$

- M' is renormalised as before
- Δ_s' and Δ_t' decrease with P^l
- Δ_o' involves the centre-of-mass m and splitting δ of loop mesons
- $\xi_S = +1/2$ and $\xi_D = -1/4$

With physical masses

Renormalisation:

$$M' = M - \sum_l \Delta M^l$$

$$\Delta_s' = \Delta_s \left(1 - \sum_l P^l\right)$$

$$\Delta_t' = \Delta_t \left(1 - \sum_l P^l\right)$$

$$\Delta_o' = \Delta_o \left(1 - \sum_l P^l\right) - \sum_l \xi_l \delta \left(\frac{\Delta M^l}{2m} - \left(\frac{\epsilon}{2m} + 1\right) P^l\right)$$

- M' is renormalised as before
- Δ_s' and Δ_t' decrease with P^l
- Δ_o' involves the centre-of-mass m and splitting δ of loop mesons
- $\xi_S = +1/2$ and $\xi_D = -1/4$

With physical masses

Renormalisation:

$$M' = M - \sum_l \Delta M^l$$

$$\Delta'_s = \Delta_s \left(1 - \sum_l P^l\right)$$

$$\Delta'_t = \Delta_t \left(1 - \sum_l P^l\right)$$

$$\Delta'_o = \Delta_o \left(1 - \sum_l P^l\right) - \sum_l \xi_l \delta \left(\frac{\Delta M^l}{2m} - \left(\frac{\epsilon}{2m} + 1\right) P^l\right)$$

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With physical masses

A potential model mass formula

$$M'_{SLJ} = M' + \Delta'_s \langle \tfrac{1}{2}\tfrac{1}{2} \rangle_S + \Delta'_t \langle \mathbf{T} \rangle_{SLJ} + \Delta'_o \langle \mathbf{L} \cdot \mathbf{S} \rangle_{SLJ}$$

Therefore

- physical states obey the non-relativistic relation:

$$\frac{1}{9}\left(M_{^3\mathrm{P}_0} + 3M_{^3\mathrm{P}_1} + 5M_{^3\mathrm{P}_2}\right) - M_{^1\mathrm{P}_1} = \Delta'_s \approx 0$$

- large mass shifts can be absorbed into an adjusted potential

Observations

		$\Delta M_{^3P_0}$	$\Delta M_{^3P_1}$	$\Delta M_{^3P_2}$	$\Delta M_{^1P_1}$	Induced H.S.
BS	(1P,$c\bar{c}$)	459	496	521	504	
K	(1P,$c\bar{c}$)	198	215	228	219	
LMC	(1P,$c\bar{c}$)	35	38	63	52	
YLCD	(1P,$c\bar{c}$)	131	152	175	162	
OT	(1P,$c\bar{c}$)	173	180	185	182	
OT	(1P,$b\bar{b}$)	43	44	45	44	
OT	(2P,$b\bar{b}$)	55	56	58	57	
LD	(1P,$b\bar{b}$)	80.777	84.823	87.388	85.785	
LD	(2P,$b\bar{b}$)	73.578	77.608	80.146	78.522	

$\Delta M_{^3P_2} > \Delta M_{^1P_1} > \Delta M_{^3P_1} > \Delta M_{^3P_0}$

Observations

		$\Delta M_{^3P_0}$	$\Delta M_{^3P_1}$	$\Delta M_{^3P_2}$	$\Delta M_{^1P_1}$	Induced H.S.
BS	(1P,$c\bar{c}$)	459	496	521	504	– 1.8
K	(1P,$c\bar{c}$)	198	215	228	219	– 1.3
LMC	(1P,$c\bar{c}$)	35	38	63	52	– 2.9
YLCD	(1P,$c\bar{c}$)	131	152	175	162	– 0.4
OT	(1P,$c\bar{c}$)	173	180	185	182	– 0.0
OT	(1P,$b\bar{b}$)	43	44	45	44	– 0.4
OT	(2P,$b\bar{b}$)	55	56	58	57	– 0.0
LD	(1P,$b\bar{b}$)	80.777	84.823	87.388	85.785	– 0.013
LD	(2P,$b\bar{b}$)	73.578	77.608	80.146	78.522	– 0.048

The induced hyperfine splitting is always negative

Observations

		$\Delta M_{^3P_0}$	$\Delta M_{^3P_1}$	$\Delta M_{^3P_2}$	$\Delta M_{^1P_1}$	Induced H.S.
BS	(1P,$c\bar{c}$)	459	496	521	504	– 1.8
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It works very well for $b\bar{b}$ because $X^{s_1 s_2}_{SLJ}$ is small

Observations

It also works for the D-wave family

$$\frac{1}{15}\left(3M_{^3\mathrm{D}_1} + 5M_{^3\mathrm{D}_2} + 7M_{^3\mathrm{D}_3}\right) - M_{^1\mathrm{D}_2} \approx 0$$

- bottomonia $^3\mathrm{D}_1$, $^3\mathrm{D}_2$ and $^3\mathrm{D}_3$ recently discovered
- prediction $M_{^1\mathrm{D}_2} = 10165.84 \pm 1.8$ MeV
  (TJB, Piccinini, Polosa & Sabelli, PRD 82,074003 (2010))

Everything depends upon the assumptions

- coupling $(Q\overline{Q}) \to (Q\overline{q})(q\overline{Q})$ has $q\overline{q}$ in spin triplet
- spin and spatial degrees of freedom factorise
- the same assumptions are supported by lattice QCD
  (TJB & Close, PRD 74,034003 (2006))