Resonances and their N_C fates in $U(3)$ chiral perturbation theory

ZHI-HUI GUO

Universidad de Murcia & Hebei Normal University

Hadron 2011, 13 June -17 June 2011, Munich, Germany

in collaboration with Jose Oller, based on arXiv:1104.2849[hep-ph]

A work dedicated to Joaquim Prades

Outline

Preface

In the chiral limit $m_u = m_d = m_s = 0$ the QCD Lagrangian is invariant under $U_L(3) \otimes U_R(3)$ symmetry at the classical level.

$U_A(1) \equiv U_{L-R}$: violated at the quantum level, i.e. $U_A(1)$ anomaly, which is also responsible for the massive η_1.

$U_V(1) \equiv U_{L+R}$: conserved baryon number.

$SU_L(3) \otimes SU_R(3) \to SU_V(3)$ is spontaneously broken. Goldstone bosons appear π, K, η_8: $SU(3)$ χPT [Gasser, Leutwyler, NPB'85].

In large N_C limit, $U_A(1)$ anomaly disappears and the η_1 mass vanishes: $M^2_{\eta_1} \sim \mathcal{O}(1/N_C)$. So η_1 together with π, K, η_8 constitute the nonet of pesudo Goldstone bosons.
[t'Hooft, NPB'74] [Witten, NPB'79] [Coleman & Witten, PRL'80]

$U(3)$ χPT takes π, K, η_8 and η_1 as its dynamical degrees of freedom and employs the triple expansion scheme: momentum, quark masses and $1/N_C$, i.e. $\delta \sim p^2 \sim m_q \sim 1/N_C$.

- Set up in: [Witten, PRL'80] [Di Vecchia & Veneziano,'80] [Rosenzweig, Schechter & Trahern, '80]
- Chiral Lagrangian to $\mathcal{O}(p^4)$ completed in: [Herrera-Siklody, Latorre, Pascual, Taron, NPB'97] . See also [Kaiser, Leutwyler, EPJC'00] .
- Applications
  Light quark masses: [Leutwyler, PLB'96]
  $\eta - \eta'$ mixing: [Herrera-Siklody, Latorre, Pascual, Taron, PLB'98] [Leutwyler, NPB(Proc.Suppl)'98]
  $\eta' \to \eta\pi\pi$ decay: [Escribano,Masjuan, Sanz-Cillero, JHEP'11]

- Our current work offers the complete one-loop amplitudes of the meson-meson scattering within $U(3)$ χPT.

  And then we study the properties of various resonances, such as their pole positions, residues and N_C behaviour, by unitarizing the $U(3)$ χPT amplitudes.

There are variant methods to treat η' in the market

- Matter filed: $M^2_{\eta'} \sim \mathcal{O}(1)$ and Infrared Regularization method used to handle the loops. [Beisert, Borasoy, NPA'02, PRD'03]
- Non-relativistic field
  [Kubis, Schneider, EPJC'09]

Relevant Chiral Lagrangian

$$\mathcal{L}^{(\delta^0)} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{3} M_0^2 \ln^2 \det u \,, \qquad (1)$$

where

$$u = e^{i\frac{\Phi}{\sqrt{2}F}} \;,\; U = u^2 \;,$$
$$u_\mu = i u^\dagger D_\mu U u^\dagger = u_\mu^\dagger \;,\; \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \,,$$

$$\Phi = \begin{pmatrix} \frac{\sqrt{3}\pi^0+\eta_8+\sqrt{2}\eta_1}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\sqrt{3}\pi^0+\eta_8+\sqrt{2}\eta_1}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8+\sqrt{2}\eta_1}{\sqrt{6}} \end{pmatrix} . \qquad (2)$$

L_is correspond to the higher order local operators.
At $\mathcal{O}(\delta)$ one has $\mathcal{O}(N_C p^4)$ and $\mathcal{O}(N_C^0 p^2)$ operators:

$$\mathcal{L}^{(\delta)} = L_2\langle u_\mu u_\nu u^\mu u^\nu\rangle + (2L_2 + L_3)\langle u_\mu u^\mu u_\nu u^\nu\rangle$$
$$+ L_5\langle u_\mu u^\mu \chi_+\rangle + L_8/2\langle \chi_+\chi_+ + \chi_-\chi_-\rangle + \ldots$$
$$+ F^2\Lambda_1/12\, D_\mu\psi D^\mu\psi - i\,F^2\Lambda_2/12\;\psi\langle U^\dagger\chi - \chi^\dagger U\rangle + \ldots$$

At $\mathcal{O}(\delta^2)$ (same order as the one-loop contribution), one then has $\mathcal{O}(N_C^{-2}p^0)$, $\mathcal{O}(N_C^{-1}p^2)$, $\mathcal{O}(N_C^0 p^4)$ and $\mathcal{O}(N_C p^6)$ operators:

$$\mathcal{L}^{(\delta^2)} = \tilde{v}_0^{(4)}X^4 + \tilde{v}_1^{(2)}X^2\langle u_\mu u^\mu\rangle + L_4\langle u_\mu u^\mu\rangle\langle\chi_+\rangle$$
$$+C_1\langle u_\rho u^\rho h_{\mu\nu}h^{\mu\nu}\rangle + \ldots\,,$$

with $\psi = -i\ln\det U$, $X = \log\det(U)$ and $h_{\mu\nu} = \nabla_\mu u_\nu + \nabla_\nu u_\mu$.
[Herrera-Siklody, Latorre, Pascual, Taron, NPB'97]
[Bijnens, Colangelo, Ecker, JHEP'99]

Alternatively, one could use resonances to estimate the higher order low energy constants:

$$\begin{aligned} \mathcal{L}_S &= c_d \langle S_8 u_\mu u^\mu \rangle + c_m \langle S_8 \chi_+ \rangle \\ &\quad + \widetilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \widetilde{c}_m S_1 \langle \chi_+ \rangle + \ldots \end{aligned} \tag{3}$$

$$\mathcal{L}_V = \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle + \ldots , \tag{4}$$

[Ecker, Gasser, Pich, de Rafael, NPB'89]

In the current discussion, we assume the resonance saturation and exploit the above resonance operators to calculate the meson-meson scattering.

The monomials proportional to Λ_1 and Λ_2 are not generated through resonance exchange. No double counting.

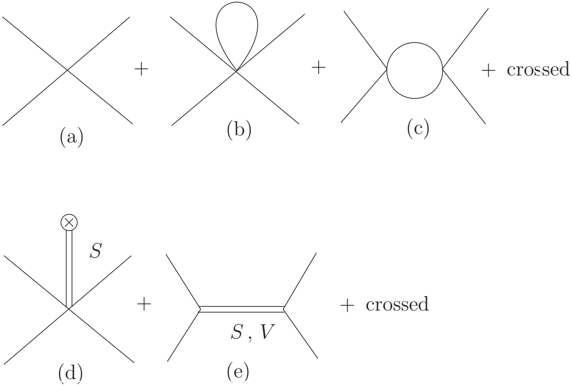
Perturbative calculation of the scattering amplitudes

Figure: Relevant Feynman diagrams for mass, wave function renormalization and $\eta - \eta'$ mixing

The leading order η-η' mixing has to be solved exactly

Figure: The dot denotes the mixing of η_8 and η_1 at leading order, which is proportional to $m_K^2 - m_\pi^2$.

Scattering amplitudes consist of



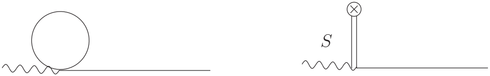


Figure: Relevant Feynman diagrams for the pseudo Goldstone decay constant. The wiggly line corresponds to the axial-vector external source.

We expressed all the amplitudes in terms of physical masses and F_π, i.e. reshuffling the leading order contributions.

Partial wave amplitude and its unitarization

Partial wave projection:

$$T_J^I(s) = \frac{1}{2(\sqrt{2})^N} \int_{-1}^{1} dx\, P_J(x)\, T^I[s, t(x), u(x)]\,, \qquad (5)$$

where $P_J(x)$ denote the Legendre polynomials and $(\sqrt{2})^N$ is a symmetry factor to account for the identical particles, such as $\pi\pi, \eta\eta, \eta'\eta'$.

The essential of the N/D method is to construct the unitarized T_J: [Chew, Mandelstam, PR'60]

$$T_J = \frac{N}{D}\,, \tag{6}$$

where

$$\begin{aligned} \mathrm{Im}D &= N\,\mathrm{Im}\,T_J = -\rho N\,, \quad \text{for } s > 4m^2\,, \\ \mathrm{Im}D &= 0\,, \quad \text{for } s < 4m^2\,, \\ \mathrm{Im}N &= D\,\mathrm{Im}\,T_J\,, \quad \text{for } s < 0\,, \\ \mathrm{Im}N &= 0\,, \quad \text{for } s > 0\,, \end{aligned} \tag{7}$$

due to the fact that the unitarity condition for the elastic channel is

$$\mathrm{Im}\,T_J^{-1} = -\rho\,, \quad s > 4m^2 \tag{8}$$

where $\rho = \sqrt{1-4m^2/s}/16\pi$.

One can now write the dispersion relations for N and D:

$$D(s) = \widetilde{a}^{SL}(s_0) - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{N(s')\ \rho(s')}{(s' - s)(s' - s_0)} ds' + \dots , \tag{9}$$

$$N(s) = \int_{-\infty}^{0} \frac{D(s')\ \mathrm{Im}\, T_J(s')}{s' - s} ds' . \tag{10}$$

It can be greatly simplified if one imposes the perturbative solution for $N(s)$ instead of the left hand discontinuity [Oller, Oset, PRD'99],

$$T_J(s) = \frac{N(s)}{1 + g(s)\ N(s)} , \tag{11}$$

where

$$g(s) = \frac{a^{SL}(s_0)}{16\pi^2} - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s' - s)(s' - s_0)} ds' . \tag{12}$$

Matching the $T_J(s) = N(s)/\left[1 + g(s)\ N(s)\right]$ with $T_J(s)|_{\chi PT} = T_2 + T_{Resonance} + T_{Loop}$ up to one-loop:

$$N(s) = T_2 + T_{Resonance} + T_{Loop} + T_2\ g(s)\ T_2\,. \tag{13}$$

The generalization to the inelastic case is straightforward:

$$T_J(s) = N(s) \cdot [1 + g(s) \cdot N(s)]^{-1}\,. \tag{14}$$

This formalism has been explored in many areas. See in this conference the talks already done by Alarcon, F.K.Guo, Magalaes, Molina, Oset.

For $IJ = 00$ case, we have 5 channels: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ and $\eta'\eta'$

$$N_0^0(s) = \begin{pmatrix} N_{\pi\pi\to\pi\pi} & N_{\pi\pi\to K\bar{K}} & N_{\pi\pi\to\eta\eta} & N_{\pi\pi\to\eta\eta'} & N_{\pi\pi\to\eta'\eta'} \\ N_{\pi\pi\to K\bar{K}} & N_{K\bar{K}\to K\bar{K}} & N_{K\bar{K}\to\eta\eta} & N_{K\bar{K}\to\eta\eta'} & N_{K\bar{K}\to\eta'\eta'} \\ N_{\pi\pi\to\eta\eta} & N_{K\bar{K}\to\eta\eta} & N_{\eta\eta\to\eta\eta} & N_{\eta\eta\to\eta\eta'} & N_{\eta\eta\to\eta'\eta'} \\ N_{\pi\pi\to\eta\eta'} & N_{K\bar{K}\to\eta\eta'} & N_{\eta\eta\to\eta\eta'} & N_{\eta\eta'\to\eta\eta'} & N_{\eta\eta'\to\eta'\eta'} \\ N_{\pi\pi\to\eta'\eta'} & N_{K\bar{K}\eta'\eta'} & N_{\eta\eta\to\eta'\eta'} & N_{\eta\eta'\to\eta'\eta'} & N_{\eta'\eta'\to\eta'\eta'} \end{pmatrix}$$

$$g_0^0(s) = \begin{pmatrix} g_{\pi\pi} & 0 & 0 & 0 & 0 \\ 0 & g_{K\bar{K}} & 0 & 0 & 0 \\ 0 & 0 & g_{\eta\eta} & 0 & 0 \\ 0 & 0 & 0 & g_{\eta\eta'} & 0 \\ 0 & 0 & 0 & 0 & g_{\eta'\eta'} \end{pmatrix} .$$

For $IJ = 1\,0$, we have 3 channels: $\pi\eta$, $K\bar{K}$ and $\pi\eta'$

$$N(s)^1_0 = \begin{pmatrix} N_{\pi\eta\to\pi\eta} & N_{\pi\eta\to K\bar{K}} & N_{\pi\eta\to\pi\eta'} \\ N_{\pi\eta\to K\bar{K}} & N_{K\bar{K}\to K\bar{K}} & N_{K\bar{K}\to\pi\eta'} \\ N_{\pi\eta\to\pi\eta'} & N_{K\bar{K}\to\pi\eta'} & N_{\pi\eta'\to\pi\eta'} \end{pmatrix},$$

$$g(s)^1_0 = \begin{pmatrix} g_{\pi\eta} & 0 & 0 \\ 0 & g_{K\bar{K}} & 0 \\ 0 & 0 & g_{\pi\eta'} \end{pmatrix}.$$

For $IJ = 1/2\ \ 0$, there are three channels: $K\pi$, $K\eta$ and $K\eta'$

$$N(s)_0^{1/2} = \begin{pmatrix} N_{K\pi\to K\pi} & N_{K\pi\to K\eta} & N_{K\pi\to K\eta'} \\ N_{K\pi\to K\eta} & N_{K\eta\to K\eta} & N_{K\eta\to K\eta'} \\ N_{K\pi\to K\eta'} & N_{K\eta\to K\eta'} & N_{K\eta'\to K\eta'} \end{pmatrix},$$

$$g(s)_0^{1/2} = \begin{pmatrix} g_{K\pi} & 0 & 0 \\ 0 & g_{K\eta} & 0 \\ 0 & 0 & g_{K\eta'} \end{pmatrix}.$$

The same expressions hold for $IJ = 1/2\ \ 1$.

For $IJ = 1\ 1$ there are 2 channels

$$N_1^1(s) = \left(\begin{array}{cc} N_{\pi\pi\to\pi\pi} & N_{\pi\pi\to K\bar{K}} \\ N_{\pi\pi\to K\bar{K}} & N_{K\bar{K}\to K\bar{K}} \end{array} \right) ,$$

$$g_1^1(s) = \left(\begin{array}{cc} g_{\pi\pi} & 0 \\ 0 & g_{K\bar{K}} \end{array} \right) .$$

For $IJ = 3/2\ 0$, it is an elastic channel

$$N(s)_0^{3/2} = N_{K\pi\to K\pi} ,$$

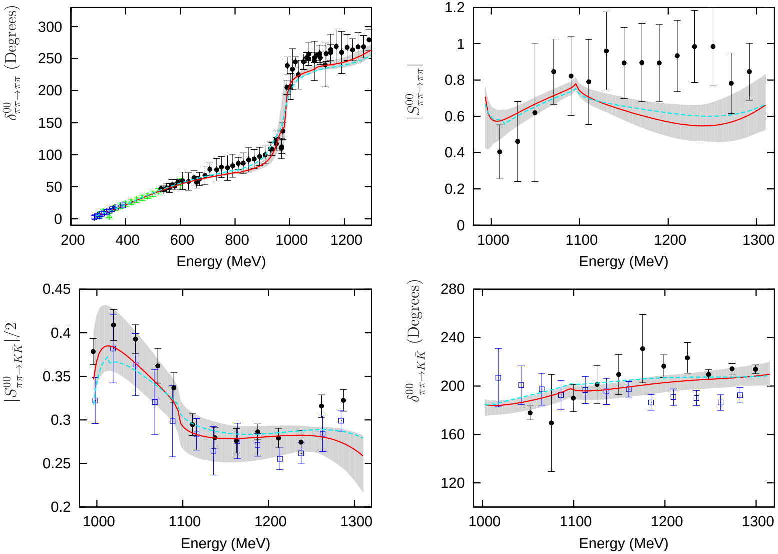
$$g(s)_0^{3/2} = g_{K\pi} .$$

For $IJ = 2\ 0$, it is

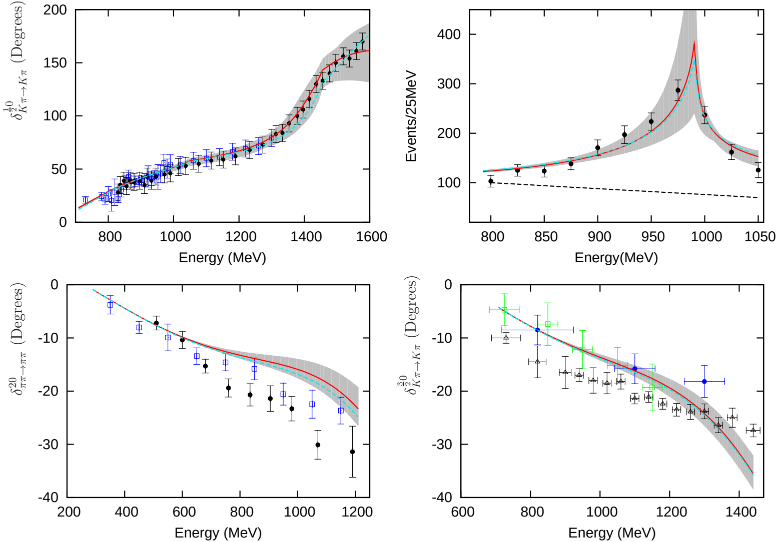
$$N(s)_0^2 = N_{\pi\pi\to\pi\pi} ,$$

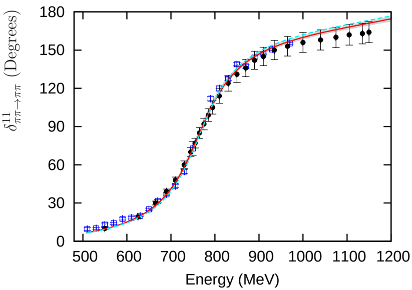
$$g(s)_0^2 = g_{\pi\pi} .$$

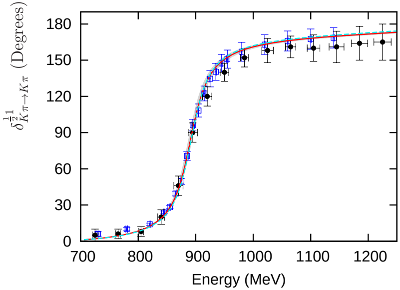
Phenomenological discussion



$\delta^{00}_{\pi\pi\to\pi\pi}$ (Degrees)
300
250
200
150
100
50
0
200
400
600
800
1000
1200
Energy (MeV)
$|S^{00}_{\pi\pi\to\pi\pi}|$
1.2
1
0.8
0.6
0.4
0.2
0
1000
1100
1200
1300
Energy (MeV)
$|S^{00}_{\pi\pi\to K\bar{K}}|/2$
0.45
0.4
0.35
0.3
0.25
0.2
1000
1100
1200
1300
Energy (MeV)
$\delta^{00}_{\pi\pi\to K\bar{K}}$ (Degrees)
280
240
200
160
120
1000
1100
1200
1300
Energy (MeV)


$\delta^{\frac{1}{2}0}_{K\pi\to K\pi}$ (Degrees)
Energy (MeV)
Events/25MeV
Energy(MeV)
$\delta^{20}_{\pi\pi\to\pi\pi}$ (Degrees)
Energy (MeV)
$\delta^{\frac{3}{2}0}_{K\pi\to K\pi}$ (Degrees)
Energy (MeV)


$\delta^{11}_{\pi\pi\to\pi\pi}$ (Degrees)
180
150
120
90
60
30
0
500 600 700 800 900 1000 1100 1200
Energy (MeV)


$\delta^{\frac{1}{2}1}_{K\pi\to K\pi}$ (Degrees)
180
150
120
90
60
30
0
700 800 900 1000 1100 1200
Energy (MeV)

We have 16 free parameters with 348 data and the fitted results are

$$
\begin{aligned}
c_d &= (15.6^{+4.2}_{-3.4})\,\mathrm{MeV}\,, & c_m &= (31.5^{+19.5}_{-22.5})\,\mathrm{MeV}\,,\\
\widetilde{c}_d &= (8.7^{+2.5}_{-1.7})\,\mathrm{MeV}\,, & \widetilde{c}_m &= (15.8^{+3.3}_{-3.0})\,\mathrm{MeV}\,,\\
M_{S_8} &= (1370^{+132}_{-57})\,\mathrm{MeV}\,, & M_{S_1} &= (1063^{+53}_{-31})\,\mathrm{MeV}\,,\\
M_\rho &= (801.0^{+7.0}_{-7.5})\,\mathrm{MeV}\,, & M_{K^*} &= (909.0^{+7.5}_{-6.9})\,\mathrm{MeV}\,,\\
G_V &= (61.9^{+1.9}_{-1.9})\,\mathrm{MeV}\,, & a_{SL}^{1\,0\,,\pi\eta} &= 2.0^{+3.1}_{-3.4}\,,\\
a_{SL}^{00} &= (-1.15^{+0.07}_{-0.09})\,, & a_{SL}^{\frac{1}{2}\,0} &= (-0.96^{+0.10}_{-0.16})\,,\\
\mathcal{N} &= (0.6^{+0.3}_{-0.3})\,\mathrm{MeV}^{-2}\,, & c &= (1.0^{+0.6}_{-0.4})\,,\\
M_0 &= (954^{+102}_{-95})\,\mathrm{MeV}\,, & \Lambda_2 &= (-0.6^{+0.5}_{-0.4})\,,
\end{aligned}
$$

with $\chi^2/\mathrm{d.o.f} = 714/(348-16) \simeq 2.15$.
$n_\sigma = \Delta\chi^2/\sqrt{2\chi^2} \leq 2$ to get the errors, $n_\sigma = 2$ Etkin *et al.* PRD'82

Poles from the unitarized amplitudes

- σ or $f_0(600)$, $IJ = 0\,0$

$$M_\sigma = 440^{+3}_{-3}\,\mathrm{MeV}\,, \quad \Gamma_\sigma/2 = 258^{+5}_{-7}\,\mathrm{MeV}\,,$$
$$|g_{\sigma\pi\pi}| = 3.02^{+0.03}_{-0.03}\,\mathrm{GeV}\,,$$
$$|g_{\sigma K\bar{K}}|/|g_{\sigma\pi\pi}| = 0.51^{+0.03}_{-0.02}\,, \; |g_{\sigma\eta\eta}|/|g_{\sigma\pi\pi}| = 0.06^{+0.03}_{-0.01}$$
$$|g_{\sigma\eta\eta'}|/|g_{\sigma\pi\pi}| = 0.16^{+0.03}_{-0.02}\,, \; |g_{\sigma\eta'\eta'}|/|g_{\sigma\pi\pi}| = 0.05^{+0.03}_{-0.03}$$

Other approaches:
$M_\sigma = 470 \pm 50$, $\Gamma_\sigma/2 = 285 \pm 25$ Zhou, *et al.* JHEP'05
$M_\sigma = 441^{+16}_{-8}$, $\Gamma_\sigma/2 = 272^{+9}_{-13}$ Caprini *et al.* PRL'06
$M_\sigma = 484 \pm 17$, $\Gamma_\sigma/2 = 255 \pm 10$ García-Martín *et al.* PRD'07
$M_\sigma = 456 \pm 6$, $\Gamma_\sigma/2 = 241 \pm 17$ Albaladejo, Oller PRL'08

- $f_0(980)$, $IJ = 00$

$$M_{f_0} = 981^{+9}_{-7}\,\mathrm{MeV}\,, \quad \Gamma_{f_0}/2 = 22^{+5}_{-7}\,\mathrm{MeV}\,,$$
$$|g_{f_0\pi\pi}| = 1.7^{+0.3}_{-0.3}\,\mathrm{GeV}$$
$$|g_{f_0K\bar{K}}|/|g_{f_0\pi\pi}| = 2.3^{+0.3}_{-0.2}\,,\ |g_{f_0\eta\eta}|/|g_{f_0\pi\pi}| = 1.6^{+0.3}_{-0.3}$$
$$|g_{f_0\eta\eta'}|/|g_{f_0\pi\pi}| = 1.2^{+0.1}_{-0.2}\,,\ |g_{f_0\eta'\eta'}|/|g_{f_0\pi\pi}| = 0.7^{+0.4}_{-0.5}$$

- $f_0(1370)$, $IJ = 00$

$$M_{f_0} = 1401^{+58}_{-37}\,\mathrm{MeV}\,, \quad \Gamma_{f_0}/2 = 106^{+36}_{-23}\,\mathrm{MeV}\,,$$
$$|g_{f_0\pi\pi}| = 2.4^{+0.2}_{-0.1}\,\mathrm{GeV}$$
$$|g_{f_0K\bar{K}}|/|g_{f_0\pi\pi}| = 0.62^{+0.04}_{-0.05}\,,\ |g_{f_0\eta\eta}|/|g_{f_0\pi\pi}| = 0.9^{+0.1}_{-0.1}$$
$$|g_{f_0\eta\eta'}|/|g_{f_0\pi\pi}| = 1.7^{+0.4}_{-0.6}\,,\ |g_{f_0\eta'\eta'}|/|g_{f_0\pi\pi}| = 1.1^{+0.4}_{-0.5}$$

Both resonances have strong couplings to states with η, η'

- κ or $K_0^*(800)$, $IJ = 1/2\,0$

$$M_\kappa = 665^{+9}_{-9}\,\mathrm{MeV}\,, \quad \Gamma_\kappa/2 = 268^{+21}_{-6}\,\mathrm{MeV}\,,$$
$$|g_{\kappa K\pi}| = 4.2^{+0.2}_{-0.2}\,\mathrm{GeV}$$
$$|g_{\kappa K\eta}|/|g_{\kappa K\pi}| = 0.7^{+0.1}_{-0.1}\,,\ |g_{\kappa K\eta'}|/|g_{\kappa K\pi}| = 0.50^{+0.1}_{-0.1}$$

Other approaches:
$\sqrt{s} = (594 \pm 79 - i\,362 \pm 166)$ MeV Zheng, *et al.* NPA'04
$\sqrt{s} = (658 \pm 13 - i\,278 \pm 12)$ MeV Descotes, Moussallam EPJC'06

- $K_0^*(1430)$, $IJ = 1/2\,0$

$$M_{K_0^*} = 1428^{+56}_{-23}\,\mathrm{MeV}\,, \quad \Gamma_{K_0^*}/2 = 87^{+53}_{-28}\,\mathrm{MeV}\,,$$
$$|g_{K_0^* K\pi}| = 3.3^{+0.5}_{-0.4}\,\mathrm{GeV}$$
$$|g_{K_0^* K\eta}|/|g_{K_0^* K\pi}| = 0.54^{+0.07}_{-0.02}\,,\ |g_{K_0^* K\eta'}|/|g_{K_0^* K\pi}| = 1.2^{+0.2}_{-0.3}$$

- $a_0(980)$, $IJ = 1\,0$

$$M_{a_0} = 1012^{+25}_{-7}\,\text{MeV}\,,\ \Gamma_{a_0}/2 = 16^{+50}_{-13}\,\text{MeV}\,,$$
$$|g_{a_0\pi\eta}| = 2.5^{+1.3}_{-0.8}\,\text{GeV}$$
$$|g_{a_0K\bar{K}}|/|g_{a_0\pi\eta}| = 1.9^{+0.2}_{-0.3}\ ,\ \ |g_{a_0\pi\eta'}|/|g_{a_0\pi\eta}| = 0.01^{+0.03}_{-0.01}$$

- $a_0(1450)$, $IJ = 1\,0$

$$M_{a_0} = 1368^{+68}_{-68}\,\text{MeV}\,,\ \Gamma_{a_0}/2 = 71^{+48}_{-23}\,\text{MeV}\,,$$
$$|g_{a_0\pi\eta}| = 2.3^{+0.4}_{-0.5}\,\text{GeV}$$
$$|g_{a_0K\bar{K}}|/|g_{a_0\pi\eta}| = 0.6^{+0.7}_{-0.2}\ ,\ \ |g_{a_0\pi\eta'}|/|g_{a_0\pi\eta}| = 0.6^{+0.2}_{-0.1}$$

- $\rho(770)$, $IJ = 1\,1$

$$M_\rho = 762^{+4}_{-4}\,\mathrm{MeV}\,,\quad \Gamma_\rho/2 = 72^{+2}_{-2}\,\mathrm{MeV}\,,$$
$$|g_{\rho\,\pi\pi}| = 2.48^{+0.03}_{-0.05}\,\mathrm{GeV}\;,\; |g_{\rho K\bar{K}}|/|g_{\rho\pi\pi}| = 0.64^{+0.01}_{-0.01}$$

- $K^*(892)$, $IJ = 1/2\;1$

$$M_{K^*} = 891^{+3}_{-4}\,\mathrm{MeV}\,,\quad \Gamma_{K^*}/2 = 25^{+2}_{-1}\,\mathrm{MeV}\,,$$
$$|g_{K^*\,\pi K}| = 1.86^{+0.05}_{-0.05}\,\mathrm{GeV}$$
$$|g_{K^*K\eta}|/|g_{K^*K\pi}| = 0.91^{+0.03}_{-0.02}\;,\; |g_{K^*K\eta'}|/|g_{K^*K\pi}| = 0.45^{+0.08}_{-0.08}$$

- $\phi(1020)$, $IJ = 0\;1$

$$M_\phi = 1019.5^{+0.3}_{-0.3}\,\mathrm{MeV}\,,\quad \Gamma_\phi/2 = 2.00^{+0.04}_{-0.08}\,\mathrm{MeV}\,,$$
$$|g_{\phi\,K\bar{K}}| = 0.85^{+0.01}_{-0.02}\,\mathrm{GeV}$$

Running of pole positions with N_C

For the first time the N_C dependence of the pseudo-Goldstone masses and mixing angle are taken into account for determining resonance properties with increasing N_C.

In $SU(3)$ χPT, there is one mixing ingredient for the large N_C limit: the singlet η_1.
The leading order behaviours of the parameters at large N_C are

$$M_0^2 \sim \Lambda_2 \sim 1/N_c$$
$$c_d \sim c_m \sim \widetilde{c}_d \sim \widetilde{c}_m \sim G_V \sim F \sim \sqrt{N_c}$$
$$M_V^2 \sim M_{S_8}^2 \sim M_{S_1}^2 \sim B \sim a_{SL} \sim \mathcal{O}(N_c^0)$$

with $\overline{m}_\pi^2 = 2Bm_u \,, \overline{m}_K^2 = B(m_u + m_s)$.
[Ecker, *et al.*, NPB'89] [Kaiser,Leutwyler, EPJC'00]

The next-to-leading order of $1/N_C$ running can be read out from our prediction for F_π

$$F_\pi = F\left\{1 + \frac{1}{16\pi^2 F_\pi^2}\left[A_0(m_\pi^2) + \frac{1}{2}A_0(m_K^2)\right] + \left[\frac{4\widetilde{c}_d\,\widetilde{c}_m(m_\pi^2 + 2m_K^2)}{F_\pi^2 M_{S_1}^2} - \frac{8c_d\,c_m\,(m_K^2 - m_\pi^2)}{3F_\pi^2 M_{S_8}^2}\right]\right\}.$$

In addition we also take the following assumptions for the next-to-leading order of $1/N_C$ pieces for the other resonance couplings

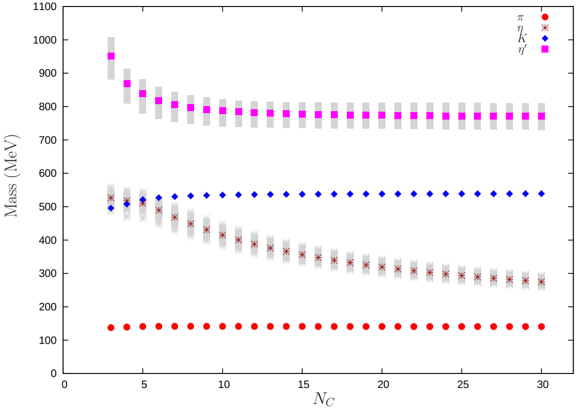
$$c_d(N_C) = c_d(N_C = 3)\frac{F_\pi(N_C)}{F_\pi(N_C = 3)}\,,$$

similar expressions also apply for $c_m\,, \widetilde{c}_d\,, \widetilde{c}_m\,, G_V$ due to the high energy constraint from QCD

$$c_d = c_m = \sqrt{3}\widetilde{c}_d = \sqrt{3}\widetilde{c}_m = \frac{F_\pi}{2}\,, \quad G_V = \frac{F_\pi}{\sqrt{2}} \text{ or } \frac{F_\pi}{\sqrt{3}}\,.$$

[Ecker, *et al.*, PLB'89] [Jamin, *et al.*, NPB'00] [Guo, *et al.*, JHEP'07]

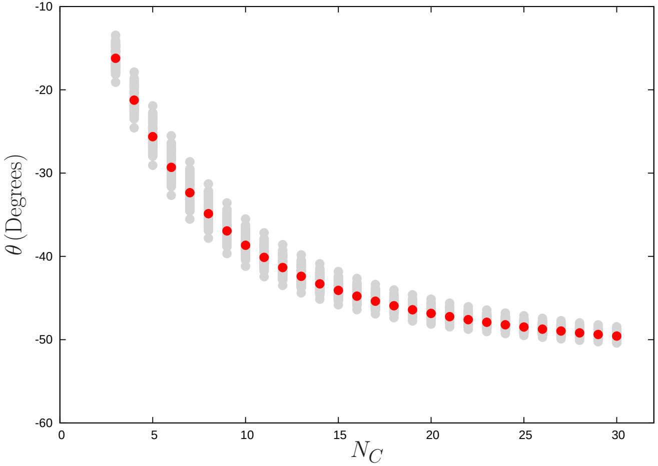
Pseudoscalar masses with varying N_C



Leading order $1/N_c \to \infty$ prediction ($M_0 \to 0$):

$$m_\eta^2 = \overline{m}_\pi^2 = (139.5^{+4.4}_{-4.6})^2 \text{ MeV}^2 \ , \ m_{\eta'}^2 = 2\overline{m}_K^2 - \overline{m}_\pi^2 = (721.5^{+17.4}_{-11.1})^2 \text{ MeV}^2 \ .$$

Ideal Mixing (OZI rule is exact): leading order mixing angle $\theta = -54.7°$

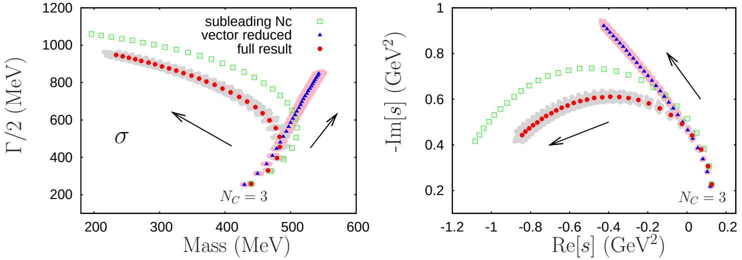


Two approximations of our full results are studied for the resonance poles

- *vector reduced* :

$$\frac{1}{M_V^2 - t} \to \frac{1}{M_V^2} ,$$

  We only includes the NLO local terms in χPT in this scheme.
- *Mimic SU(3)* : Mixing is set to zero and η_1 is kept in the loops. π, K, η_8, η_1 masses are frozen. Differences highlight the role of η and η'.



- The results from one-loop inverse amplitude (IAM) are quite similar with the *vector reduced* case. [Pelaez, '04][Sun, *et al.* '07][Ruiz-Arriolla, Nieves, '09]
- Two-loop($SU(2)$) IAM shows a quite different picture: σ moves to a pole with zero width at 1 GeV. [Pelaez, Rios, '06][Sun, *et al.* '07] We also obtain such a pole but it comes from the bare scalar singlet $M_{S_1} \simeq 1$ GeV (At $N_C = 3$ it contributes to the $f_0(980)$.)

A short summary of our finding for σ:

- The one-loop IAM study reflects a specific approximation of our full result: *vector reduced*. Whereas the *scalar reduced* approximation perfectly agrees with the full result.
- The *mimic SU(3)* approximation turns out to be quite similar to the full result of the σ trajectory, indicating σ is insensitive to η and η' even for large N_C.
- The possible source of the disagreement of our result and the two-loop IAM is the higher order local terms, because much more resonance operators will be involved to produce the $\mathcal{O}(p^6)$ LECs.
  [Cirigliano, *et al.*, NPB'06]

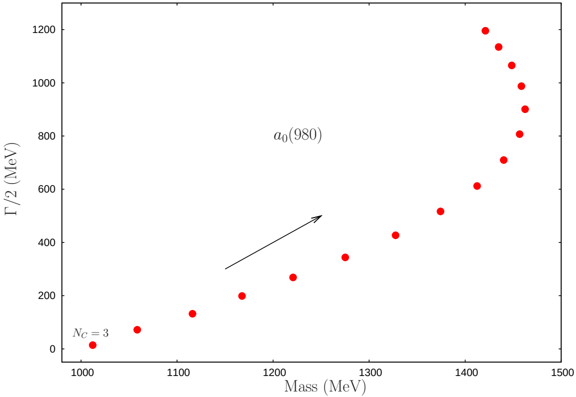


Figure: N_C trajectory for $a_0(980)$

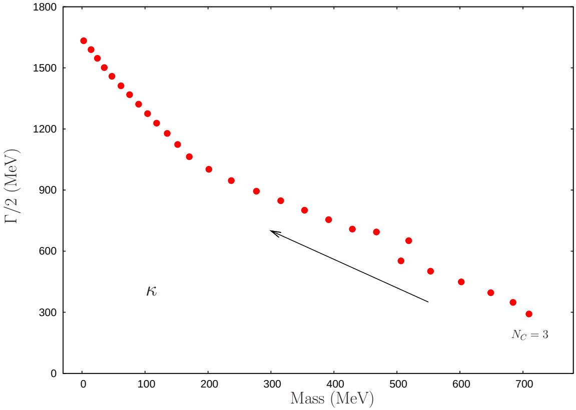
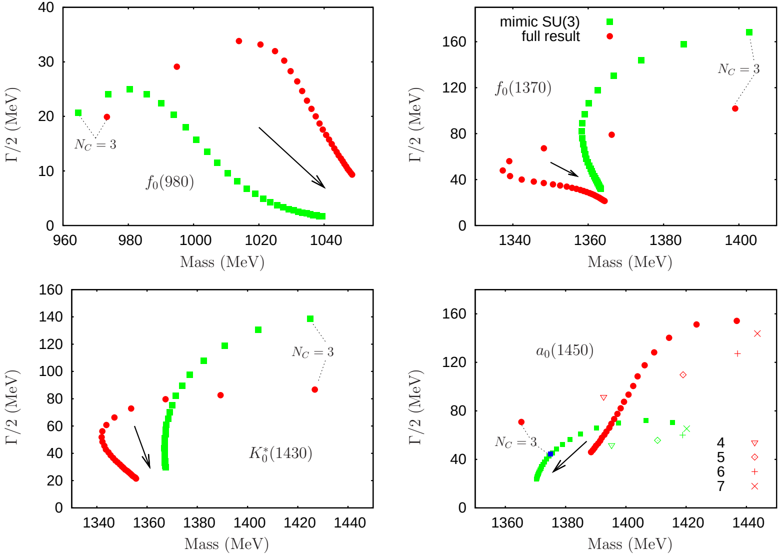


Figure: N_C trajectory for κ



40
30
20
10
0
960
980
1000
1020
1040
Mass (MeV)
Γ/2 (MeV)
$N_C = 3$
$f_0(980)$
mimic SU(3)
full result
160
120
80
40
0
1340
1360
1380
1400
$f_0(1370)$
$N_C = 3$
160
140
120
100
80
60
40
20
0
1340
1360
1380
1400
1420
1440
$N_C = 3$
$K_0^*(1430)$
160
120
80
40
0
1360
1380
1400
1420
1440
$a_0(1450)$
$N_C = 3$
4
5
6
7

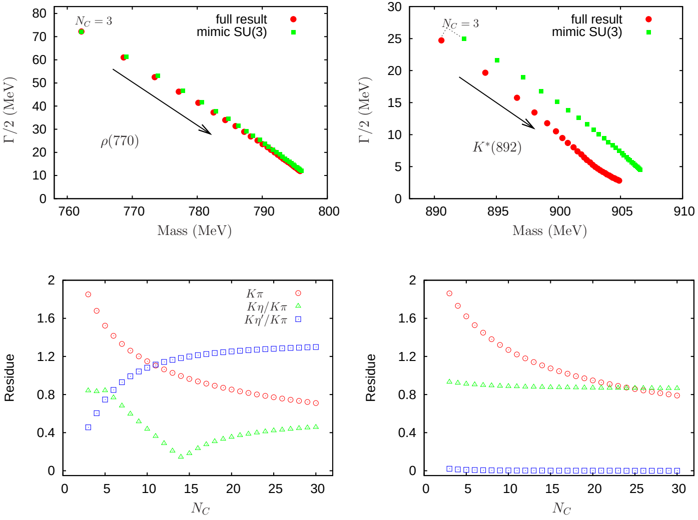


Figure: N_C running of the residues for K^*

Conclusions

- A complete one-loop calculation of all meson-meson scattering amplitudes within $U(3)$ χPT has been worked out for the first time in literature.
- A variant N/D method has been employed to resum the *s*-channel loops. Various resonance poles in the complex plane and their residues have been calculated.
- N_C dependence of the resonance pole positions and the residuals, are studied, also for the first time in literature, by taking into account the N_C running of the pseudo-Goldstone masses and the $\eta - \eta'$ mixing angle.

Danke !

$$\bar{\eta} = \cos\theta\,\eta_8 - \sin\theta\,\eta_1\,,$$
$$\bar{\eta}' = \sin\theta\,\eta_8 + \cos\theta\,\eta_1\,,$$

$$m_{\bar{\eta}}^2 = \frac{M_0^2}{2} + \overline{m}_K^2 - \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^2}}{2}\,,$$
$$m_{\bar{\eta}'}^2 = \frac{M_0^2}{2} + \overline{m}_K^2 + \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^2}}{2}\,,$$

$$\sin\theta = -1/\sqrt{1 + \left(3M_0^2 - 2\Delta^2 + \sqrt{9M_0^4 - 12M_0^2\Delta^2 + 36\Delta^4}\,\right)^2/32\Delta^4}$$

$\Delta^2 = \overline{m}_K^2 - \overline{m}_\pi^2$, $\sin\theta \to 0$ for $\Delta^2 \to 0$, i.e. in $SU(3)$ limit.

The NLO $\bar{\eta}$-$\bar{\eta}'$ mixing can be treated perturbatively

$$\mathcal{L} = \frac{1+\delta_{\overline{\eta}}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\delta_{\overline{\eta}'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \delta_k\,\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' - \frac{m^2_{\overline{\eta}}+\delta_{m^2_{\overline{\eta}}}}{2}\overline{\eta}\,\overline{\eta} - \frac{m^2_{\overline{\eta}'}+\delta_{m^2_{\overline{\eta}'}}}{2}\overline{\eta}'\overline{\eta}' - \delta_{m^2}\,\overline{\eta}\,\overline{\eta}'\,.$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_\delta & -\sin\theta_\delta \\ \sin\theta_\delta & \cos\theta_\delta \end{pmatrix} \begin{pmatrix} 1+\frac{\delta_{\overline{\eta}}}{2} & \frac{\delta_k}{2} \\ \frac{\delta_k}{2} & 1+\frac{\delta_{\overline{\eta}'}}{2} \end{pmatrix} \begin{pmatrix} \overline{\eta} \\ \overline{\eta}' \end{pmatrix}\,.$$

Observables fitted:

- $I = J = 0$: $\delta^{00}_{\pi\pi\to\pi\pi}$, $|S^{00}_{\pi\pi\to\pi\pi}|$, $\frac{1}{2}|S^{00}_{\pi\pi\to K\bar{K}}|$, $\delta^{00}_{\pi\pi\to K\bar{K}}$
- $I = J = 1$: $\delta^{11}_{\pi\pi\to\pi\pi}$
- $I = 1/2\ J = 0\ ,1$: $\delta^{\frac{1}{2}0}_{\pi K\to\pi K}$, $\delta^{\frac{1}{2}1}_{\pi K\to\pi K}$
- $I = 2\ J = 0$: $\delta^{20}_{\pi\pi\to\pi\pi}$
- $I = 3/2\ J = 0$: $\delta^{\frac{3}{2}0}_{\pi K\to\pi K}$
- $I = 1\ J = 0$: $\pi\eta$ event distribution around $a_0(980)$

$$\frac{dN_{\pi\eta}}{dE_{\pi\eta}} = q_{\pi\eta}\,\mathcal{N}\Big|\ T_{K\bar{K}\to\pi\eta}(s) + c\ T_{\pi\eta\to\pi\eta}(s)\ \Big|^2 .$$

- $m_\eta,\ m_{\eta'}$

Subtraction Constants: The number of free ones can be reduced enormously by applying Isospin and U(3) symmetry.
Jido,Oller,Oset,Ramos,Meißner, NPA'03

- Isospin Symmetry requires that all the a_{SL}^{IJ} are the same separately for $\pi\pi$, $K\bar{K}$ and $K\pi$
- U(3) Symmetry requires that all a_{SL}^{IJ} are the same for a given J

$$a_{SL}^{00} = a_{SL}^{00\,,\,\pi\pi} = a_{SL}^{00\,,\,K\bar{K}} = a_{SL}^{00\,,\,\eta\eta} = a_{SL}^{00\,,\,\eta\eta'} = a_{SL}^{00\,,\,\eta'\eta'} = a_{SL}^{20\,,\,\pi\pi}$$
$$= a_{SL}^{1\,0\,,\,\pi\eta'} = a_{SL}^{1\,0\,,\,K\bar{K}}\,,$$
$$a_{SL}^{\frac{1}{2}\,0} = a_{SL}^{\frac{1}{2}\,0\,,\,K\pi} = a_{SL}^{\frac{1}{2}\,0\,,\,K\eta} = a_{SL}^{\frac{1}{2}\,0\,,\,K\eta'} = a_{SL}^{\frac{3}{2}\,0\,,\,K\pi}$$
$$a_{SL}^{1\,0\,,\,\pi\eta}$$

All the subtraction constants in the vector channels are set equal to a_{SL}^{00} (play a little role).

We have 16 free parameters with 348 data and the fitted results are

$$c_d = (15.6^{+4.2}_{-3.4})\,\mathrm{MeV}\,, \qquad c_m = (31.5^{+19.5}_{-22.5})\,\mathrm{MeV}\,,$$

$$\widetilde{c}_d = (8.7^{+2.5}_{-1.7})\,\mathrm{MeV}\,, \qquad \widetilde{c}_m = (15.8^{+3.3}_{-3.0})\,\mathrm{MeV}\,,$$

$$M_{S_8} = (1370^{+132}_{-57})\,\mathrm{MeV}\,, \qquad M_{S_1} = (1063^{+53}_{-31})\,\mathrm{MeV}\,,$$

$$M_\rho = (801.0^{+7.0}_{-7.5})\,\mathrm{MeV}\,, \qquad M_{K^*} = (909.0^{+7.5}_{-6.9})\,\mathrm{MeV}\,,$$

$$G_V = (61.9^{+1.9}_{-1.9})\,\mathrm{MeV}\,, \qquad a_{SL}^{1\,0\,,\pi\eta} = 2.0^{+3.1}_{-3.4}\,,$$

$$a_{SL}^{00} = (-1.15^{+0.07}_{-0.09})\,, \qquad a_{SL}^{\frac{1}{2}\,0} = (-0.96^{+0.10}_{-0.16})\,,$$

$$\mathcal{N} = (0.6^{+0.3}_{-0.3})\,\mathrm{MeV}^{-2}\,, \qquad c = (1.0^{+0.6}_{-0.4})\,,$$

$$M_0 = (954^{+102}_{-95})\,\mathrm{MeV}\,, \qquad \Lambda_2 = (-0.6^{+0.5}_{-0.4})\,,$$

with $\chi^2/\mathrm{d.o.f} = 714/(348-16) \simeq 2.15$.
$n_\sigma = \Delta\chi^2/\sqrt{2\chi^2} \leq 2$ to get the errors, $n_\sigma = 2$ Etkin *et al.* PRD'82

Another strategy to perform the fit

The number of parameters can be reduced by imposing the following constraints [Ecker, Gasser, Pich, de Rafael, NPB'88]

$$\widetilde{c}_d = \frac{c_d}{\sqrt{3}}\,, \quad \widetilde{c}_m = \frac{c_m}{\sqrt{3}}\,, \tag{15}$$

and some of the parameters can be taken from other works:
$M_{S_1} = 1020$ MeV, $M_{S_8} = 1390$ MeV [Oller, Oset, PRD'99];
$M_0 = 850$ MeV from [Feldmann, IJMPLA'00];
$G_V = 60.0$ MeV, average value from
[Ecker, Gasser, Pich, de Rafael, PLB'89]
[Guo, Sanz-Cillero, Zheng, JHEP'07]
[Guo, Sanz-Cillero, PRD'09].

We have 10 free parameters with 348 data now and the fitted results are

$$c_d = 17.4\,\mathrm{MeV}\,, \qquad c_m = 28.1\,\mathrm{MeV}\,,$$
$$M_\rho = 800.4\,\mathrm{MeV}\,, \qquad M_{K^*} = 910.0\,\mathrm{MeV}\,,$$
$$a_{SL}^{00} = -1.14\,, \qquad a_{SL}^{\frac{1}{2}0} = -0.89\,,$$
$$\Lambda_2 = -0.22\,, \qquad a_{SL}^{1\,0\,,\pi\eta} = 2.0\,,$$
$$\mathcal{N} = 0.55\,\mathrm{MeV}^{-2}\,, \qquad c = 0.84\,,$$

with $\chi^2/\mathrm{d.o.f} = 842/(348-10) \simeq 2.5$.