Mesons and baryons in holographic soft-wall model

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Introduction

- Holographic QCD (HQCD) – approximation to QCD:
  Hadron Physics in terms of fields/strings living in extra dimensions (AdS space)

- Motivation: AdS/CFT correspondence 1998 (Maldacena, Polyakov, Witten et al)

  Dynamics of the superstring theory in AdS$_{d+1}$ background is encoded in d conformal field theory living on the AdS boundary.

- AdS metric $ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$ Poincaré form

  z is extra dimension (holographic) coordinate; $z = 0$ is UV boundary

AdS/CFT dictionary

Gauge	Gravity
Operator $\hat{\mathcal{O}}$	Bulk field $\Phi(x,z)$
Δ — scaling dimension of $\hat{\mathcal{O}}$	m — mass of $\Phi(x,z)$
Source of $\hat{\mathcal{O}}$	Non-normalizable bulk profile near $z=0$
$\langle\hat{\mathcal{O}}\rangle$	Normalizable bulk profile near $z=0$

Introduction

- Towards to QCD:
  – Break conformal invariance and generate mass gap
  – Tower of normalized bulk fields (Kaluza-Klein modes) $\leftrightarrow$ Hadron wave functions
  – Spectrum of Kaluza-Klein modes $\leftrightarrow$ Hadrons spectrum

- HQCD: Description of low-energy QCD

- Bottom-up HQCD: hard-wall and soft-wall models

- Hard-wall:
  AdS geometry is cutted by two branes UV $(z = \epsilon \to 0)$ and IR $(z = z_{\rm IR})$
  Analogue of quark bag model, linear dependence on $J(L)$ of hadron masses

- Soft-wall:
  Soft cuttoff of AdS space by dilaton field $\exp(-\varphi(z))$
  Analytical solution of EOM, $M^2 \sim J(L)$ (Regge behavior)

Introduction

- Brodsky and de Téramond:
  Semiclassical 1st approximation to QCD based on combination of LF holography and correspondence of String Theory in AdS_5 and CFT in Min_4.

- LF holography EOM for propagation of spin-J modes in AdS are equivalent to Hamiltonian formulation of QCD on LF

- Mapping of string mode in AdS 5th dimension z to the hadron LFWF depending on impact variable ζ — separation of quark and gluons inside hadron.

- Objective:

  SW holographic approach for mesons and baryons with any n, J, L, S.

Approach: Fields propagating in AdS

- Conformal group contains 15 generators:

  10 Poincaré (translations P_μ, Lorentz transformations $M_{\mu\nu}$),
  5 conformal (conformal boosts K_μ, dilatation D):

$M_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$	rotational symmetry
$D = i(x\,\partial)$	energy
$P_\mu = i\partial_\mu$	raising energy
$K_\mu = 2ix_\mu(x\,\partial) - ix^2\partial_\mu$	lowering energy

- Isomorphic to $SO(4,2)$ – the isometry group of AdS_5 space

- Fields in AdS_5 are classified by unitary, irreducible representations of $SO(4,2)$

Approach: Scalar Field

- Action for scalar field
  Brodsky, Téramond

$$S_\Phi^+ = \frac{1}{2}\int d^dx dz\, \sqrt{g}\, e^{\varphi(z)} \left(\partial_N \Phi_+ \partial^N \Phi_+ - m^2\, \Phi_+^2\right)$$

  Our

$$S_\Phi^- = \frac{1}{2}\int d^dx dz\, \sqrt{g}\, e^{-\varphi(z)} \left(\partial_N \Phi_- \partial^N \Phi_- - (m^2 + \Delta U(z))\, \Phi_-^2\right)$$

- dilaton $\varphi(z) = \kappa^2 z^2$ (Regge behavior of hadron masses),
- metric $g_{MN}(z) = \epsilon_M^a(z)\epsilon_N^b(z)\eta_{ab}, \quad g = |\mathrm{det} g_{MN}|$
- vielbein $\epsilon_M^a(z) = e^{A(z)}\delta_M^a, \quad A(z) = \log(R/z)$ (conformal)
- interval $ds^2 = g_{MN} dx^M dx^N = e^{2A(z)}(g_{\mu\nu} dx^\mu dx^\nu - dz^2)$
- equivalence: bulk field redefinition $\Phi_\pm = e^{\mp\varphi(z)}\Phi_\mp$
- potential $\Delta U(z) = e^{-2A(z)}[\varphi''(z) + \frac{1-d}{z}\varphi'(z)]$

Approach: Scalar Field

- Klebanov, Witten

$$\Phi(x,z)\Big|_{z\to 0} \to z^{d-\Delta}\left[\Phi_0(x) + O(z^2)\right] + z^{\Delta}\left[A(x) + O(z^2)\right]$$

$\Phi_0(x)$ is source of the CFT operator $\hat{\mathcal{O}}$

$A(x) \sim \langle\hat{\mathcal{O}}\rangle$ is physical fluctuation

- Towards to QCD Brodsky, Téramond

$\Delta \equiv \tau = 2 + L$ scaling dimension of two-parton state with $L = 0, 1$.
extended to any J and independent on J

Approach: Scalar Field

- Kaluza-Klein expansion $\Phi(x,z) = \sum_n \int \frac{d^d p}{(2\pi)^d}\, e^{-ipx}\, \Phi_n(p)\, \Phi_n(p,z)$
- Substitution $\Phi_n(p,z) = e^{-B(z)/2}\phi_n(p,z)$
- Schrödinger-type EOM for $\phi_n(z) = \phi_n(p,z)|_{p^2=M_n^2}$:

  $$\Big[- \frac{d^2}{dz^2} + \frac{4L^2-1}{4z^2} + U(z)\Big]\phi_n(z) = M_n^2 \phi_n(z)$$

- $\phi_n(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+1)}}\, \kappa^{L+1}\, z^{L+1/2}\, e^{-\kappa^2 z^2/2}\, L_n^L(\kappa^2 z^2)$
- $M_n^2 = 4\kappa^2 \Big(n + \frac{L}{2}\Big)$,
- Massless pion $M_\pi^2 = 0$ for $n = L = 0$ (Brodsky, Téramond)
- $\Phi_n(z) = z^{3/2}\, \phi_n(z) \, \sim \, z^{2+L}$ (at small z)
- $\Phi_n(z) \to 0$ (at large z)

Approach: Higher J boson fields

- $\Phi_J = \Phi_{M_1 \cdots M_J}(x,z)$ – a symmetric, traceless tensor:

  Brodsky, Téramond

  $$S^{+}_{\Phi} = \frac{1}{2}\int d^dx dz \sqrt{g}\, e^{\varphi(z)} \left(\partial_N \Phi^{+}_J \partial^N \Phi^{J,+} - \mu^2_J\, \Phi^{+}_J \Phi^{J,+} \right)$$

  Our

  $$S^{-}_{\Phi} = \frac{1}{2}\int d^dx dz \sqrt{g}\, e^{-\varphi(z)} \left(\partial_N \Phi^{-}_J \partial^N \Phi^{J,-} - (\mu^2_J + \Delta U_J(z))\, \Phi^{-}_J \Phi^{J,-} \right)$$

- $\mu^2_J R^2 = (\Delta - J)(\Delta + J - d)$

- $\Delta = 2 + L$ and $\mu^2_J = L^2 - (2-J)^2$ for $d = 4$

- Effective potential $\Delta U_J(z) = e^{-2A(z)}[\varphi''(z) + \frac{1+2J-d}{z}\varphi'(z)]$

Approach: Higher J boson fields

- Axial gauge $\Phi_{z\ldots}(x,z) = 0$
- KK decomposition $\Phi_{\nu_1\cdots\nu_J}(x,z) = \sum_n \Phi_{nJ}(z) \int \frac{d^d P}{(2\pi)^d} e^{-iPx} \epsilon^n_{\nu_1\cdots\nu_J}(P)$
- Substitution

$$\Phi_{nJ}(z) = \left(\frac{R}{z}\right)^{\frac{1-d}{2}} \varphi_{nJ}(z)$$

- Schrödinger EOM for $\Phi_{nJ}(z)$:

$$\left[-\frac{d^2}{dz^2} + U_J(z)\right]\varphi_{nJ}(z) = M^2_{nJ}\varphi_{nJ}(z)$$

- Effective potential $U_J(z)$

$$U_J(z) = \kappa^4 z^2 + \frac{4a^2-1}{4z^2} + 2\kappa^2\Big(b_J - 1\Big)\,.$$

- $$a = \frac{1}{2}\sqrt{d^2 + 4(\mu R)^2} = \Delta - \frac{d}{2}\,, \qquad b_J = J + \frac{4-d}{2}$$

Approach: Higher J boson fields

- Solutions at $d = 4$:

$$\varphi_{nJ}(z) = \sqrt{\frac{2n!}{(n+L)!}}\, \kappa^{1+L}\, z^{1/2+L}\, e^{-\kappa^2 z^2/2} L_n^L(\kappa^2 z^2)$$

$$M_{nJ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right)$$

- At $J(L) \to \infty \quad M_{nJ}^2 = 4\kappa^2(n+J)$

- Scaling $\Phi_{nJ} = z^{3/2}\,\varphi_{nJ} \sim z^{\tau}$, twist $\tau = 2 + L$

Approach: Higher J boson fields (gauge-invariant)

- Fradkin, Vasiliev, Metsaev, Buchbinder et al, Karch et al, $\cdots$

$$S_\Phi = \frac{1}{2}\int d^dx dz\,\sqrt{g}\,e^{-\varphi(z)}\left(\nabla_N \Phi_{M_1\ldots M_J}\nabla^N\Phi^{M_1\ldots M_J} - \left(\mu_J^2 + U_J(\varphi)\right)\Phi_{M_1\ldots M_J}\Phi^{M_1\ldots M_J}\right) + \cdots$$

- $\nabla_N \Phi_{M_1\ldots M_J} = \partial_N \Phi_{M_1\ldots M_J} - \Gamma^K_{N M_1}\Phi_{K M_2\ldots M_J} - \Gamma^K_{N M_J}\Phi_{M_1\ldots M_{J-1}K}$

- Affine connection $\Gamma^K_{MN} = \frac{1}{2}g^{KL}\left(\frac{\partial g_{LM}}{\partial x^N} + \frac{\partial g_{LN}}{\partial x^M} - \frac{\partial g_{MN}}{\partial x^K}\right)$

- Gauge constraints (transversity, traceless)

  $\nabla^{M_1}\Phi_{M_1 M_2\ldots M_J} = 0$ and $g^{M_1 M_2}\Phi_{M_1 M_2\ldots M_J} = 0$

- Bulk mass $\mu_J^2 R^2 = \Delta_J(\Delta_J - d) - J = J^2 + J(d-5) + 4 - 2d$

  with $\Delta_J = J + d - 2$

- Mass spectrum $M_{nJ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right)$ Scaling $\Phi_{nJ} \sim z^{2+L}$

Approach: Higher J fermion fields (gauge-invariant)

- $$S_\Psi = \int d^dx dz \sqrt{g}\, e^{-\varphi(z)}\, \overline{\Psi}^{M_1\ldots M_{J-1/2}} \left(\epsilon_a^M \Gamma^a \mathcal{D}_M - \mu_J - \frac{\varphi(z)}{R}\right) \Psi_{M_1\ldots M_{J-1/2}} + \cdots$$

- $$\mathcal{D}_M = \nabla_M - \tfrac{1}{8}\omega_M^{ab}[\Gamma_a, \Gamma_b], \quad \omega_M^{ab} = A'(z)\,(\delta_z^a \delta_M^b - \delta_z^b \delta_M^a)$$

- Relation of spin and affine connection

$$\omega_M^{ab} = \epsilon_K^a \left(\partial_M \epsilon^{Kb} + \epsilon^{Nb}\Gamma_{MN}^K\right)$$

- Gauge constraints (transversity, traceless)

$$\nabla^{M_1}\Psi_{M_1 M_2\ldots M_{J-1/2}} = 0, \qquad \Gamma^{M_1}\Psi_{M_1 M_2\ldots M_{J-1/2}} = 0,$$

$$g^{M_1 M_2}\Psi_{M_1 M_2\ldots M_{J-1/2}} = 0$$

- Bulk mass $\mu_J R = \Delta_J - d/2$ with $\Delta_J = J + d - 2$ Metsaev

- Toward QCD: $\Delta_J \equiv \tau + 1/2 = 7/2 + L$

independent on J and gives correct scaling of nucleon FF

Approach: Higher J fermion fields (gauge-invariant)

- EOM $\left[iz\not{\partial} + \gamma^5 z\partial_z - \frac{d}{2}\gamma^5 - \mu R - \varphi(z)\right]\Psi_{a_1\cdots a_{J-1/2}}(x,z) = 0$

- $\Psi(x,z) = \Psi_L(x,z) + \Psi_R(x,z)\,, \quad \Psi_{L/R} = \frac{1\mp\gamma^5}{2}\Psi$

- $\Psi_{L/R}(x,z) = \sum_n \int \frac{d^d p}{(2\pi)^d}\; e^{-ipx}\; \Psi_{L/R}(p)\; F^n_{L/R}(p,z)$

- $F^n_{L/R}(p,z) = e^{-A(z)\cdot d/2}\, f^n_{L/R}(p,z)$

- $\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2\left(\mu R \mp \frac{1}{2}\right) + \frac{\mu R(\mu R \pm 1)}{z^2}\right] f^n_{L/R}(z) = M_n^2\, f^n_{L/R}(z)$

- For $d = 4$ and $\mu R = L + 3/2$

$$f^n_L(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+3)}}\; \kappa^{L+3}\; z^{L+5/2}\; e^{-\kappa^2 z^2/2}\; L_n^{L+2}(\kappa^2 z^2)$$

$$f^n_R(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+2)}}\; \kappa^{L+2}\; z^{L+3/2}\; e^{-\kappa^2 z^2/2}\; L_n^{L+1}(\kappa^2 z^2)$$

$$M_n^2 = 4\kappa^2\left(n + L + 2\right), \qquad F^n_L(z) \sim z^{9/2+L}\,, \qquad F^n_R(z) \sim z^{7/2+L}$$

Approach: Hadronic Wave Function

- Correspondence of holographic coordinate z to the impact variable ζ in LF

- Two parton case: $q_1\bar{q}_2$ mesons $\quad z \to \zeta, \qquad \zeta^2 = \mathbf{b}_\perp^2 x(1-x)$

  ζ - impact variable; $\mathbf{b}_\perp$ - impact separation (conjugate to $\mathbf{k}_\perp$)

- Mapping $\Phi_{nJ}(z)$ to the transverse mode of LFWF

- $\psi_{nJ}(x,\zeta,m_1,m_2) = \psi_{\rm T}(\zeta)\cdot\psi_{\rm L}(x)\cdot\psi_{\rm A}(\varphi)$

  $\psi_T = \Phi_{nJ}(\zeta)$ — transverse (from AdS/QCD)
  $\psi_L = f(x,m_1,m_2) = e^{-m_1^2/(2x\lambda^2)-m_2^2/(2(1-x)\lambda^2)}$ — longitudinal
  $\psi_A = e^{im\varphi}$ — angular mode
  λ - additional scale parameter

$$M_{nJ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right) + \int\limits_0^1 dx\left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right) f^2(x,m_1,m_2)$$

Approach: Choice of parameters

- Constituent quark massses:
  $m = 420 \text{ MeV}\,, \quad m_s = 570 \text{ MeV}\,, \quad m_c = 1.6 \text{ GeV}\,, \quad m_b = 4.8 \text{ GeV}$

- dilaton parameter $\kappa = 550$ MeV

- Dimensional parameters λ in the longitudinal WF are fitted as:

$$\lambda_{qq} = 0.63 \text{ GeV}\,,\ \lambda_{qs} = 1.20 \text{ GeV}\,,\ \lambda_{ss} = 1.68 \text{ GeV}\,,\ \lambda_{qc} = 2.50 \text{ GeV}\,, \lambda_{sc} = 3.00 \text{ GeV}$$
$$\lambda_{qb} = 3.89 \text{ GeV}\,,\ \lambda_{sb} = 4.18 \text{ GeV}\,,\ \lambda_{cc} = 4.04 \text{ GeV}\,,\ \lambda_{cb} = 4.82 \text{ GeV}\,,\ \lambda_{bb} = 6.77 \text{ GeV}$$

Approach: HQET Constraints

- Heavy–light mesons

$$M_{qQ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right) + \int_0^1 dx \left(\frac{m_q^2}{x} + \frac{m_Q^2}{1-x}\right) f^2(x, m_q, m_Q)$$

$$= \left(m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q)\right)^2$$

- Scaling of dimensional parameters: $\kappa = \mathcal{O}(1), \quad \lambda_{qQ} = \mathcal{O}(\sqrt{m_Q})$

- Mass splitting: $\Delta M_{qQ} = \frac{2\kappa^2}{M_{qQ}^V + M_{qQ}^P} \sim \frac{1}{m_Q}$

- Leptonic decay constants

$$f_P = f_V = \kappa \frac{\sqrt{6}}{\pi} \int_0^1 dx \sqrt{x(1-x)}\, f(x, m_1, m_2) \sim \frac{1}{\sqrt{m_Q}}$$

- Heavy quarkonia

$$M_{Q_1\bar{Q}_2} = m_{Q_1} + m_{Q_2} + E + \mathcal{O}(1/m_{Q_{1,2}})$$

Results: Mass spectrum

Masses of light mesons

Meson	n	L	S	Mass [MeV]			
π	0	0,1,2,3	0	140	1355	1777	2099
π	0,1,2,3	0	0	140	1355	1777	2099
K	0	0,1,2,3	0	496	1505	1901	2207
η	0,1,2,3	0	0	544	1552	1946	2248
$f_0[\bar{n}n]$	0,1,2,3	1	1	1114	1600	1952	2244
$f_0[\bar{s}s]$	0,1,2,3	1	1	1304	1762	2093	2372
$a_0(980)$	0,1,2,3	1	1	1114	1600	1952	2372
$\rho(770)$	0,1,2,3	0	1	804	1565	1942	2240
$\rho(770)$	0	0,1,2,3	1	804	1565	1942	2240
$\omega(782)$	0,1,2,3	0	1	804	1565	1942	2240
$\omega(782)$	0	0,1,2,3	1	804	1565	1942	2240
$\phi(1020)$	0,1,2,3	0	1	1019	1818	2170	2447
$a_1(1260)$	0,1,2,3	1	1	1358	1779	2101	2375

Results: Mass spectrum

Masses of heavy-light mesons

Meson	J^{P}	n	L	S	Mass [MeV]			
$D(1870)$	0^-	0	0,1,2,3	0	1857	2435	2696	2905
$D^*(2010)$	1^-	0	0,1,2,3	1	2015	2547	2797	3000
$D_s(1969)$	0^-	0	0,1,2,3	0	1963	2621	2883	3085
$D_s^*(2107)$	1^-	0	0,1,2,3	1	2113	2725	2977	3173
$B(5279)$	0^-	0	0,1,2,3	0	5279	5791	5964	6089
$B^*(5325)$	1^-	0	0,1,2,3	1	5336	5843	6015	6139
$B_s(5366)$	0^-	0	0,1,2,3	0	5360	5941	6124	6250
$B_s^*(5413)$	1^-	0	0,1,2,3	1	5416	5992	6173	6298

Results: Mass spectrum

Masses of heavy quarkonia $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$

Meson	J^{P}	n	L	S	Mass [MeV]			
$\eta_c(2986)$	0^-	0,1,2,3	0	0	2997	3717	3962	4141
$\psi(3097))$	1^-	0,1,2,3	0	1	3097	3798	4038	4213
$\chi_{c0}(3414)$	0^+	0,1,2,3	1	1	3635	3885	4067	4226
$\chi_{c1}(3510)$	1^+	0,1,2,3	1	1	3718	3963	4141	4297
$\chi_{c2}(3555)$	2^+	0,1,2,3	1	1	3798	4038	4213	4367
$\eta_b(9300)$	0^-	0,1,2,3	0	0	9428	10190	10372	10473
$\Upsilon(9460)$	1^-	0,1,2,3	0	1	9460	10219	10401	10502
$\chi_{b0}(9860)$	0^+	0,1,2,3	1	1	10160	10343	10444	10521
$\chi_{b1}(9893)$	1^+	0,1,2,3	1	1	10190	10372	10473	10550
$\chi_{b2}(9912)$	2^+	0,1,2,3	1	1	10219	10401	10502	10579
$B_c(6276)$	0^-	0,1,2,3	0	0	6276	6911	7092	7209

Results: Nucleon FFs and GPDs

- Nucleon form factors in AdS/QCD Abidin-Carlson, Brodsky-Teramond

$$\begin{aligned} F_1^p(Q^2) &= C_1(Q^2) + \eta_p C_2(Q^2) \sim 1/Q^4 \\ F_2^p(Q^2) &= \eta_p C_3(Q^2) \sim 1/Q^6 \\ F_1^n(Q^2) &= \eta_n C_2(Q^2) \sim 1/Q^4 \\ F_2^n(Q^2) &= \eta_n C_3(Q^2) \sim 1/Q^6 , \quad \eta_N = \kappa\, k_N/(2m_N \sqrt{2}) \end{aligned}$$

- Structure integrals

$$\begin{aligned} C_1(Q^2) &= \int dz e^{-\varphi(z)} \frac{V(Q,z)}{2z^4} (\psi_L^2(z) + \psi_R^2(z)) \\ C_2(Q^2) &= \int dz e^{-\varphi(z)} \frac{\partial_z V(Q,z)}{2z^3} (\psi_L^2(z) - \psi_R^2(z)) \\ C_3(Q^2) &= \int dz e^{-\varphi(z)} \frac{2m_N V(Q,z)}{2z^3} \psi_L(z)\psi_R(z) \end{aligned}$$

$\psi_{L/R}(z)$ — KK modes dual to L/R-handed nucleon fields:
$\psi_L(z) = \kappa^3 z^{9/2}$, $\psi_R(z) = \kappa^2 z^{7/2}\sqrt{2}$

$V(Q,z) = \Gamma(1 + \frac{Q^2}{4\kappa^2})U(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2)$ bulk-to-boundary propagator of the vector field (holograhic analogue of EM current)

Results: Nucleon FFs and GPDs

- EM radii

$$\langle r_E^2\rangle^p = \frac{147}{64\kappa^2}\left(1+\frac{13}{147}\mu_p\right) = 0.910\ \mathrm{fm}^2\ (\mathrm{our})\,, \quad 0.766\ \mathrm{fm}^2\ (\mathrm{data})$$

$$\langle r_E^2\rangle^n = \frac{13}{64\kappa^2}\mu_n = -0.123\ \mathrm{fm}^2\ (\mathrm{our})\,, \quad -0.116\ \mathrm{fm}^2\ (\mathrm{data})$$

$$\langle r_M^2\rangle^p = \frac{177}{64\kappa^2}\left(1-\frac{17}{177\mu_p}\right) = 0.849\ \mathrm{fm}^2\ (\mathrm{our})\,, \quad 0.731\ \mathrm{fm}^2\ (\mathrm{data})$$

$$\langle r_M^2\rangle^n = \frac{177}{64\kappa^2} = 0.849\ \mathrm{fm}^2\ (\mathrm{our})\,, \quad 0.731\ \mathrm{fm}^2\ (\mathrm{data})$$

Results: Nucleon FFs and GPDs

- Sum rules relating EM FF and GPDs Ji, Radyushkin

$$F_1^p(t) = \int_0^1 dx \left(\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t)\right)$$

$$F_1^n(t) = \int_0^1 dx \left(\frac{2}{3} H_v^d(x,t) - \frac{1}{3} H_v^u(x,t)\right)$$

$$F_2^p(t) = \int_0^1 dx \left(\frac{2}{3} E_v^u(x,t) - \frac{1}{3} E_v^d(x,t)\right)$$

$$F_2^n(t) = \int_0^1 dx \left(\frac{2}{3} E_v^d(x,t) - \frac{1}{3} E_v^u(x,t)\right)$$

- Grigoryan-Radyushkin integral representation for bulk-to-boundary propagator

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\frac{x}{1-x}\kappa^2 z^2}$$

- LF mapping (Brodsky-Teramond): x is equivalent to LC momentum fraction

Results: Nucleon FFs and GPDs

- GPDs $H_v^q(x,Q^2) = q(x)\, x^{\frac{Q^2}{4\kappa^2}}\,, \quad E_v^q(x,Q^2) = e^q(x)\, x^{\frac{Q^2}{4\kappa^2}}$

- Distribution functions $q(x)$ and $e^q(x)$

$$q(x) = \alpha^q \gamma_1(x) + \beta^q \gamma_2(x)\,, \quad e^q(x) = \beta^q \gamma_3(x)$$

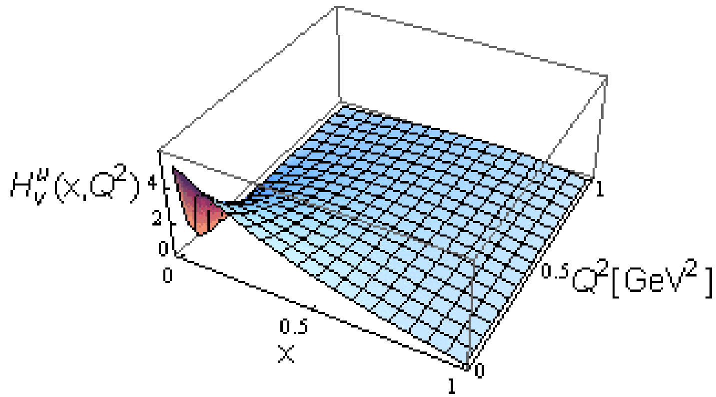
Flavor couplings α^q, β^q and functions $\gamma_i(x)$ are written as

$$\alpha^u = 2\,,\ \alpha^d = 1\,,\ \beta^u = 2\eta_p + \eta_n\,,\ \beta^d = \eta_p + 2\eta_n$$

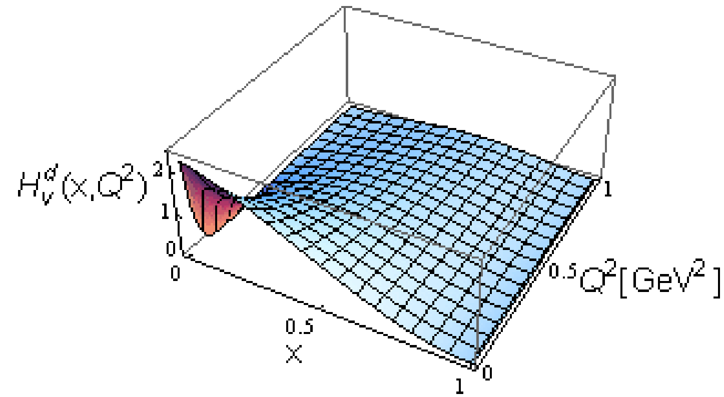
and

$$\begin{aligned}
\gamma_1(x) &= \frac{1}{2}(5 - 8x + 3x^2) \\
\gamma_2(x) &= 1 - 10x + 21x^2 - 12x^3 \\
\gamma_3(x) &= \frac{6m_N\sqrt{2}}{\kappa}(1-x)^2
\end{aligned}$$

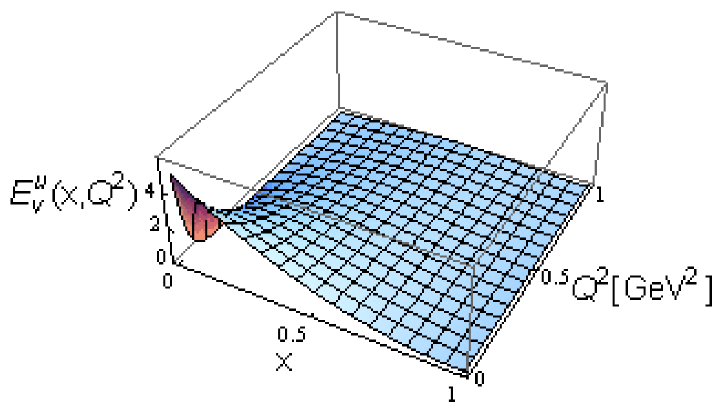
Results: Nucleon FFs and GPDs



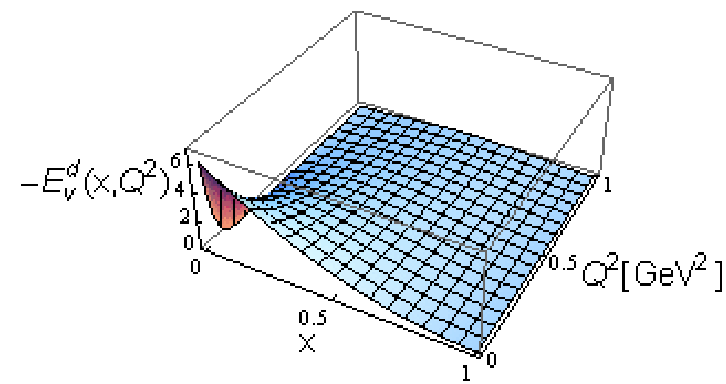
$H_v^u(x, Q^2)$



$H_d^u(x, Q^2)$



$E_v^u(x, Q^2)$



$E_d^u(x, Q^2)$

Results: Nucleon FFs and GPDs

- Nucleon GPDs in impact space   Burkardt, Miller, Diehl et al

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} H_q(x,\mathbf{k}_\perp^2) e^{-i\mathbf{b}_\perp \mathbf{k}_\perp} = q(x) \frac{\kappa^2}{\pi \log(1/x)} e^{-\frac{\mathbf{b}_\perp^2 \kappa^2}{\log(1/x)}}$$

$$e^q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} E_q(x,\mathbf{k}_\perp^2) e^{-i\mathbf{b}_\perp \mathbf{k}_\perp} = e^q(x) \frac{\kappa^2}{\pi \log(1/x)} e^{-\frac{\mathbf{b}_\perp^2 \kappa^2}{\log(1/x)}}$$

- Parton charge and magnetization densities in transverse impact space

$$\rho_E^N(\mathbf{b}_\perp) = \sum_q e_q^N \int_0^1 dx q(x,\mathbf{b}_\perp) = \frac{\kappa^2}{\pi} \sum_q e_q^N \int_0^1 \frac{dx}{\log(1/x)} q(x) e^{-\frac{\mathbf{b}_\perp^2 \kappa^2}{\log(1/x)}}$$

$$\rho_M^N(\mathbf{b}_\perp) = \sum_q e_q^N \int_0^1 dx e^q(x,\mathbf{b}_\perp) = \frac{\kappa^2}{\pi} \sum_q e_q^N \int_0^1 \frac{dx}{\log(1/x)} e^q(x) e^{-\frac{\mathbf{b}_\perp^2 \kappa^2}{\log(1/x)}}$$

Results: Nucleon FFs and GPDs

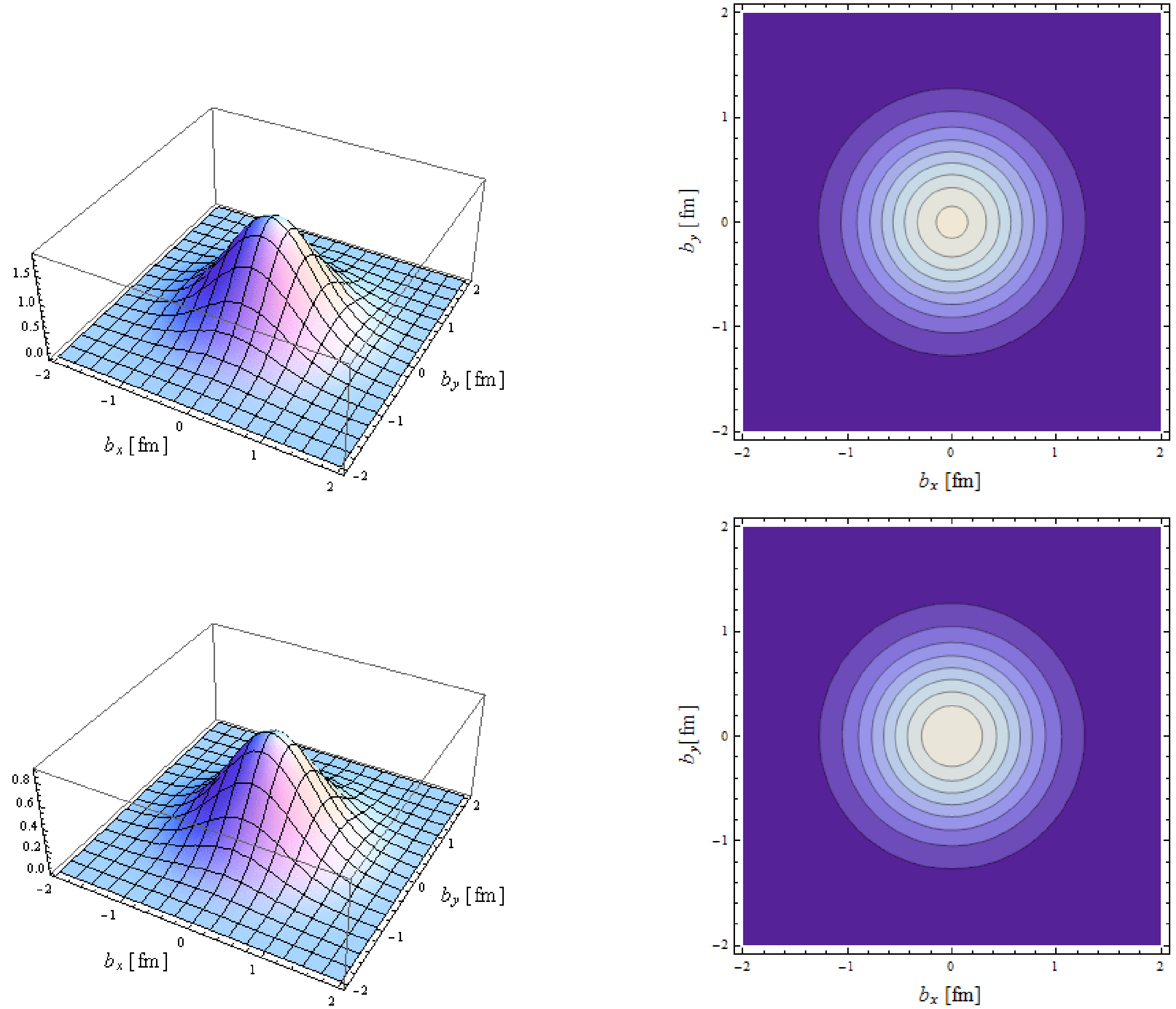
- Transverse width of impact parameter dependent GPD

$$\langle R_\perp^2(x)\rangle_q = \frac{\int d^2\mathbf{b}_\perp \mathbf{b}_\perp^2\, q(x,\mathbf{b}_\perp)}{\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)} = -4\frac{\partial \log H_v^q(x,Q^2)}{\partial Q^2}\Big|_{Q^2=0} = \frac{\log(1/x)}{\kappa^2}$$

- Transverse rms radius

$$\langle R_\perp^2\rangle_q = \frac{\int d^2\mathbf{b}_\perp \mathbf{b}_\perp^2 \int_0^1 dx q(x,\mathbf{b}_\perp)}{\int d^2\mathbf{b}_\perp \int_0^1 dx q(x,\mathbf{b}_\perp)} = \frac{1}{\kappa^2}\left(\frac{5}{3}+\frac{\beta^q}{12\alpha^q}\right) \simeq 0.527\ \mathrm{fm}^2$$

Results: Nucleon FFs and GPDs



Plots for $q(x, \mathbf{b}_\perp)$ for $x = 0.1$: $u(x, \mathbf{b}_\perp)$ - upper pannels, $d(x, \mathbf{b}_\perp)$ - lower pannels

Summary

- Soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Mass spectrum, decay constansts, form factors, GPDs
- Current and Future work:
  – GPDs and Deeply Virtual Exclusive Processes
  – Baryon excitation spectrum and form factors
  – Mesons and baryons: including multiparton states
  – Hybrid and exotic states