Quark mass dependence of light resonances and phase shifts in elastic $\pi\pi$ and $\pi K$ scattering

Jenifer Nebreda, J. R. Peláez and G. Ríos
Universidad Complutense de Madrid

Hadron 2011, Munich
June 13-17, 2011
Motivation

Phase shifts $M_\pi$ dependence in **Standard ChPT**

Phase shifts $M_\pi$ dependence in **Unitarized ChPT**

Comparison of **ChPT and lattice** results

Light resonances dependence on $\hat{m}$

Summary
**Motivation**

**Lattice**: rigorous QCD results with quarks and gluons. Growing interest in scattering and scalar sector. Caveat: small, realistic quark masses are difficult to implement.

**ChPT**: QCD dependence on quark masses as an expansion.

We can compare:

- Lattice multi-hadron states calculations → phase shifts and scattering lengths vs. standard ChPT (model independent) or UChPT (to go higher in $\sqrt{s}$)
- Lattice spectrum calculations → masses vs. UChPT

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Standard Chiral Perturbation Theory
Chiral Perturbation Theory  Weinberg, Gasser & Leutwyler

Low energy effective theory of QCD with:

- **DOF:** Pseudo-Goldstone Bosons of the spontaneous chiral symmetry breaking

\[ SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \]

- \( N_f = 2 \rightarrow \pi \)’s
- \( N_f = 3 \rightarrow \pi \)’s, \( K \)’s and \( \eta \)

- expansion in masses and momenta

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots \]

- **parameters:** Low Energy Constants (LECs)
  - \( N_f = 2 \rightarrow 4 \ l \)’s (one loop) and 7 \( r \)’s (two loops)
  - \( N_f = 3 \rightarrow 8 \ l \)’s (one loop)
\( \pi \pi \) scattering in SU(2) standard ChPT:

- Already calculated to 1 and 2 loops*, we study the phases dependence on \( \hat{m} = \frac{m_u + m_d}{2} \).

Advantages:

- **SISTEMATIC EXPANSION, MODEL INDEPENDENT**
- some lattice groups already giving results for I=2 phases and scattering lengths**

Limitations:

- only low energy region
- no resonances.


Standard $SU(2)$ ChPT amplitudes with LECs from


<table>
<thead>
<tr>
<th>$O(p^4)$ LECs ($\times 10^{-3}$)</th>
<th>$O(p^6)$ LECs ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_1^r$</td>
<td>-3.98 ± 0.62</td>
</tr>
<tr>
<td>$l_2^r$</td>
<td>1.89 ± 0.23</td>
</tr>
<tr>
<td>$l_3^r$</td>
<td>0.18 ± 1.11</td>
</tr>
<tr>
<td>$l_4^r$</td>
<td>6.17 ± 1.39</td>
</tr>
<tr>
<td>$r_1^r$</td>
<td>-0.60 ± 0.35</td>
</tr>
<tr>
<td>$r_2^r$</td>
<td>1.28 ± 0.74</td>
</tr>
<tr>
<td>$r_3^r$</td>
<td>-1.68 ± 0.97</td>
</tr>
<tr>
<td>$r_4^r$</td>
<td>-1.00 ± 0.58</td>
</tr>
<tr>
<td>$r_5^r$</td>
<td>1.52 ± 0.42</td>
</tr>
<tr>
<td>$r_6^r$</td>
<td>0.40 ± 0.04</td>
</tr>
</tbody>
</table>

Statistical error, not systematic.

Change $\hat{m} \Rightarrow$ change on $M_{\pi}^2 = 2\hat{m}B_0 \Rightarrow$ change on $f_\pi$

(one more $O(p^6)$ parameter: $r_f^r \approx 0 \pm 1.2 \times 10^{-4}$)

Uncertainties in phase shifts: Montecarlo Gaussian Sampling.
Phase shifts vs. Momentum, increasing $M_\pi$

Phases vs. energy $\rightarrow \hat{m}$ dependence from the threshold’s shift. Better to plot phases vs. momentum.
One loop
Standard ChPT

Quark mass dependence of light resonances and phase shifts
One loop
Standard ChPT
Motivation

Standard ChPT

Unitarized ChPT

ChPT vs. lattice

Resonances

Summary

Standard ChPT $\delta$ dependence on $M_\pi$

One loop

Standard ChPT

Quark mass dependence of light resonances and phase shifts
Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
One loop

If threshold is "subtracted": VERY SOFT $M_\pi$ dependence!
If threshold is "subtracted": VERY SOFT $M_\pi$ dependence!
Standard ChPT \( \delta \) dependence on \( M_\pi \)

Motivation  
Standard ChPT  
Unitarized ChPT  
ChPT vs. lattice  
Resonances  
Summary

\[ \delta_{00} \mathcal{O}(p^6) \]

\[ \delta_{02} \mathcal{O}(p^6) \]

\[ \delta_{20} \mathcal{O}(p^6) \]

\[ \delta_{22} \mathcal{O}(p^6) \]

\[ M_\pi = 139.57 \text{ MeV} \]

Quark mass dependence of light resonances and phase shifts
Attention: 2 loops is just NLO for D waves

Two loops Standard ChPT

Standard ChPT $\delta$ dependence on $M_\pi$
Standard ChPT $\delta$ dependence on $M_\pi$

Two loops

Standard ChPT

Quark mass dependence of light resonances and phase shifts
Motivation

Standard ChPT

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Motivation

Standard ChPT

Unitarized ChPT

ChPT vs. lattice

Resonances

Summary

Standard ChPT $\delta$ dependence on $M_\pi$

Quark mass dependence of light resonances and phase shifts

Two loops

Standard ChPT

Jennifer Nebreda, U. Complutense de Madrid
Standard ChPT $\delta$ dependence on $M_\pi$

Two loops

Standard ChPT

Still soft $M_\pi$ dependence

Motivation  Standard ChPT  Unitarized ChPT  ChPT vs. lattice  Resonances  Summary

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Motivation | Standard ChPT | Unitarized ChPT | ChPT vs. lattice | Resonances | Summary

Standard ChPT

Unitarized ChPT

ChPT vs. lattice

Resonances

Summary

Quark mass dependence of light resonances and phase shifts

Two loops

Still soft $M_\pi$ dependence

$\rho$ channel phase shift decreases??

$\delta_{00}$ $O(p^6)$

$\delta_{20}$ $O(p^6)$

$\delta_{02}$ $O(p^6)$

$\delta_{22}$ $O(p^6)$

$M_\pi = 139.57$ MeV

$M_\pi = 230$ MeV

$M_\pi = 300$ MeV

$M_\pi = 420$ MeV
Unitarized ChPT

Quark mass dependence of light resonances and phase shifts
Elastic IAM partial waves satisfy exact unitarity

$$SS^\dagger = 1 \Rightarrow \text{Im} \ t^{-1} = -\sigma$$

\(O(p^4)\) IAM partial waves:

$$t(s) \approx \frac{t_2^2(s)}{t_2(s) - t_4(s)}$$

It is derived from a dispersion relation:

- exact on the elastic right cut,
- left cut and subtraction constants approximated within NLO ChPT,
- fully renormalized,
- no spurious parameters.
Elastic IAM partial waves satisfy exact unitarity

\[ SS^\dagger = 1 \Rightarrow \text{Im} \, t^{-1} = -\sigma \]

\( O(\rho^6) \) IAM partial waves:

\[ t(s) \simeq \frac{t_2^2(s)}{t_2(s) - t_4(s) + \frac{t_4^2}{t_2} - t_6} \]

It is derived from a dispersion relation:

- exact on the elastic right cut,
- left cut and subtraction constants approximated within NLO ChPT,
- fully renormalized,
- no spurious parameters.
SU(2) Unitarized ChPT phase shifts vs. Momentum
Unitarized SU(2) ChPT amplitudes with LECs:

**One loop**

| $O(p^4)$ LECs ($\times 10^{-3}$) | 
|-----------------|-----------------|---------------------|
| $l_r^1(\mu)$    | $-3.7 \pm 0.2$  |
| $l_r^2(\mu)$    | $5.0 \pm 0.4$   |
| $l_r^3(\mu)$    | $0.8 \pm 3.8$   |
| $l_r^4(\mu)$    | $6.2 \pm 5.7$   |

**Two loops**

<table>
<thead>
<tr>
<th>$O(p^4)(x10^{-3})$</th>
<th>Set A</th>
<th>Set D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_r^1(\mu)$</td>
<td>-5.0</td>
<td>-4.0</td>
</tr>
<tr>
<td>$l_r^2(\mu)$</td>
<td>1.7</td>
<td>1.2</td>
</tr>
<tr>
<td>$l_r^3(\mu)$</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$l_r^4(\mu)$</td>
<td>6.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$O(p^6)(x10^{-4})$</th>
<th>Set A</th>
<th>Set D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_r^1(\mu)$</td>
<td>-0.6</td>
<td>-0.6</td>
</tr>
<tr>
<td>$r_r^2(\mu)$</td>
<td>1.3</td>
<td>1.5</td>
</tr>
<tr>
<td>$r_r^3(\mu)$</td>
<td>-1.7</td>
<td>-3.3</td>
</tr>
<tr>
<td>$r_r^4(\mu)$</td>
<td>2.0</td>
<td>0.9</td>
</tr>
<tr>
<td>$r_r^5(\mu)$</td>
<td>2.0</td>
<td>1.7</td>
</tr>
<tr>
<td>$r_r^6(\mu)$</td>
<td>-0.6</td>
<td>-0.7</td>
</tr>
<tr>
<td>$r_f^r(\mu)$</td>
<td>-1.4</td>
<td>-1.8</td>
</tr>
</tbody>
</table>
Unitarized ChPT

$\delta_{00} \, O(p^4)$

$\delta_{20} \, O(p^4)$

$\delta_{11} \, O(p^4)$

$M_\pi = 139.57 \text{ MeV}$

One loop

Quark mass dependence of light resonances and phase shifts
Unitarized ChPT

\[
\delta_{00} O(p^4) \quad \delta_{20} O(p^4) \quad \delta_{11} O(p^4)
\]

\[
M_\pi = 139.57 \text{ MeV} \\
M_\pi' = 230 \text{ MeV}
\]

One loop

Quark mass dependence of light resonances and phase shifts
Unitarized ChPT

\[ \delta_{00} O(p^4) \]
\[ \delta_{20} O(p^4) \]
\[ \delta_{11} O(p^4) \]

One loop

- \( M_\pi = 139.57 \text{ MeV} \)
- \( M_\pi = 230 \text{ MeV} \)
- \( M_\pi = 300 \text{ MeV} \)

Motivation  Standard ChPT  Unitarized ChPT  ChPT vs. lattice  Resonances  Summary

Unitarized ChPT \( \delta \) dependence on \( M_\pi \)

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Unitarized ChPT

Motivation Standard ChPT

Unitarized ChPT

ChPT vs. lattice

Resonances Summary

Unitarized ChPT $\delta$ dependence on $M_\pi$

$\delta_{00} \propto p^4$

$\delta_{20} \propto p^4$

$\delta_{11} \propto p^4$

One loop

$M_\pi = 139.57$ MeV
$M_\pi = 230$ MeV
$M_\pi = 300$ MeV
$M_\pi = 420$ MeV
One loop

$\delta_0 O(p^4)$

$\delta_2 O(p^4)$

$\rho$ channel phase shift increases!! contradiction with ChPT??

$M_\pi = 139.57$ MeV

$M_\pi = 230$ MeV

$M_\pi = 300$ MeV

$M_\pi = 420$ MeV
Unitarized ChPT

\[ \delta_{00} \propto O(p^6) \]

\[ \delta_{20} \propto O(p^6) \]

\[ \delta_{11} \propto O(p^6) \]

\[ M_\pi = 139.57 \text{ MeV} \]

Two loops

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Unitarized ChPT

Two loops

$M_\pi = 139.57 \text{ MeV}$
$M_\pi = 230 \text{ MeV}$
Motivation Standard ChPT Unitarized ChPT ChPT vs. lattice Resonances Summary

Unitarized ChPT

Quark mass dependence of light resonances and phase shifts

Unitarized ChPT

Two loops

\( \delta_{00} O(p^6) \)

\( \delta_{20} O(p^6) \)

\( \delta_{11} O(p^6) \)

\( M_\pi = 139.57 \text{ MeV} \)

\( M_\pi = 230 \text{ MeV} \)

\( M_\pi = 300 \text{ MeV} \)
Unitarized ChPT

\[ \delta_{00} \text{ O}(p^6) \]

\[ \delta_{20} \text{ O}(p^6) \]

\[ \delta_{11} \text{ O}(p^6) \]

Two loops

\[ M_\pi = 139.57 \text{ MeV} \]
\[ M_\pi = 230 \text{ MeV} \]
\[ M_\pi = 300 \text{ MeV} \]
\[ M_\pi = 420 \text{ MeV} \]
Motivation
Standard ChPT
Unitarized ChPT
ChPT vs. lattice
Resonances
Summary
Unitarized ChPT \(\delta\) dependence on \(M_\pi\)

**Motivation**
Standard ChPT
Unitarized ChPT
ChPT vs. lattice
Resonances
Summary
Unitarized ChPT \(\delta\) dependence on \(M_\pi\)

**Motivation**

- **Standard ChPT**
- **Unitarized ChPT**
- **ChPT vs. lattice**
- **Resonances**
- **Summary**

**Motivation**

- **Standard ChPT**
- **Unitarized ChPT**
- **ChPT vs. lattice**
- **Resonances**
- **Summary**

**Unitarized ChPT \(\delta\) dependence on \(M_\pi\)**

---

**Bound state: phase jumps \(2\pi\)**
*(Levinson’s theorem)*

---

**\(\rho\) channel phase shift increases!!**
**contradiction with ChPT??**

---

**Two loops**

- \(M_\pi = 139.57\ MeV\)
- \(M_\pi = 230\ MeV\)
- \(M_\pi = 300\ MeV\)
- \(M_\pi = 420\ MeV\)

---

**Jenifer Nebreda, U. Complutense de Madrid**

**Quark mass dependence of light resonances and phase shifts**
Crude, intuitive model of $I=1 \ J=1$ channel behavior
For a simple Breit-Wigner parametrization:

\[ t(s) = \frac{-\sqrt{s}M \Gamma(p)/2\rho}{s - M^2 + iM \Gamma(p)} \quad \text{with} \quad \Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^3 \]

we get a positive phase shift derivative:

\[ \frac{\partial \delta(p)}{\partial (M^2_\pi)} = -\frac{\partial \delta(p)}{\partial (p^2_R)} = \frac{4M \Gamma(p)}{(4p^2 - 4p^2_R)^2 + M^2 \Gamma(p)^2} > 0. \]

The phase shift grows as the \( \rho \) approaches threshold.

Intuitive behavior but opposed to ChPT at low momentum.
For a simple Breit-Wigner parametrization:

\[
t(s) = \frac{-\sqrt{s} M \Gamma(p)/2p}{s - M^2 + iM\Gamma(p)} \quad \text{with} \quad \Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^3
\]

we get a positive phase shift derivative:

\[
\frac{\partial \delta(p)}{\partial (M^2_\pi)} = -\frac{\partial \delta(p)}{\partial (p_R^2)} = \frac{4M\Gamma(p)}{(4p^2 - 4p_R^2)^2 + M^2\Gamma(p)^2} > 0.
\]

The phase shift grows as the $\rho$ approaches threshold.

Intuitive behavior but opposed to ChPT at low momentum.
For a simple Breit-Wigner parametrization:

\[
t(s) = \frac{-\sqrt{s}M \Gamma(p)/2p}{s - M^2 + iM \Gamma(p)} \quad \text{with} \quad \Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^3
\]

we get a positive phase shift derivative:

\[
\frac{\partial \delta(p)}{\partial (M^2)} = -\frac{\partial \delta(p)}{\partial (p_R^2)} = \frac{4M \Gamma(p)}{(4p^2 - 4p_R^2)^2 + M^2 \Gamma(p)^2} > 0.
\]

The phase shift grows as the \( \rho \) approaches threshold.

Intuitive behavior but opposed to ChPT at low momentum.
Introducing Blatt-Weisskopf modification:

\[ \Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^{2l+1} \frac{D_l(p_R r)}{D_l(pr)} \equiv \tilde{\Gamma}(p) \frac{D_l(p_R r)}{D_l(pr)} \]

the phase shift derivative is given by:

\[ \frac{\partial \delta(p)}{\partial (M^2/\pi^2)} \approx \frac{1 + p_R^4 (r^2)'}{4p_R^4} M \tilde{\Gamma}(p) \]

Estimation of \( r^2 \) matching LO ChPT at low \( p \):

\[ r^2 = \frac{1}{g^2 f_\pi^2} \frac{M}{M_\pi} + O(M_\pi^0) \Rightarrow 1 + p_R^4 (r^2)' = 1 - \frac{M p_R^4}{2g^2 f_\pi^2 M_\pi^3} < 0 \]
Introducing Blatt-Weisskopf modification:

\[ \Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^{2l+1} \frac{D_l(p_Rr)}{D_l(pr)} \equiv \tilde{\Gamma}(p) \frac{D_l(p_Rr)}{D_l(pr)} = \tilde{\Gamma}(p) \frac{1 + (p_Rr)^2}{1 + (pr)^2} \]

the phase shift derivative is given by:

\[ \frac{\partial \delta(p)}{\partial (M^2_{\pi})} \approx \frac{1 + p^4_R (r^2)'}{4p^4_R} M \tilde{\Gamma}(p) \]

Estimation of \( r^2 \) matching LO ChPT at low \( p \):

\[ r^2 = \frac{1}{g^2 f^2_{\pi}} \frac{M}{M_{\pi}} + O(M^0_{\pi}) \Rightarrow 1 + p^4_R (r^2)' = 1 - \frac{M p^4_R}{2g^2 f^2_{\pi} M^3_{\pi}} < 0 \]
Introducing Blatt-Weisskopf modification:

\[
\Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^{2l+1} \frac{D_l(p_R r)}{D_l(pr)} \equiv \tilde{\Gamma}(p) \frac{D_l(p_R r)}{D_l(pr)} = \tilde{\Gamma}(p) \frac{1 + (p_R r)^2}{1 + (pr)^2}
\]

The phase shift derivative is given by:

\[
\frac{\partial \delta(p)}{\partial (M_\pi^2)} \approx \frac{1 + p_R^4 (r^2)'}{4p_R^4} M \tilde{\Gamma}(p)
\]

Estimation of \( r^2 \) matching LO ChPT at low \( p \):

\[
r^2 = \frac{1}{g^2 f_\pi^2} \frac{M}{M_\pi} + O(M_\pi^0) \Rightarrow 1 + p_R^4 (r^2)' = 1 - \frac{M p_R^4}{2 g^2 f_\pi^2 M_\pi^3} < 0
\]
Introducing Blatt-Weisskopf modification:

$$\Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^{2l+1} \frac{D_l(p_R r)}{D_l(pr)} \equiv \tilde{\Gamma}(p) \frac{D_l(p_R r)}{D_l(pr)} = \tilde{\Gamma}(p) \frac{1 + (p_R r)^2}{1 + (pr)^2}$$

the phase shift derivative is given by:

$$\frac{\partial \delta(p)}{\partial (M^2/\pi)} \approx \frac{1 + p_R^4 (r^2)'}{4p_R^4} M\tilde{\Gamma}(p)$$

Estimation of $r^2$ matching LO ChPT at low $p$:

$$r^2 = \frac{1}{g^2 f_\pi^2} \frac{M}{M_\pi} + O(M_\pi^0) \Rightarrow 1 + p_R^4 (r^2)' = 1 - \frac{M p_R^4}{2 g^2 f_\pi^2 M_\pi^3} < 0$$
Introducing Blatt-Weisskopf modification:

$$\Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^{2l+1} \frac{D_l(p_R r)}{D_l(pr)} \equiv \tilde{\Gamma}(p) \frac{D_l(p_R r)}{D_l(pr)} = \tilde{\Gamma}(p) \frac{1 + (p_R r)^2}{1 + (pr)^2}$$

the phase shift derivative is given by:

$$\frac{\partial \delta(p)}{\partial (M^2_\pi)} \simeq \frac{1 + p_R^4 (r^2)'}{4p_R^4} M \tilde{\Gamma}(p) < 0$$

Estimation of $r^2$ matching LO ChPT at low $p$:

$$r^2 = \frac{1}{g^2 f^2_\pi} \frac{M}{M_\pi} + O(M_\pi^0) \Rightarrow 1 + p_R^4 (r^2)' = 1 - \frac{M p_R^4}{2g^2 f^2_\pi M_\pi^3} < 0$$
Introducing Blatt-Weisskopf modification:

\[
\Gamma(p) = \Gamma_R \left( \frac{p}{p_R} \right)^{2l+1} \frac{D_l(p_Rr)}{D_l(pr)} \equiv \tilde{\Gamma}(p) \frac{D_l(p_Rr)}{D_l(pr)}.
\]

the phase shift derivative is given by:

\[
\frac{\partial \delta(p)}{\partial (M^2_\pi)} \simeq \frac{1 + p^4_R (r^2)'}{4p^4_R} M\tilde{\Gamma}(p) < 0
\]

The phase shift goes down for low \( p \) and near \( M_\pi = M_\pi^{\text{phys}} \)

Agreement with standard and unitarized ChPT.
At low $p$ the phase shift decreases as in standard ChPT.
However at higher $p$ the phase shift \textbf{grows}

At low $p$ the phase shift \textit{decreases} as in standard ChPT

\begin{align*}
\delta_{11} \text{ (deg.)} & \quad O(p^4) \\
& \quad m_{\pi} \text{ phys} \\
& \quad m_{\pi} = 350 \text{ MeV}
\end{align*}
Standard and unitarized ChPT phase shifts vs. lattice results

**ChPT**

**Lattice**
Scalar I=2 wave - one loop
I=2 J=0 phase shift at one loop

\( M_\pi = 139.57 \text{ MeV} \)

**Standard ChPT**

**Unitarized ChPT**

\( m_\pi = m_\pi \text{phys} \)

\( \delta_{20} (\text{deg.}) \)

\( p (\text{MeV}) \)

**Quark mass dependence of light resonances and phase shifts**

Jenifer Nebreda, U. Complutense de Madrid
I=2 J=0 phase shift at one loop

$M_\pi = 396$ MeV

**Standard ChPT**

$m_\pi = m_\pi \text{ phys}$

$m_\pi = 396$ MeV

**Unitarized ChPT**

$m_\pi = m_\pi \text{ phys}$

$m_\pi = 396$ MeV

Quark mass dependence of light resonances and phase shifts
I=2 J=0 phase shift at one loop

$M_\pi = 420$ MeV

Standard ChPT

Unitarized ChPT

Quark mass dependence of light resonances and phase shifts
Scalar I=2 wave

\[ M_\pi = 444 \text{ MeV} \]

**Motivation** Standard ChPT  Unitarized ChPT  ChPT vs. lattice  Resonances  Summary

**I=2 J=0 phase shift at one loop**

\[ \delta_{20} \text{ (deg.)} \]

Standard ChPT

- \( m_\pi = m_\pi \text{ phys} \)
- \( m_\pi = 396 \text{ MeV} \)
- \( m_\pi = 420 \text{ MeV} \)
- \( m_\pi = 444 \text{ MeV} \)

Unitarized ChPT

- \( m_\pi = m_\pi \text{ phys} \)
- \( m_\pi = 396 \text{ MeV} \)
- \( m_\pi = 420 \text{ MeV} \)
- \( m_\pi = 444 \text{ MeV} \)

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
$I=2$ $J=0$ phase shift at one loop

$M_\pi = 524$ MeV

Standard ChPT

Unitarized ChPT

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
**Scalar I=2 wave**

**L=2 J=0 phase shift at one loop**

\[ M_\pi = 524 \text{ MeV} \]

### Standard ChPT

- Limited to very low momenta

### Unitarized ChPT

- Improves behavior at higher momenta

**Graphs:**
- **p (MeV)** vs. **\( \delta_{20} \) (deg.)**
- Lines for different values of \( m_\pi \):
  - \( m_\pi = m_\pi \text{ phys} \)
  - \( m_\pi = 396 \text{ MeV} \)
  - \( m_\pi = 420 \text{ MeV} \)
  - \( m_\pi = 444 \text{ MeV} \)
  - \( m_\pi = 524 \text{ MeV} \)

---

**Jenifer Nebreda, U. Complutense de Madrid**

Quark mass dependence of light resonances and phase shifts
Scalar I=2 wave - two loops
$I=2$ $J=0$ phase shift at two loops

$M_\pi = 139.57$ MeV

Standard ChPT

Unitarized ChPT

$m_\pi = m_\pi \text{ phys}$

$\delta_{20}$ (deg.)

$p$ (MeV)

Quark mass dependence of light resonances and phase shifts

Jenifer Nebreda, U. Complutense de Madrid
I=2 J=0 phase shift at two loops

$M_\pi = 396$ MeV

**Standard ChPT**

- $m_\pi = m_\pi^{\text{phys}}$
- $m_\pi = 396$ MeV

**Unitarized ChPT**

- $m_\pi = m_\pi^{\text{phys}}$
- $m_\pi = 396$ MeV

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
**I=2 J=0 phase shift at two loops**

\[ M_\pi = 420 \text{ MeV} \]

**Standard ChPT**

\[ \delta_{20} \text{ (deg.)} \]

- \[ m_\pi = m_\pi \text{ phys} \]
- \[ m_\pi = 396 \text{ MeV} \]
- \[ m_\pi = 420 \text{ MeV} \]

**Unitarized ChPT**

\[ \delta_{20} \text{ (deg.)} \]

- \[ m_\pi = m_\pi \text{ phys} \]
- \[ m_\pi = 396 \text{ MeV} \]
- \[ m_\pi = 420 \text{ MeV} \]

Quark mass dependence of light resonances and phase shifts

Jenifer Nebreda, U. Complutense de Madrid
$I=2 \ J=0$ phase shift at two loops

$M_\pi = 444 \text{ MeV}$

**Standard ChPT**

- $m_\pi = m_\pi \text{ phys}$
- $m_\pi = 396 \text{ MeV}$
- $m_\pi = 420 \text{ MeV}$
- $m_\pi = 444 \text{ MeV}$

**Unitarized ChPT**

- $m_\pi = m_\pi \text{ phys}$
- $m_\pi = 396 \text{ MeV}$
- $m_\pi = 420 \text{ MeV}$
- $m_\pi = 444 \text{ MeV}$

$\delta_{20} \text{ (deg.)}$

$0 \ 100 \ 200 \ 300 \ 400 \ 500 \ 600$

$p \text{ (MeV)}$
I=2 J=0 phase shift at two loops

$M_\pi = 524$ MeV

Standard ChPT

Unitarized ChPT

Quark mass dependence of light resonances and phase shifts
Scalar $I=2$ wave

$M_\pi = 524$ MeV

$I=2$ J=0 phase shift at two loops

Bends down faster than 1 loop

No improvement

Works better than Standard ChPT at high $p$

No clear improvement either

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Tensor I=2 wave

D waves are zero at tree level:
- IAM cannot be applied at one or two loops
- one and two-loops amplitudes are only LO and NLO
I=2 J=2 phase shift in standard ChPT $M_\pi=139.57$ MeV

One loop

Two loops

Quark mass dependence of light resonances and phase shifts
$l=2$ $J=2$ phase shift in standard ChPT

$M_\pi = 396$ MeV

One loop

Two loops

$\delta_{22}$ (deg.)

$p$ (MeV)

Quark mass dependence of light resonances and phase shifts

Jenifer Nebreda, U. Complutense de Madrid
I=2 J=2 phase shift in standard ChPT

$M_\pi = 444$ MeV

One loop

Two loops

$\delta_{22}$ (deg.)

$m_\pi = m_\pi^{\text{phys}}$
$m_\pi = 396$ MeV
$m_\pi = 444$ MeV

$p$ (MeV)

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
I=2 J=2 phase shift in standard ChPT

$M_\pi = 524$ MeV

One loop

Two loops

Quark mass dependence of light resonances and phase shifts
$I=2$ $J=2$ phase shift in standard ChPT

$M_\pi = 524$ MeV

**One loop**

\[
\delta_{22} \text{ (deg.)}
\]

- $m_\pi = m_\pi^{\text{phys}}$
- $m_\pi = 396$ MeV
- $m_\pi = 444$ MeV
- $m_\pi = 524$ MeV

**Two loops**

\[
\delta_{22} \text{ (deg.)}
\]

- $m_\pi = m_\pi^{\text{phys}}$
- $m_\pi = 396$ MeV
- $m_\pi = 444$ MeV
- $m_\pi = 524$ MeV

- **Works up to higher $p$**
- **No improvement**

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Scalar and vector mesons dependence on $M_\pi$
Quark mass dependence

Generalization to $SU(3)$ of previous work on $SU(2)^*$. 

Elastic channels:

- $\pi\pi \to \pi\pi$: resonances $\rho$ and $\sigma$ (comparison to $SU(2)$ results)

- $\pi K \to \pi K$: resonances $K^*(892)$ and $\kappa$.

Change of $\hat{m} = \frac{m_u + m_d}{2}$ and $m_s \Rightarrow$

change of $M_\pi^2, M_K^2, M_\eta^2, f_\pi, f_K, f_\eta$.

Applicability in $SU(3)$: $0 < M_\pi \lesssim 400 \text{ MeV} \Rightarrow M_K \lesssim 600 \text{ MeV}$ (Being optimistic!)

Light vector mesons: $\rho$ and $K^*(892)$
Both masses increase slower than $M_\pi$

Agreement with SU(2) analysis (blue line)*

Quark mass dependence of light resonances and phase shifts

\[ \hat{m} \text{ dependence - Light vector mesons - Width} \]

\[ \frac{\hat{m}}{m_{\text{phys}}} \]

\[ \frac{\Gamma_{\rho}}{\Gamma_{\rho \text{ phys}}} \]

\[ \frac{\Gamma_{K^*}}{\Gamma_{K^* \text{ phys}}} \]

Width decrease in accordance with phase space reduction:

\[ \Gamma_V = g^2 \frac{1}{8\pi} \frac{|p|^3}{M_V^2} \]

(black lines)
Coupling to two mesons independent of $\hat{m}$ (assumption in some lattice works)
Fulfill the KSFR relation for different $\hat{m}$:

$$g \simeq \frac{M_V}{2\sqrt{2}f_\pi}$$
Light scalar mesons: $\sigma$ and $\kappa$
Motivation Standard ChPT Unitarized ChPT ChPT vs. lattice Resonances Summary

\( \hat{m} \) dependence - Light scalar mesons - Mass

\[ \frac{M_\sigma}{M_\sigma \text{phys}} \]

\[ \frac{M_\pi}{M_\pi \text{phys}} \]

\[ \frac{\hat{m}}{\hat{m}_{\text{phys}}} \]

Mass split into two branches
Agreement with SU(2) analysis

Quark mass dependence of light resonances and phase shifts

Width decrease not explained by phase space reduction:

\[ \Gamma_S = g^2 \frac{1}{8\pi} \frac{|p|}{M_S^2} \]
Motivation Standard ChPT Unitarized ChPT ChPT vs. lattice Resonances Summary

$\hat{m}$ dependence - Light scalar mesons - Coupling

Motivation

$\hat{m}$ / $\hat{m}_{phys}$

0.5 1 2 3 4 5 6 7 8 9

$g_{\sigma\pi\pi}/g_{\sigma\pi\pi\phys}$

$g_{\kappa\pi K}/g_{\kappa\pi K\phys}$

Strong $\hat{m}$ dependence of coupling to two mesons

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Chiral extrapolation of the parameters of the $\sigma (f_0(600))$, $\kappa(800)$, $\rho(770)$ and $K^*(892)$ resonances increasing $\hat{m}$.

Vector mesons

Scalar mesons
Chiral extrapolation of the parameters of the $\sigma (f_0(600))$, $\kappa(800)$, $\rho(770)$ and $K^*(892)$ resonances increasing $\hat{m}$.

Vector mesons

- vector resonances mass grows slower than $M_\pi$,

Scalar mesons
Chiral extrapolation of the parameters of the $\sigma$ ($f_0(600)$), $\kappa(800)$, $\rho(770)$ and $K^*(892)$ resonances increasing $\hat{m}$.

**Vector mesons**

- vector resonances mass grows slower than $M_\pi$,
- coupling to two mesons almost independent of $M_\pi$,

**Scalar mesons**
Chiral extrapolation of the parameters of the $\sigma (f_0(600))$, $\kappa(800)$, $\rho(770)$ and $K^*(892)$ resonances increasing $\hat{m}$.

**Vector mesons**
- vector resonances mass grows slower than $M_\pi$,
- coupling to two mesons almost independent of $M_\pi$,
- KSFR is well satisfied for different quark masses.

**Scalar mesons**
Summary

Chiral extrapolation of the parameters of the $\sigma$ ($f_0(600)$), $\kappa(800)$, $\rho(770)$ and $K^*(892)$ resonances increasing $\hat{m}$.

Vector mesons

- vector resonances mass grows slower than $M_\pi$,
- coupling to two mesons almost independent of $M_\pi$,
- KSFR is well satisfied for different quark masses.

Scalar mesons

- very different behavior from vector mesons: two branches,
Summary

Chiral extrapolation of the parameters of the $\sigma$ ($f_0(600)$), $\kappa(800)$, $\rho(770)$ and $K^*(892)$ resonances increasing $\hat{m}$.

Vector mesons

- vector resonances mass grows slower than $M_\pi$,  
- coupling to two mesons almost independent of $M_\pi$, 
- KSFR is well satisfied for different quark masses.

Scalar mesons

- very different behavior from vector mesons: two branches, 
- $\sigma$ and $\kappa$ show different quantitative but similar qualitative behavior,
Summary

Chiral extrapolation of the parameters of the $\sigma$ ($f_0(600)$), $\kappa(800)$, $\rho(770)$ and $K^*(892)$ resonances increasing $\hat{m}$.

Vector mesons

- vector resonances mass grows slower than $M_\pi$,
- coupling to two mesons almost independent of $M_\pi$,
- KSFR is well satisfied for different quark masses.

Scalar mesons

- very different behavior from vector mesons: two branches,
- $\sigma$ and $\kappa$ show different quantitative but similar qualitative behavior,
- coupling to two mesons shows stronger $M_\pi$ dependence.
We have presented recent results for the phase shifts $M_\pi$ dependence:

- Standard ChPT
- Unitarized ChPT
We have presented recent results for the phase shifts $M_\pi$ dependence:

**Standard ChPT**
- very soft $M_\pi$ dependence once threshold is "subtracted",

**Unitarized ChPT**
We have presented recent results for the phase shifts $M_\pi$ dependence:

**Standard ChPT**
- very soft $M_\pi$ dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,

**Unitarized ChPT**
We have presented recent results for the phase shifts $M_\pi$ dependence:

**Standard ChPT**
- very soft $M_\pi$ dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,
- S2 wave: agreement with lattice only at very low $p$,

**Unitarized ChPT**
We have presented recent results for the phase shifts $M_\pi$ dependence:

**Standard ChPT**
- very soft $M_\pi$ dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,
- S2 wave: agreement with lattice only at very low $p$,
- D2 wave: fair agreement with lattice at 1 loop, spoilt at 2 loops.

**Unitarized ChPT**
We have presented recent results for the phase shifts $M_\pi$ dependence:

**Standard ChPT**
- very soft $M_\pi$ dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,
- S2 wave: agreement with lattice only at very low $p$,
- D2 wave: fair agreement with lattice at 1 loop, spoilt at 2 loops.

**Unitarized ChPT**
- S2 wave: better agreement with lattice at high $p$,
We have presented recent results for the phase shifts $M_\pi$ dependence:

**Standard ChPT**
- very soft $M_\pi$ dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,
- S2 wave: agreement with lattice only at very low $p$,
- D2 wave: fair agreement with lattice at 1 loop, spoilt at 2 loops.

**Unitarized ChPT**
- S2 wave: better agreement with lattice at high $p$,
- similar results at one and two loops,
Summary

We have presented recent results for the phase shifts $M_\pi$ dependence:

Standard ChPT

- very soft $M_\pi$ dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,
- S2 wave: agreement with lattice only at very low $\rho$,
- D2 wave: fair agreement with lattice at 1 loop, spoilt at 2 loops.

Unitarized ChPT

- S2 wave: better agreement with lattice at high $\rho$,
- similar results at one and two loops,
- reconciles $\rho$ ChPT behavior with naive expectation,
We have presented recent results for the phase shifts $M_\pi$ dependence:

**Standard ChPT**
- very soft $M_\pi$ dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,
- S2 wave: agreement with lattice only at very low $p$,
- D2 wave: fair agreement with lattice at 1 loop, spoilt at 2 loops.

**Unitarized ChPT**
- S2 wave: better agreement with lattice at high $p$,
- similar results at one and two loops,
- reconciles $\rho$ ChPT behavior with naive expectation,
- bound states seen as $2\pi$ jump in phase shift (Levinson’s).
We have presented recent results for the phase shifts $M_\pi$ dependence *:

**Standard ChPT**
- very soft $M_\pi$ dependence once threshold is "subtracted",
- surprising decrease of phase in vector channel,
- S2 wave: agreement with lattice only at very low $p$,
- D2 wave: fair agreement with lattice at 1 loop, spoilt at 2 loops.

**Unitarized ChPT**
- S2 wave: better agreement with lattice at high $p$,
- similar results at one and two loops,
- reconciles $\rho$ ChPT behavior with naive expectation,
- bound states seen as $2\pi$ jump in phase shift (Levinson’s).

$m_s$ dependence of $\sigma, \rho, \kappa$ and $K^*(892)$
Light vector mesons: $\rho$ and $K^*(892)$
$m_s$ dependence - Light vector mesons - Mass & Width

$M_\rho / M_\rho \text{phys}$

$M_{K^*} / M_{K^*} \text{phys}$

$\Gamma_\rho / \Gamma_\rho \text{phys}$

$\Gamma_{K^*} / \Gamma_{K^*} \text{phys}$

Quark mass dependence of light resonances and phase shifts
Quark mass dependence of light resonances and phase shifts

- Light vector mesons
- Mass & Width

$m_s$ dependence - Motivation Standard ChPT - Unitarized ChPT - ChPT vs. lattice - Resonances - Summary

Graphs showing $m_s / m_s \text{phys}$ vs. $M_\rho / M_\rho \text{phys}$, $M_{K^*} / M_{K^*} \text{phys}$, $\Gamma_\rho / \Gamma_\rho \text{phys}$, and $\Gamma_{K^*} / \Gamma_{K^*} \text{phys}$.
Quark mass dependence of light resonances and phase shifts

Coupling to two mesons constant
Quark mass dependence of light resonances and phase shifts

KSFR relation well satisfied for different $m_s$
Light scalar mesons: $\sigma$ and $\kappa$
Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Summary

$m_s$ dependence - Light scalar mesons - Mass & Width

- $M_\sigma/M_\sigma \text{phys}$
- $M_\kappa/M_\kappa \text{phys}$
- $\Gamma_\sigma/\Gamma_\sigma \text{phys}$
- $\Gamma_\kappa/\Gamma_\kappa \text{phys}$

Jenifer Nebreda, U. Complutense de Madrid

Quark mass dependence of light resonances and phase shifts
Motivation
Standard ChPT
Unitarized ChPT
ChPT vs. lattice
Resonances
Summary

$m_s$ dependence - Light scalar mesons - Coupling

Quark mass dependence of light resonances and phase shifts