The role of final-state interactions in Dalitz plot studies

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The role of final-state interactions in Dalitz plot studies

Introduction

- Dalitz plots and CP violation
- the usefulness of hadronic input

What do (low-energy) hadron physicists have on offer?

- scattering consistent with analyticity and unitarity: Roy equations
- decays linked to scattering: form factors and Omnès solution
- low-energy constraints: amplitudes consistent with chiral symmetry (only mentioned in passing)
- many-particle dynamics for the example of $\eta \to 3\pi$

Les Nabis group

input from C. Hanhart gratefully acknowledged

CP violation in three-body decays

Advantage of 3-body decays:

- resonance-rich environment
- larger branching fractions

here: $B^\pm \to K^\pm \pi^\mp \pi^\pm$

e.g. 3.7σ signal in $K\rho$

BELLE 2006, BABAR 2008

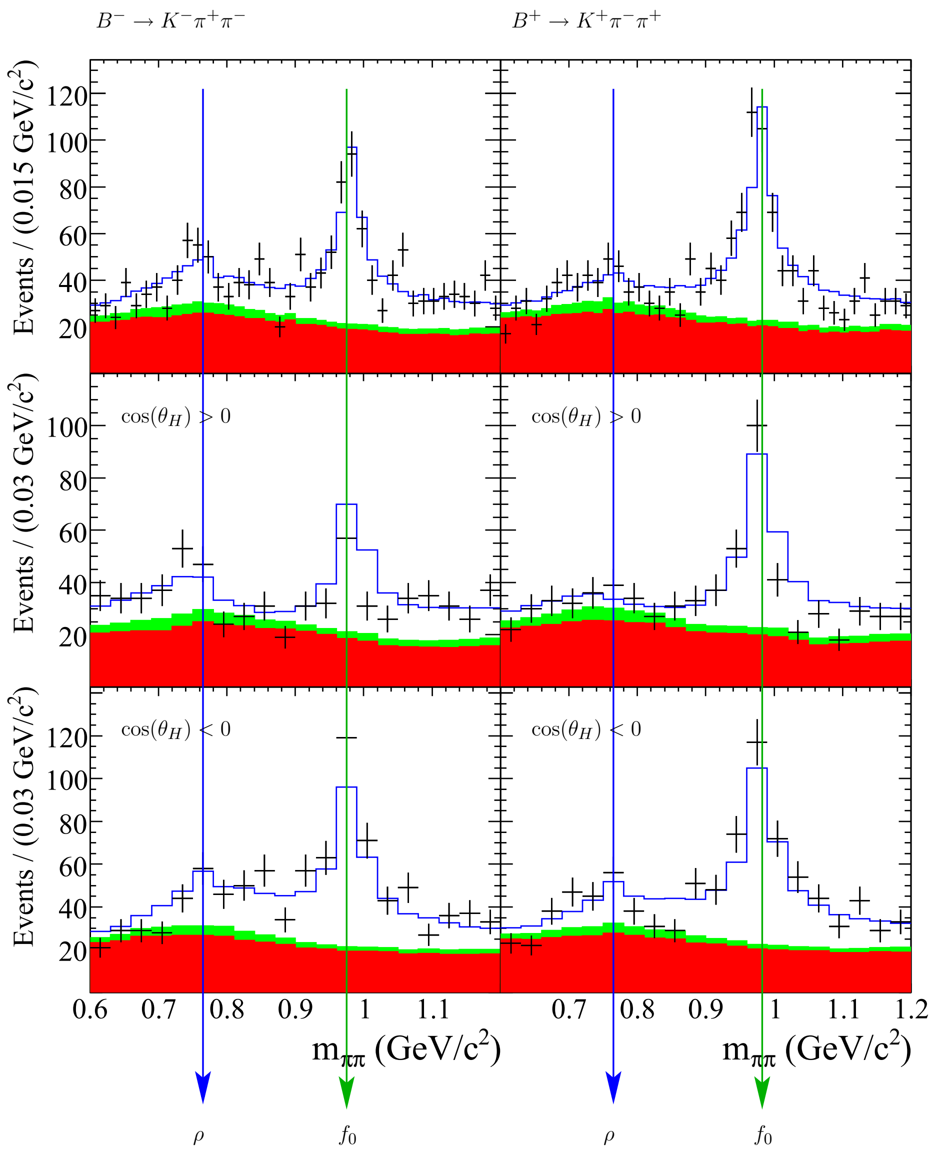
How to analyse CP violation in Dalitz plots?

1. strictly model-independent extraction from data directly

   Gardner et al. 2003, 2004

   Bediaga et al. 2009

2. theoretical information on strong amplitudes as input!



Direct data analysis: significance

Bediaga et al. 2009

Significance in Dalitz plot distributions:

$$^{\mathrm{Dp}}S_{\mathrm{CP}}(i) \doteq \frac{N(i) - \bar{N}(i)}{\sqrt{N(i) + \bar{N}(i)}}$$

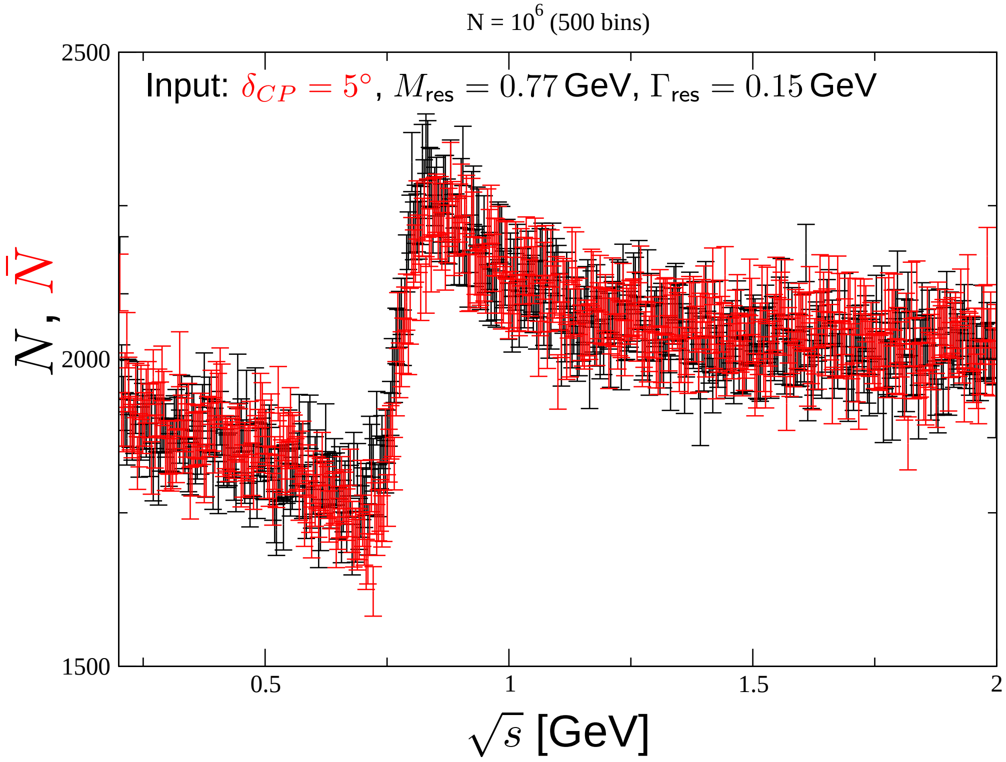
N, $\bar{N}$: CP-conjugate decays; i: label of a specific Dalitz plot bin

- allows to study local asymmetries
- no theoretical input required at all — strictly model-independent
- B decays: clear evidence, in particular in $B^\pm \to K^\pm \rho^0 (\to \pi^\pm \pi^\mp)$
  consistent with Standard Model BELLE 2006, BABAR 2008
- D decays: only upper limits (at few-percent level)
  Standard Model prediction tiny BABAR 2008

Illustration: the use of hadronic amplitudes (1)

- model: resonance plus CP-violating phase   provided by C. Hanhart

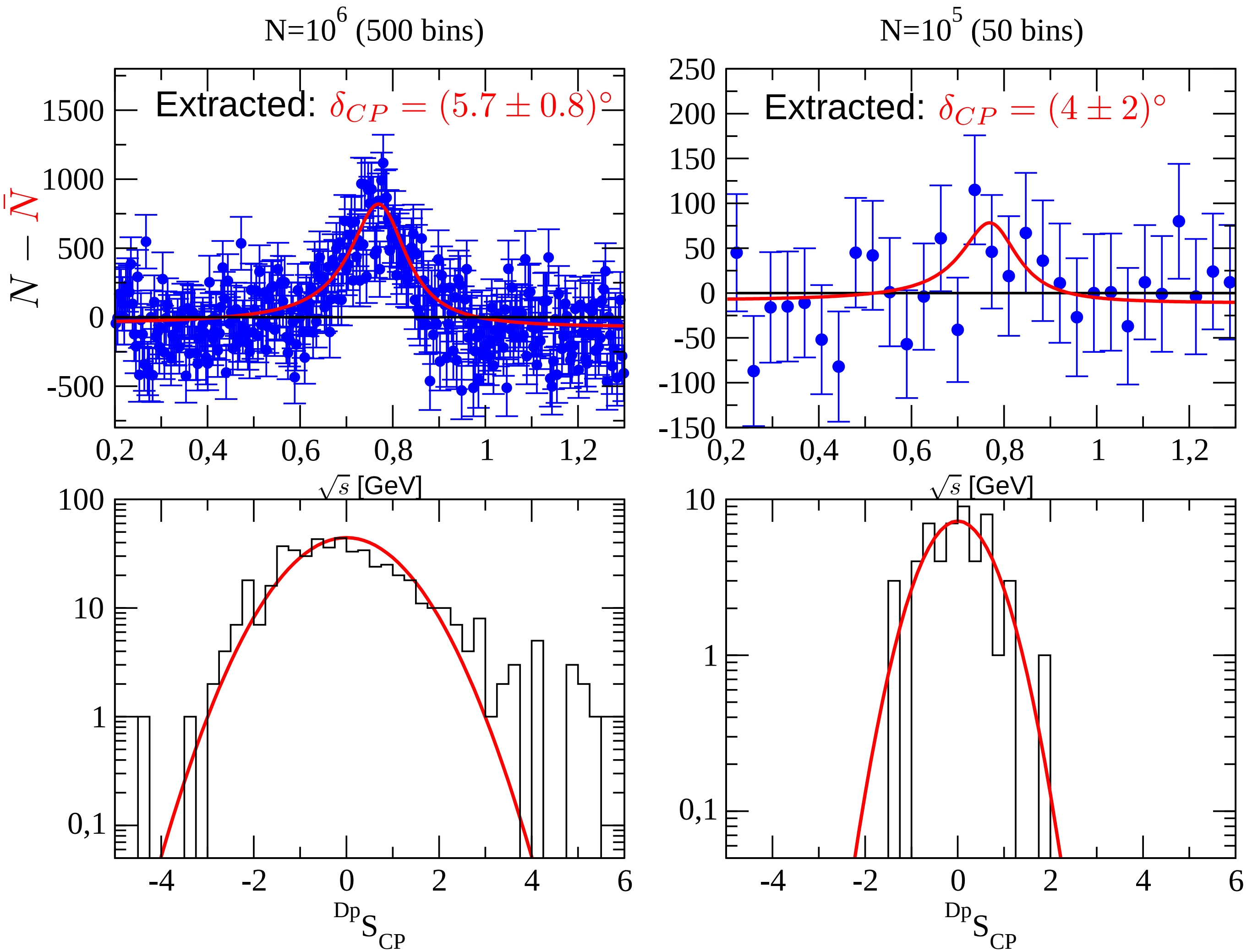
$$N,\ \bar{N} = \alpha + \beta\,\mathrm{Re}\left\{\frac{\exp(\pm i\delta_{CP})}{s - M_{\mathrm{res}}^2 + iM_{\mathrm{res}}\Gamma_{\mathrm{res}}}\right\}$$



- asymmetry: $N - \bar{N} = \sin\delta_{CP} \times \dfrac{2\beta M_{\mathrm{res}}\Gamma_{\mathrm{res}}}{(s - M_{\mathrm{res}}^2)^2 + (M_{\mathrm{res}}\Gamma_{\mathrm{res}})^2}$

Illustration: the use of hadronic amplitudes (2)

Input: $\delta_{CP} = 5°$, $M_{\text{res}} = 0.77\,\text{GeV}$, $\Gamma_{\text{res}} = 0.15\,\text{GeV}$



- no signal in significance
  hadronic amplitudes still allow to extract phase δ_{CP}

$\pi\pi$ scattering constrained by analyticity and unitarity

compare also talk by M. Hoferichter on Tuesday

Roy equations = coupled system of partial-wave dispersion relations
+ crossing symmetry + unitarity

- twice-subtracted fixed-t dispersion relation:

$$T(s,t) = c(t) + \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \left\{ \frac{s^2}{s'^2(s'-s)} + \frac{u^2}{s'^2(s'-u)} \right\} \mathrm{Im}T(s',t)$$

- subtraction function $c(t)$ determined from crossing symmetry

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- subtraction function $c(t)$ determined from crossing symmetry
- project onto partial waves $t_J^I(s)$ (angular momentum J, isospin I)
$\Rightarrow$ coupled system of partial-wave integral equations

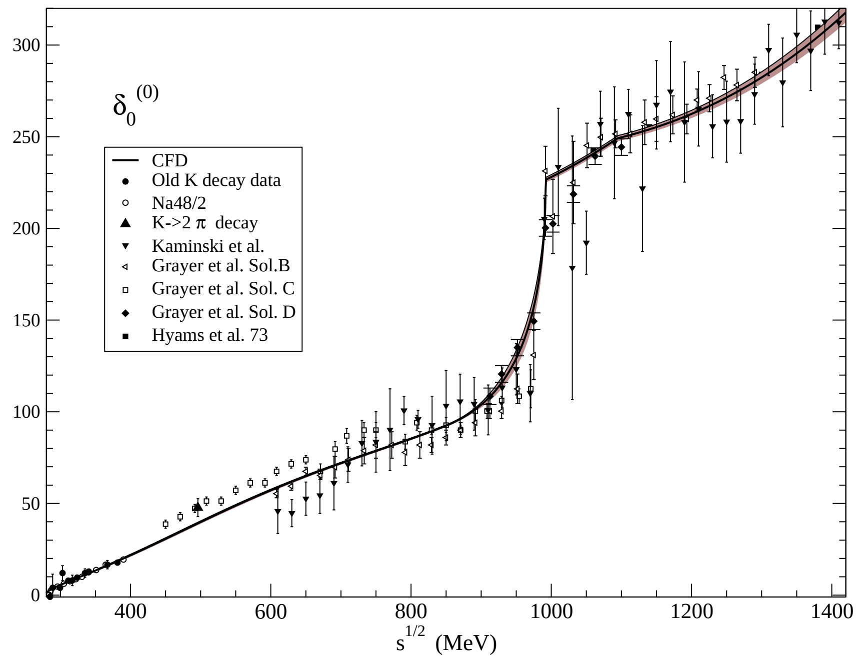
$$t_J^I(s) = k_J^I(s) + \sum_{I'=0}^{2}\sum_{J'=0}^{\infty}\int_{4M_\pi^2}^{\infty} ds' K_{JJ'}^{II'}(s,s')\mathrm{Im}t_{J'}^{I'}(s')$$

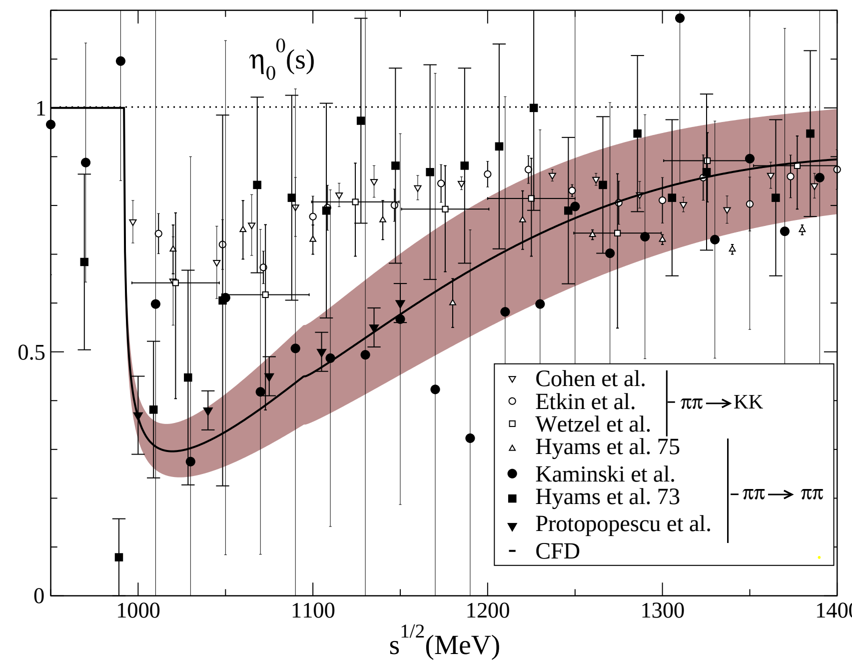
Roy 1971

- subtraction polynomial $k_J^I(s)$: $\pi\pi$ scattering lengths
can be matched to chiral perturbation theory Colangelo et al. 2001
- kernel functions $K_{JJ'}^{II'}(s,s')$ known analytically

$\pi\pi$ scattering constrained by analyticity and unitarity

- elastic unitarity $\Rightarrow$ coupled integral equations for phase shifts
- modern precision analyses:
  - ▷ $\pi\pi$ scattering — Ananthanarayan et al. 2001, García-Martín et al. 2011
  - ▷ πK scattering — Büttiker et al. 2004
- example: $\pi\pi$ $I = 0$ S-wave phase shift & inelasticity



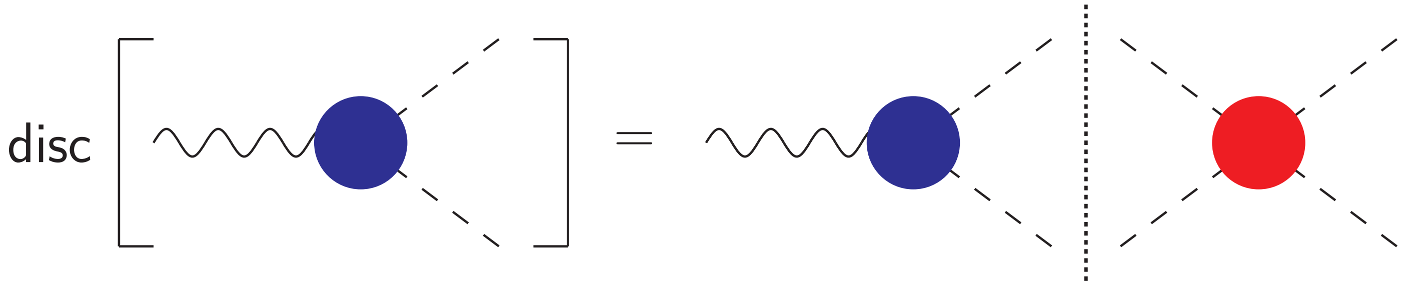


García-Martín et al. 2011

- strong constraints on data from analyticity and unitarity!

Analyticity and unitarity: form factor

- just two particles in final state (form factor); from unitarity:



$$\text{Im}\, F_I(s) = F_I(s) \times \theta(s - 4\,M_\pi^2) \times \sin\delta_I(s)\, e^{-i\delta_I(s)}$$

$\Rightarrow$ Watson's final-state theorem — Watson 1954

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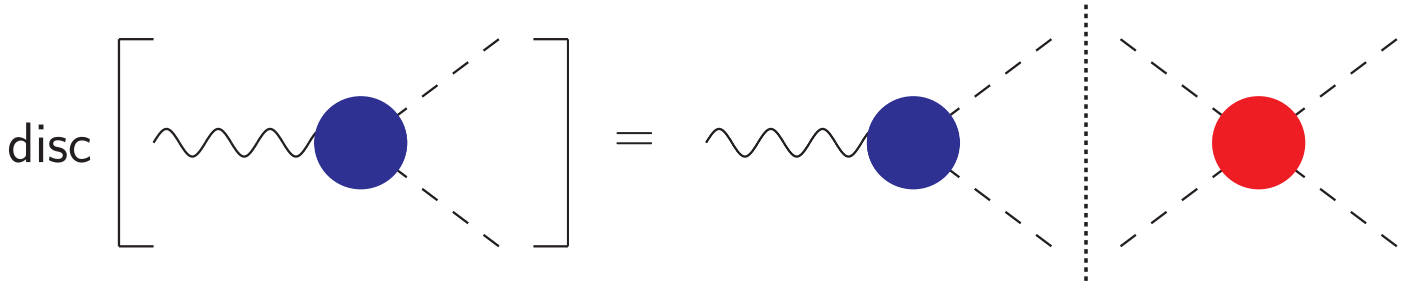

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- solution to this homogeneous integral equation known:

$$F_I(s) = P_I(s)\Omega_I(s)\;, \quad \Omega_I(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s)}\right\}$$

$P_I(s)$ polynomial, $\Omega_I(s)$ Omnès function — Omnès 1958

completely given in terms of phase shift $\delta_I(s)$

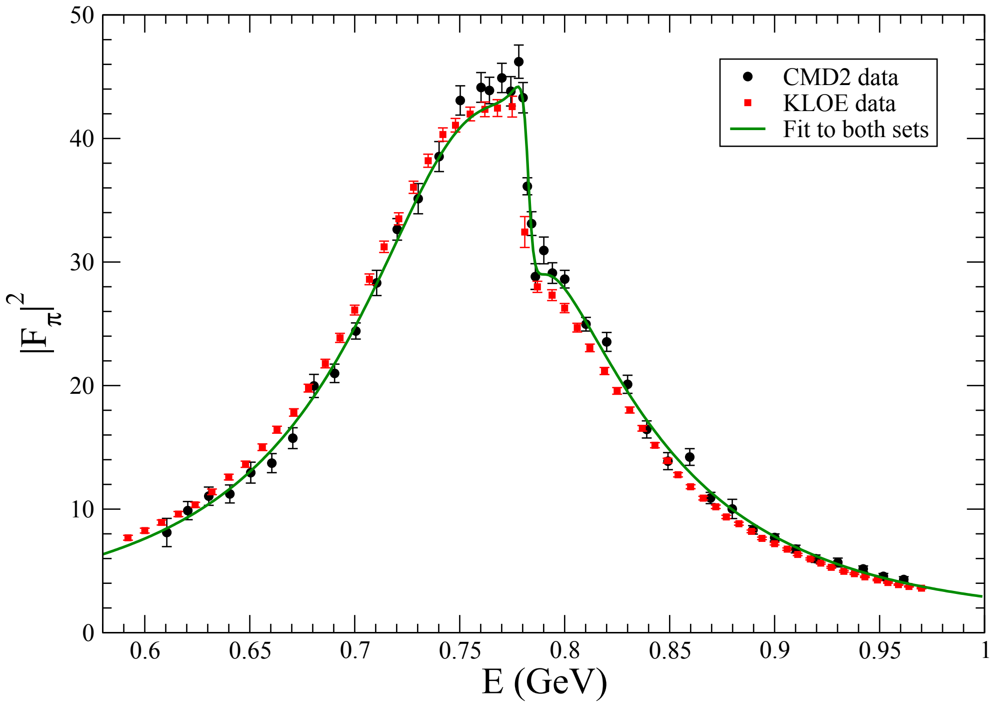
Pion vector form factor and $a_\mu^{\rm hvp}$

- more refined representation: taken from talk by G. Colangelo 2008

$$F_V^\pi(s) = \Omega_1(s)\Omega_{\rm inel}(s)G_\omega(s)$$

$\Omega_{\rm inel}(s)$: inelastic for $\sqrt{s} \gtrsim (M_\pi + M_\omega)$, parametrized using conformal mapping techniques Trocóniz, Ynduráin 2002

$G_\omega(s)$: $\rho - \omega$ mixing



- achieve amazing precision for hadronic contribution to a_μ below 1 GeV:

$$a_\mu^{\rm hvp}(\sqrt{s} \leq 2M_K)$$
$$= (493.7 \pm 1.0) \times 10^{-10}$$

Colangelo et al. (preliminary)

- check of data compatibility with analyticity / unitarity

Dispersion relations for three-body decays

compare also following talk by P. Magalhães

Example: $\eta \to 3\pi$

- interesting due to relation to light quark mass ratios
- $\mathcal{M}(s,t,u) \propto \mathcal{A}(\eta \to \pi^+\pi^-\pi^0)$ can be decomposed according to

$$\mathcal{M}(s,t,u) = \mathcal{M}_0(s)+(s-t)\mathcal{M}_1(u)+(s-u)\mathcal{M}_1(t)+\mathcal{M}_2(t)+\mathcal{M}_2(u)-\frac{2}{3}\mathcal{M}_2(s)$$

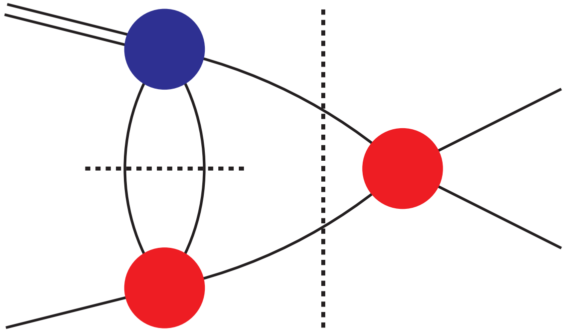
$\mathcal{M}_I(s)$ functions of one variable with only a right-hand cut

Stern, Sazdjian, Fuchs 1993; Anisovich, Leutwyler 1998

- I: isospin, i.e. $\mathcal{M}_{0,2}$ S-waves, $\mathcal{M}_1$ P-wave
- decomposition exact if discontinuities in D- and higher partial waves neglected

From unitarity to integral equations: inhomogeneities

- more complicated unitarity relation for 4-point functions:



$$\text{Im}\,\mathcal{M}_I(s) = \{\mathcal{M}_I(s) + \hat{\mathcal{M}}_I(s)\} \times \theta(s - 4\,M_\pi^2) \times \sin\,\delta_I(s)\,e^{-i\delta_I(s)}$$

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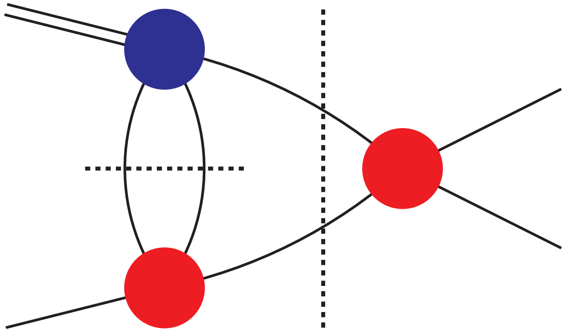

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- inhomogeneities $\hat{\mathcal{M}}_I(s)$: angular averages over the $\mathcal{M}_I(s)$: e.g.

$$\hat{\mathcal{M}}_0 = \frac{2}{3}\langle\mathcal{M}_0\rangle + \frac{20}{9}\langle\mathcal{M}_2\rangle + 2(s - s_0)\langle\mathcal{M}_1\rangle + \frac{2}{3}\kappa\langle z\mathcal{M}_1\rangle\ , \quad s_0 = \frac{M_\eta^2 + 3M_\pi^2}{3}$$

$$\langle z^n f\rangle = \frac{1}{2}\int_{-1}^{1} dz\, z^n f\big(t(s,z)\big)\ , \quad \kappa = \sqrt{1 - \frac{4\,M_\pi^2}{s}} \times \lambda^{1/2}(s, M_\eta^2, M_\pi^2)$$

From unitarity to integral equations: inhomogeneities

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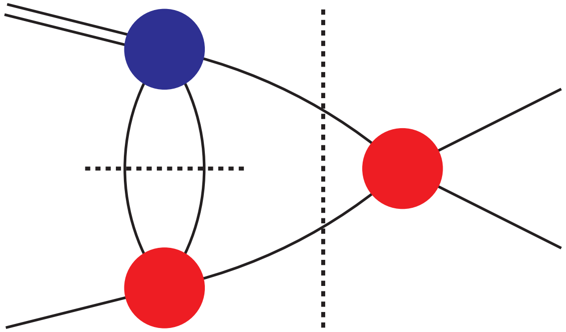

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- allows for cross-channel scattering between s-, t-, and u-channel
- "angular averaging" non-trivial
  $\Rightarrow$ generates complex analytic structure (3-particle cuts)

From unitarity to integral equations: solution

- integral equations including the inhomogeneities $\hat{\mathcal{M}}_I$:

$$\mathcal{M}_0(s) = \Omega_0(s)\left\{\alpha_0+\beta_0\, s+\gamma_0\, s^2+\frac{s^3}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^3}\frac{\sin\delta_0(s')\hat{\mathcal{M}}_0(s')}{|\Omega_0(s')|(s'-s-i\epsilon)}\right\}$$

  + 2 similar for $\mathcal{M}_{1,2}(s)$; 4 subtraction constants to be fixed

  Khuri, Treiman 1960; Aitchison 1977; Anisovich, Leutwyler 1998

- solve these equations iteratively by a numerical procedure

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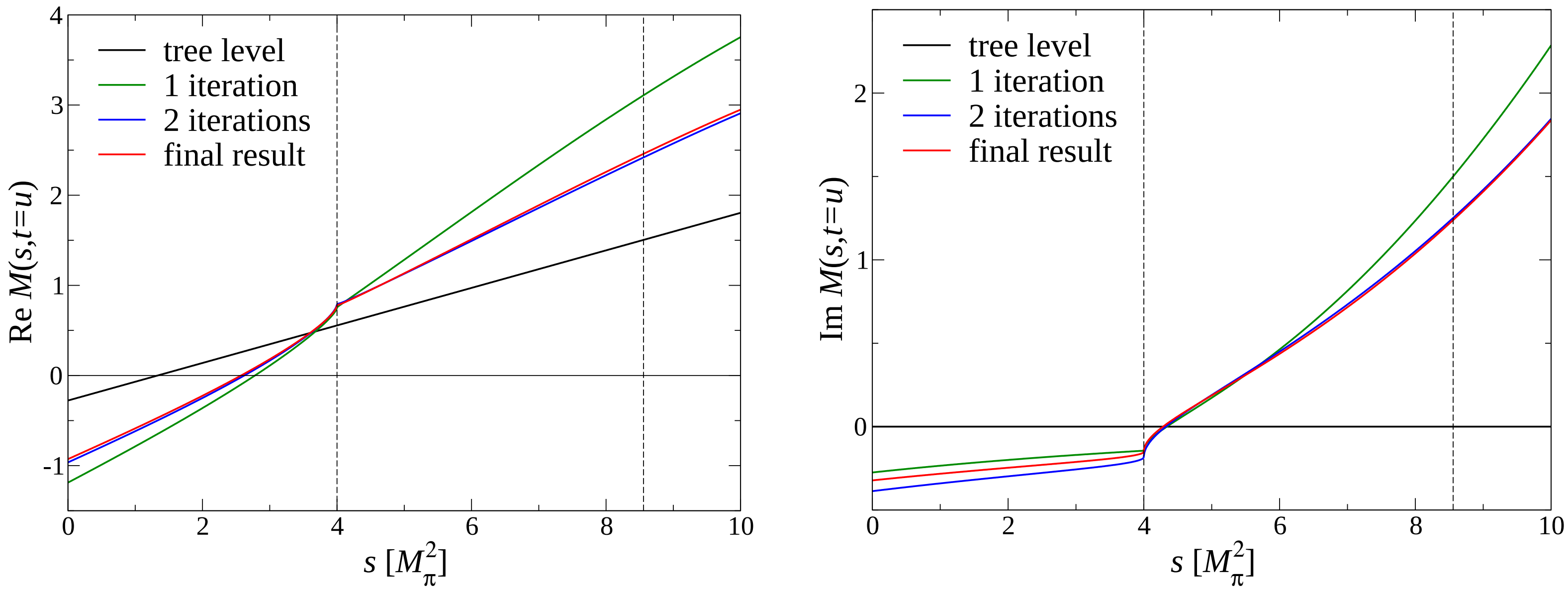

Schneider, Kubis; compare Colangelo et al. 2010

- fast convergence: close to final result after 2 iterations

Extensions to heavier decays

- currently extended to other decays: $\eta' \to \eta\pi\pi$, $\eta' \to 3\pi$, $\omega \to 3\pi$

  Schneider, Niecknig, Kubis

- challenges for going to heavier meson decays:
  - ▷ at higher energies: coupled-channel integral equation
  - ▷ inelasticities certainly not negligible
  - ▷ perturbative treatment of crossed-channel effects reliable?
  - ▷ when are higher partial waves non-negligible?

Les Nabis

Paul Serusier, *The talisman* (1888)

Les Nabis

informal network to bring together

particle ("heavy-quark") and

hadron ("light-quark") physicists

from theory

I. Bigi, S. Gardner, C. Hanhart, B. Kubis, T. Mannel, U.-G. Meißner, J.R. Peláez, M.R. Pennington. . .

and experiment

I. Bediaga, A.E. Bondar, A. Denig, T.J. Gershon, W. Gradl, B.T. Meadows, K. Peters, U. Wiedner, G. Wilkinson. . .

to optimize future Dalitz plot CP-studies along these lines!