Heavy – quark masses and heavy – meson decay constants from Borel sum rules in QCD

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A new extraction of the decay constants of D, D_s, B, and B_s mesons from the two-point function of heavy-light pseudoscalar currents is presented. Our main emphasis is laid on the uncertainties in these quantites, both related to the OPE for the relevant correlators and to the extraction procedures of the method of sum rules.

Based on "Heavy-meson decay constants from QCD sum rules" arXiv:1008.3129
"OPE, charm-quark mass, and decay constants of D and D_s mesons from QCD sum rules" arXiv:1101.5986, Phys. Lett. **B 701**, *in press*

A QCD sum-rule calculation of hadron parameters involves two steps:

I. Calculating the operator product expansion (OPE) series for a relevant correlator

One observes a very strong dependence of the OPE for the correlator (and, consequently, of the extracted decay constant) on the heavy-quark mass used, i.e., on-shell (pole), or running $\overline{MS}$ mass.

We make use of the three-loop OPE for the correlator by Chetyrkin et al, reshuffled in terms of $\overline{MS}$ mass, in which case OPE exhibits a reasonable convergence.

II. Extracting the parameters of the ground state by a numerical procedure

NEW :

(a) Make use of the new more accurate duality relation based on Borel-parameter-dependent threshold.

Allows a more accurate extraction of the decay constants and provides realistic estimates of the intrinsic (systematic) errors — those related to the limited accuracy of sum-rule extraction procedures.

(b) Study the sensitivity of the extracted value of f_P to the OPE parameters (quark masses, condensates,…). The corresponding error is referred to as OPE uncertainty, or statistical error.

Basic object: OPE for $\Pi(p^2) = i\int dx e^{ipx}\langle 0|T\left(j_5(x)j_5^\dagger(0)\right)|0\rangle$, $\qquad j_5(x) = (m_Q + m)\bar{q}i\gamma_5 Q(x)$
and its Borel transform ($p^2 \to \tau$).

Quark – hadron duality assumption :

$$f_Q^2 M_Q^4 e^{-M_Q^2\tau} = \int_{(m_Q+m_u)^2}^{s_{\rm eff}} e^{-s\tau}\rho_{\rm pert}(s,\alpha,m_Q,\mu)\,ds + \Pi_{\rm power}(\tau,m_Q,\mu) \equiv \Pi_{\rm dual}(\tau,\mu,s_{\rm eff})$$

In order the l.h.s. and the r.h.s. have the same τ**-behavior**

$s_{\rm eff}$ **is a function of** τ **(and** μ**) :** $s_{\rm eff}(\tau,\mu)$

The "dual" mass: $$M_{\rm dual}^2(\tau) = -\frac{d}{d\tau}\log\Pi_{\rm dual}(\tau, s_{\rm eff}(\tau)).$$

If quark-hadron duality is implemented "perfectly", then $M_{\rm dual}$ **should be equal to** M_Q**;**
The deviation of $M_{\rm dual}$ **from the actual meson mass** M_Q **measures the contamination of the dual correlator by excited states. Better reproduction of** $M_Q \to$ **more accurate extraction of** f_Q**.**

Taking into account τ**-dependence of** $s_{\rm eff}$
improves the accuracy of the duality approximation.

Obviously, in order to predict f_Q**, we need to fix** $s_{\rm eff}$**. How to fix** $s_{\rm eff}$**?**

Our new algorithm for extracting ground – state parameters when M_Q is known

For a given trial function $s_{\rm eff}(\tau)$ there exists a variational solution which minimizes the deviation of the dual mass from the actual meson mass in the τ-"window" (only a few lowest-dimension power corrections are known, work at $\tau m_Q \le 1$).

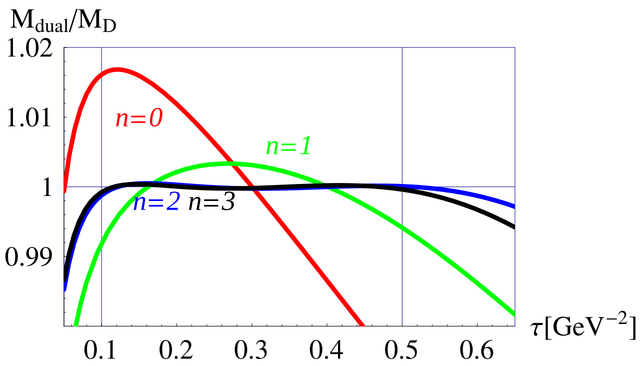
(i) Consider a set of Polynomial τ-dependent Ansaetze for $s_{\rm eff}$: $s_{\rm eff}^{(n)}(\tau) = \sum_{j=0}^{n} s_j^{(n)}(\tau)^j$.

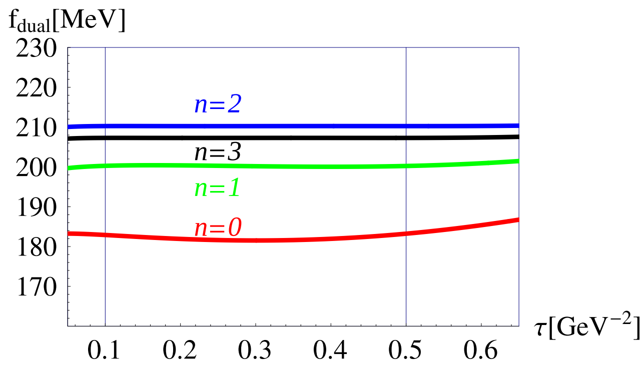
(ii) Minimize the squared difference between the "dual" mass $M^2_{\rm dual}$ and the known value M_Q^2 in the τ-window. This gives us the parameters of the effective continuum threshold.

(iii) Making use of the obtained thresholds, calculate the decay constant.

(iv) Take the band of values provided by the results corresponding to <u>linear</u>, <u>quadratic</u>, and <u>cubic</u> effective thresholds as the characteristic of the intrinsic uncertainty of the extraction procedure.

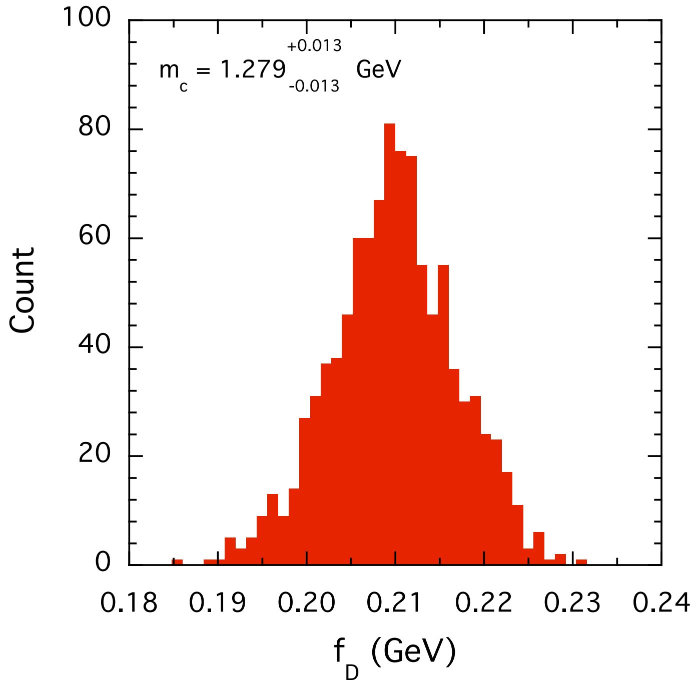
Illustration: D-meson

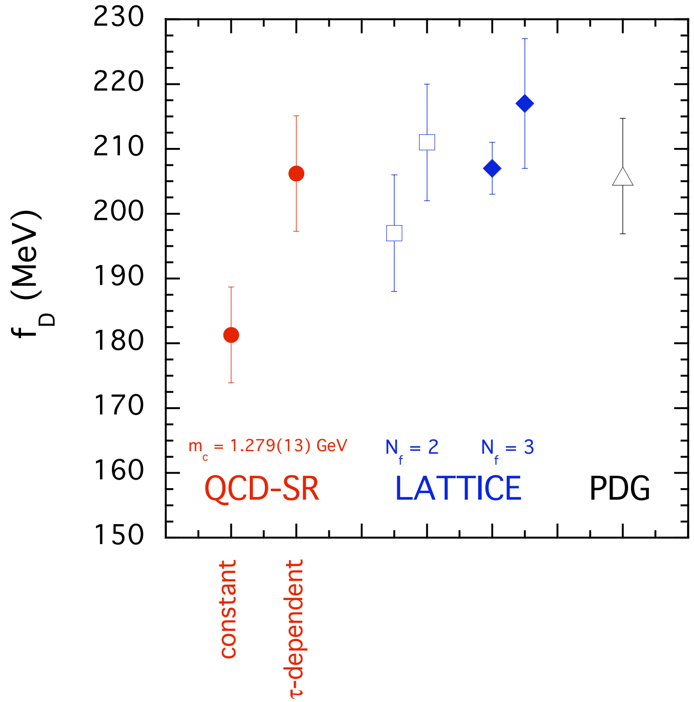




Extraction of f_D

$m_c(m_c) = 1.279 \pm 0.013\,\text{GeV}, \mu = 1 - 3\,\text{GeV}.$





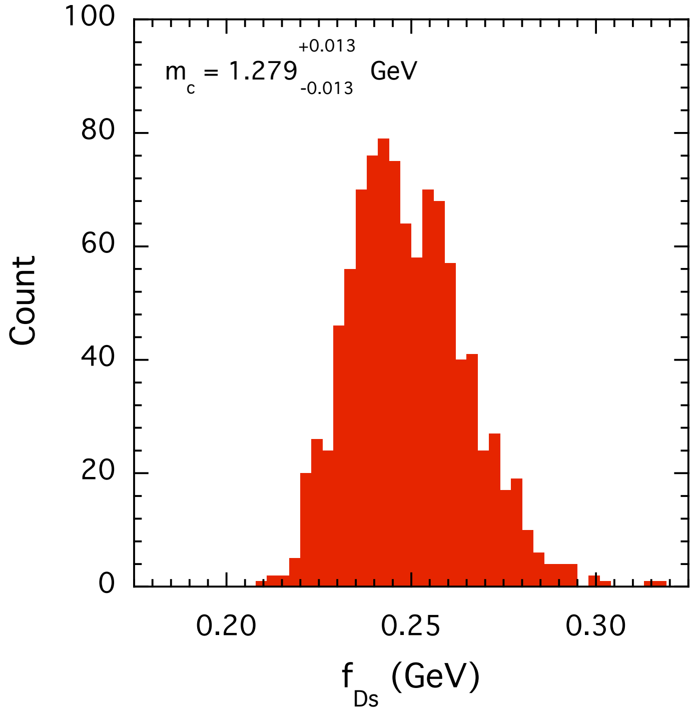
$f_D = 206.2 \pm 7.3_{OPE} \pm 5.1_{syst}$ MeV

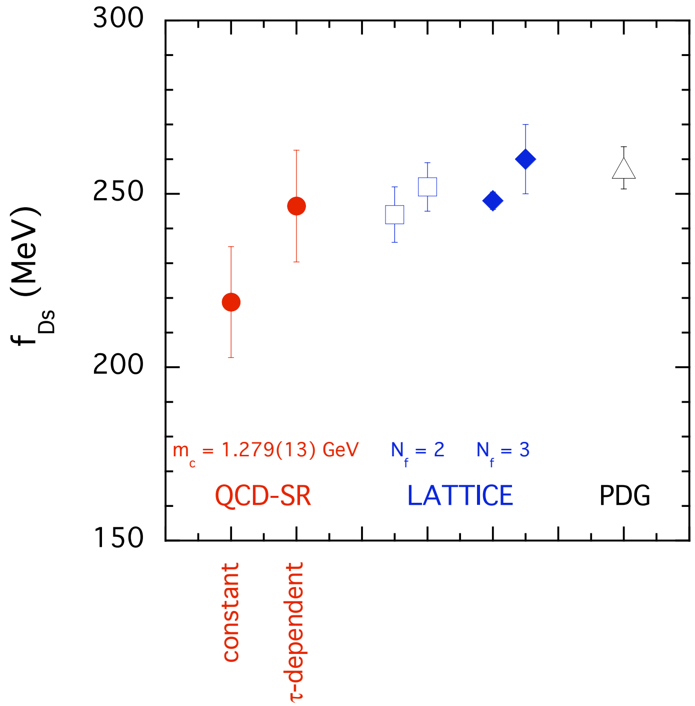
f_D (const) = 181.3 $\pm$ 7.4$_{OPE}$ MeV

The effect of τ-dependent threshold is visible!

Extraction of f_{Ds}

$m_c(m_c) = 1.279 \pm 0.013\,\text{GeV}, \mu = 1 - 3\,\text{GeV}.$

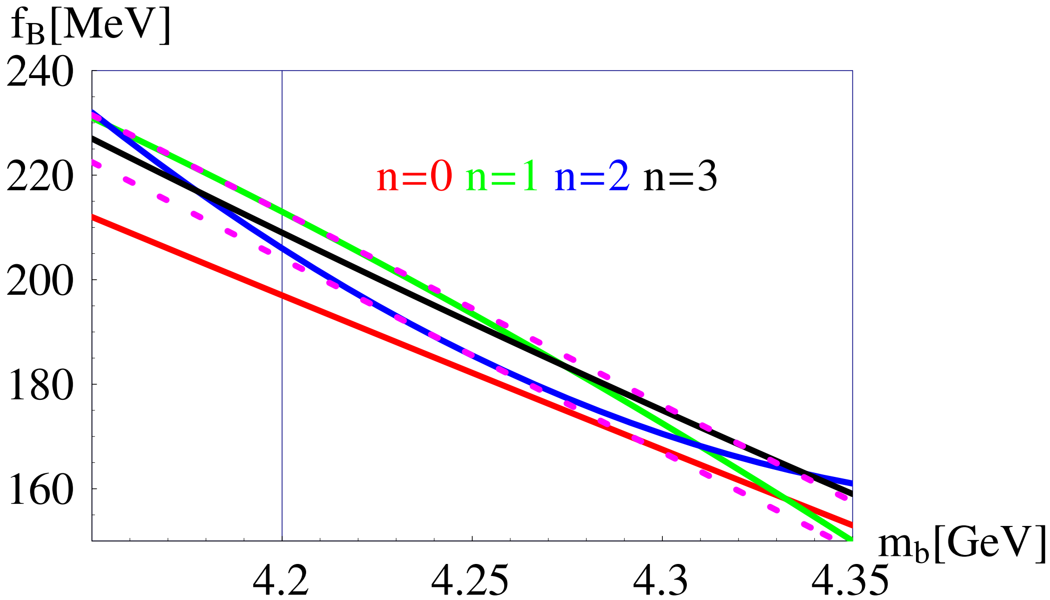




$$f_{D_s} = 246.5 \pm 15.7_{OPE} \pm 5_{syst}\ \text{MeV}$$

$$f_{Ds}\ (\text{const}) = 218.8 \pm 16.1_{OPE}\ \text{MeV}$$

Extraction of f_B: a very strong sensitivity to $m_b(m_b)$

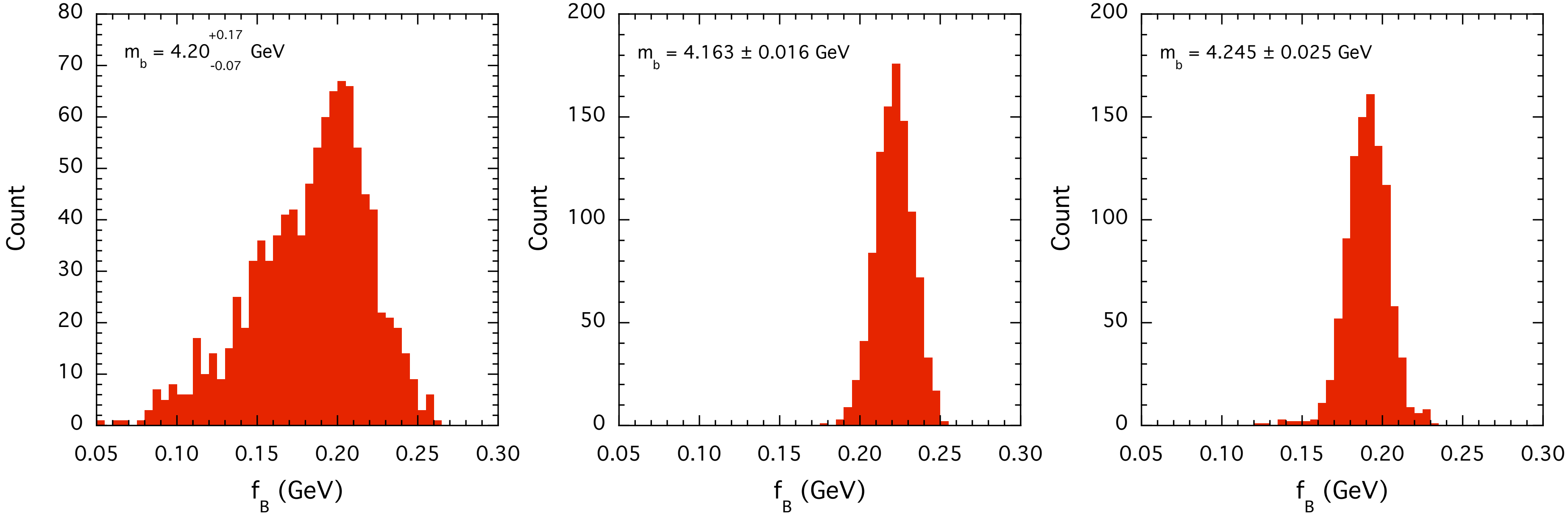


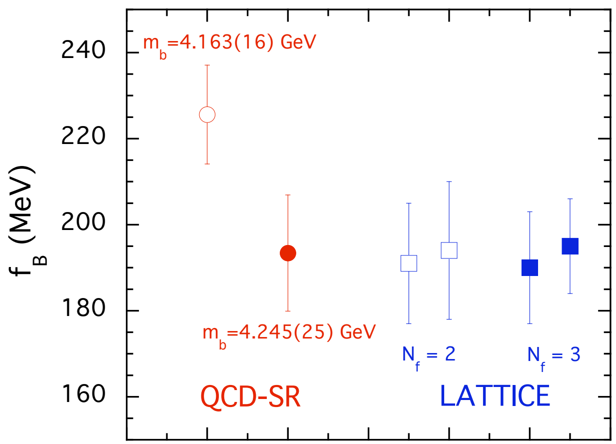
τ-dependent effective threshold:

$$f_B^{\text{dual}}(m_b, \langle\bar{q}q\rangle, \mu = m_b) = \left[206.5 \pm 4 - 37\left(\frac{m_b - \mathbf{4.245\ GeV}}{\mathbf{0.1\ GeV}}\right) + 4\left(\frac{\langle\bar{q}q\rangle^{1/3} - \mathbf{0.267\ GeV}}{\mathbf{0.01\ GeV}}\right)\right] MeV,$$

$\pm$ 10 MeV on $m_b \to \mp$ 37 MeV on f_B!

The prediction for f_B is not feasible without a very precise knowledge of m_b:



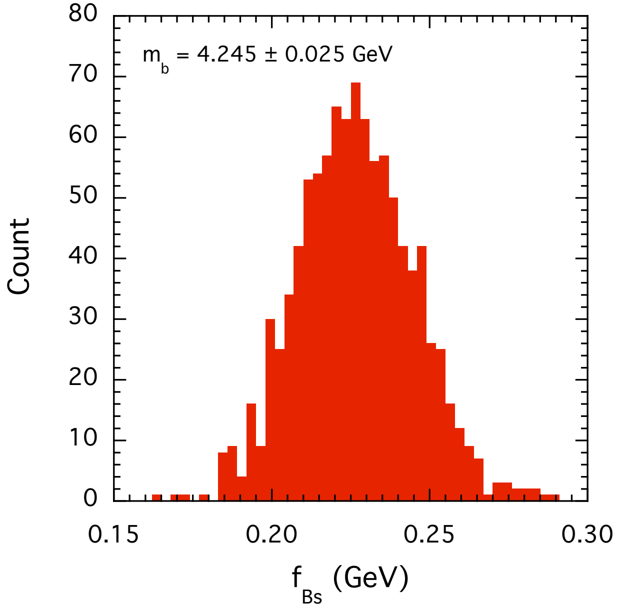


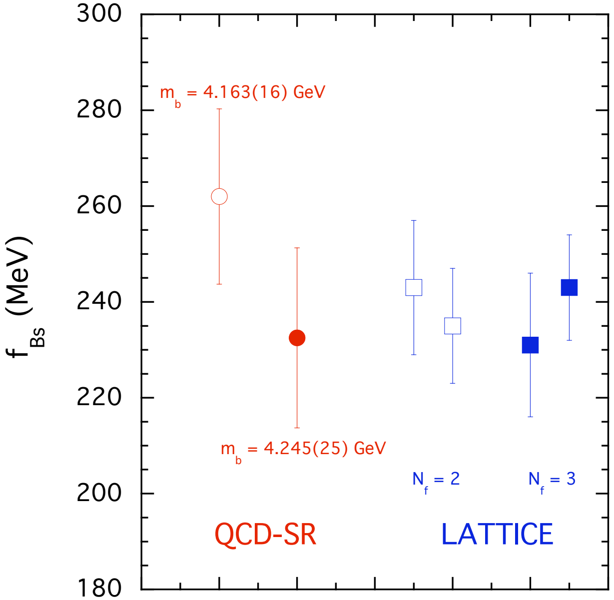
Our estimate : $\overline{m}_b(\overline{m}_b) = 4.245 \pm 0.025$ GeV

$$f_B = 193.4 \pm 12.3_{OPE} \pm 4.3_{syst}\ \text{MeV}$$

$$f_B\ (\text{const}) = 184 \pm 13_{OPE}\ \text{MeV}$$

Extraction of f_{Bs}





$f_{Bs} = 232.5 \pm 18.6_{OPE} \pm 2.4_{syst}$ MeV

f_{Bs} (const) = $218 \pm 18_{OPE}$ MeV

Conclusions

The effective continuum threshold $s_{\rm eff}$ is an important ingredient of the method which determines to a large extent the numerical values of the extracted hadron parameters. Finding a criterion for fixing $s_{\rm eff}$ poses a problem in the method of sum rules.

- **τ-dependence of $s_{\rm eff}$ emerges naturally when trying to make quark-hadron duality more accurate. For those cases where the ground-state mass M_Q is known, we proposed a new algorithm for fixing $s_{\rm eff}$. We have tested that our algorithm leads to the extraction of more accurate values of bound-state parameters than the standard algorithms used in the context of sum rules before.**

- **τ-dependent $s_{\rm eff}$ is a useful concept as it allows one to probe realistic intrinsic uncertainties of the extracted parameters of the bound states.**

- **We obtained predictions for the decay constants of heavy mesons f_Q which along with the "statistical" errors related to the uncertainties in the QCD parameters, <u>for the first time</u> include <u>realistic "systematic" errors</u> related to the uncertainty of the extraction procedure of the method of QCD sum rules.**

- **Matching our SR estimate to the average of the recent lattice results for f_B allowed us to obtain a rather accurate estimate**

$$\overline{m}_b(\overline{m}_b) = 4.245 \pm 0.025\,\mathrm{GeV}.$$

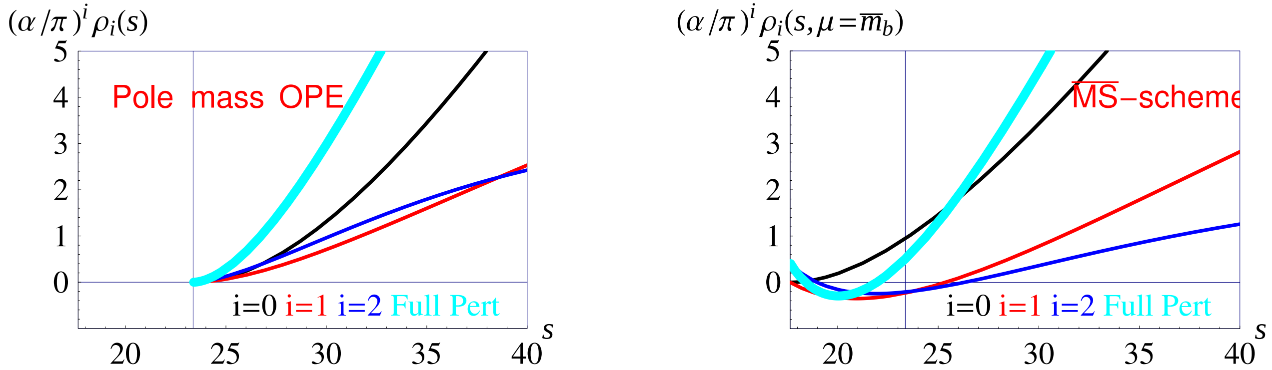
Interestingly, this range does not overlap with a very accurate range reported by Chetyrkin et al. Why?

OPE : heavy – quark pole mass or running mass?

To α_s^2-accuracy, $m_{b,pole} = 4.83$ GeV $\leftrightarrow \overline{m}_b(\overline{m}_b) = 4.20$ GeV:

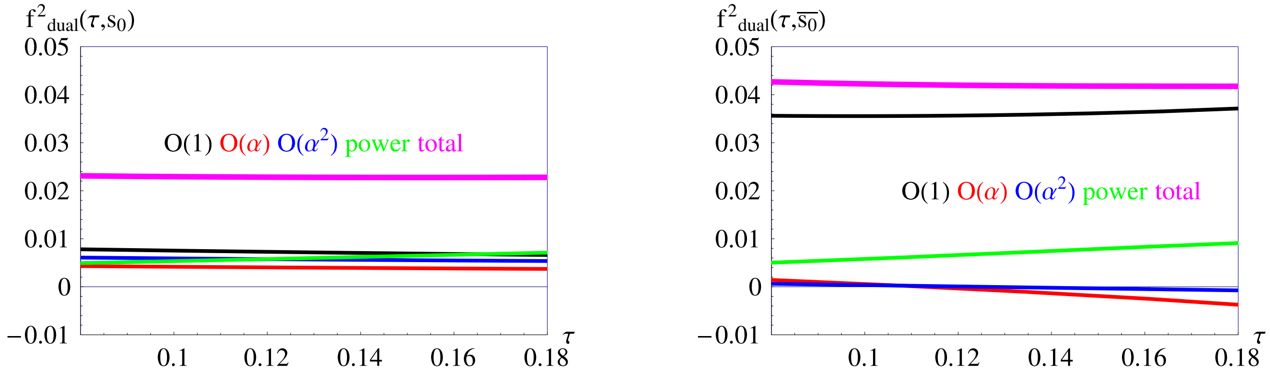
Spectral densities

$$\rho(m_b, \alpha_s, s) \to \Pi(m_b, \alpha_s, \tau) \to \Pi(m_b(\overline{m}_b, \alpha_s), \alpha_s, \tau) \to \Pi(\overline{m}_b, \alpha_s, \tau) \to \rho(\overline{m}_b, \alpha_s, s)$$



- **In pole mass scheme poor convergence of perturbative expansion**
- **In $\overline{MS}$ scheme the pert. spectral density has negative regions $\to$ higher orders NOT negligible**

Extracted decay constant



- **Decay constant in pole mass shows NO hierarchy of perturbative contributions**
- **Decay constant in $\overline{MS}$-scheme shows such hierarchy. Numerically, f_P using pole mass $\ll f_P$ using $\overline{MS}$ mass.**

	OPE	mQ, GeV		fB, MeV
Aliev [1983]	O (α)	pole : 4.8		130 $(\pm 20\,\%)$
Narison [2001]	O (α^2)	pole : 4.7	$\overline{MS}$: 4.05	$203 \pm 23_{OPE}$
Jamin [2001]	O (α^2)	pole : 4.83	$\overline{MS}$: 4.21 ± 0.05	$215 \pm 19_{OPE}$
Our results	O (α^2)		$\overline{MS}$: 4.25 ± 0.025	$193 \pm 13_{OPE} \pm 4_{syst}$