The Two Nucleon System in Chiral Effective Field Theory: Searching for the Power Counting

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- The NN Potential in ChPT (Weinberg Counting):
  - Power Counting in the Chiral NN Potentials.
  - However, breakdown of counting in NN Observables.
- Building a Power Counting for the Two-Nucleon System:
  - Perturbative Treatment of NLO and $\mathrm{N^2LO}$
  - Cut-off independence: modifications to W counting.
  - Results for S- and P-waves.
- Conclusions

Based on: PRC83, 024003 (2011), arXiv:0912.0699

The Nucleon-Nucleon Chiral Potential (I)

- The nuclear force is a fundamental problem in nuclear physics
  - Many phenomenological descriptions available which are, however, not grounded in QCD.
- Chiral Perturbation Theory (Weinberg counting):
  - Problem: NN interaction is non-perturbative
  - Weinberg's solution:
    - apply ChPT to construct the nuclear potential (instead of the scattering amplitude)
    - insert the potential into the Schrödinger equation, as traditionally done in nuclear physics.

The Nucleon-Nucleon Chiral Potential (I)

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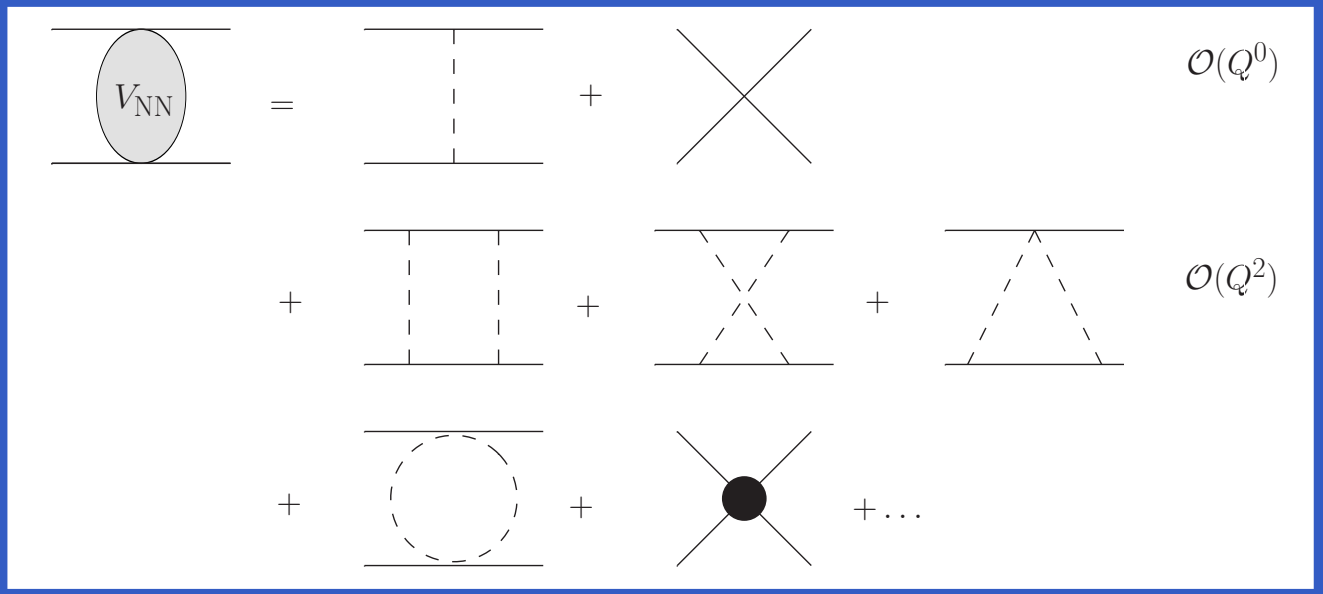
- Chiral Perturbation Theory (Weinberg counting):



Weinberg (90); Ray, Ordoñez, van Kolck (93,94); etc.

The Nucleon-Nucleon Chiral Potential (II)

The two essential ingredients:

- Chiral Symmetry provides the connection with QCD.
  It constraints the nature of pion exchanges (specially TPE).
- Power counting allows to express the NN potential as a low energy expansion in terms of a ratio of scales Q/Λ_0:

$$V_\chi(\vec{q}) = V_\chi^{(0)}(\vec{q}) + V_\chi^{(2)}(\vec{q}) + V_\chi^{(3)}(\vec{q}) + \mathcal{O}(\frac{Q^4}{\Lambda_0^4})$$

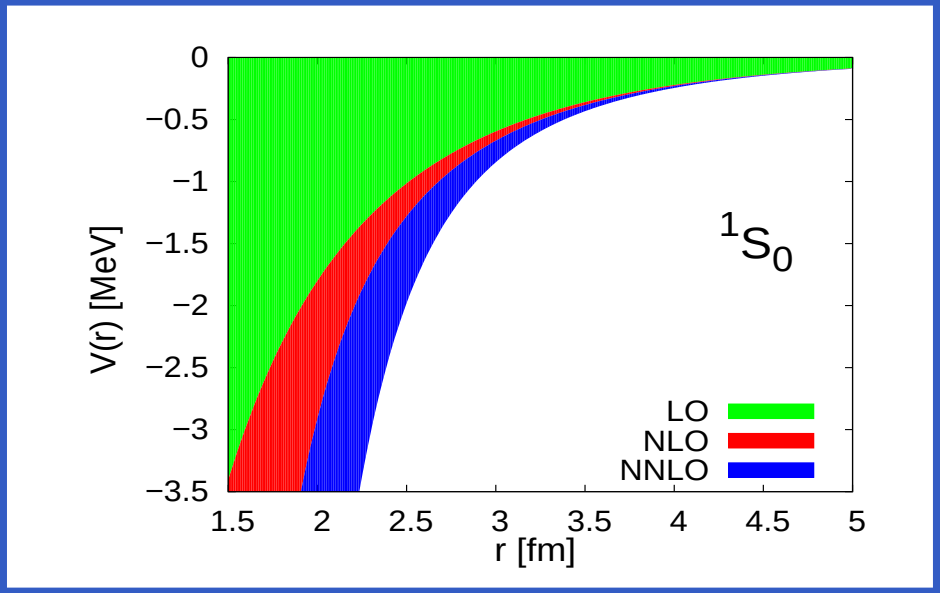
$Q \sim |\vec{q}| \sim p \sim m_\pi \sim 100 - 200\,\mathrm{MeV}$ (low energy scale)
$\Lambda_0 \sim m_\rho \sim M_N \sim 4\pi f_\pi \sim 0.5 - 1\mathrm{GeV}$ (high energy scale)

The resulting potential should convergence quickly at low energies / large distances (and diverge at high energies).

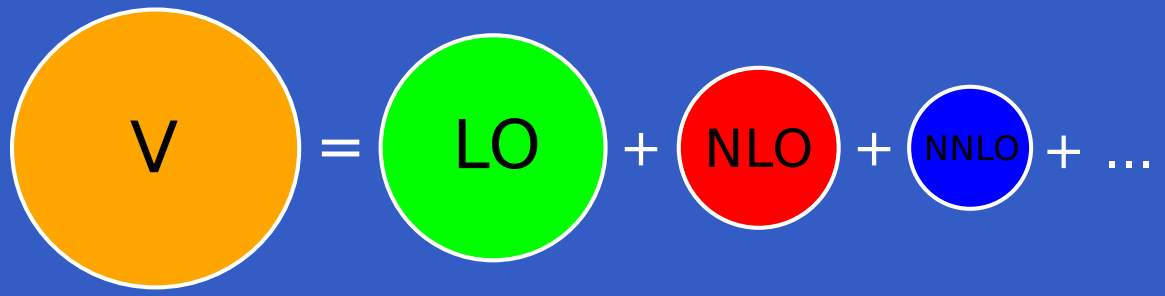
Power counting is essential for having a systematic scheme!

The Nucleon-Nucleon Chiral Potential (III)

The NN chiral potential in coordinate space:

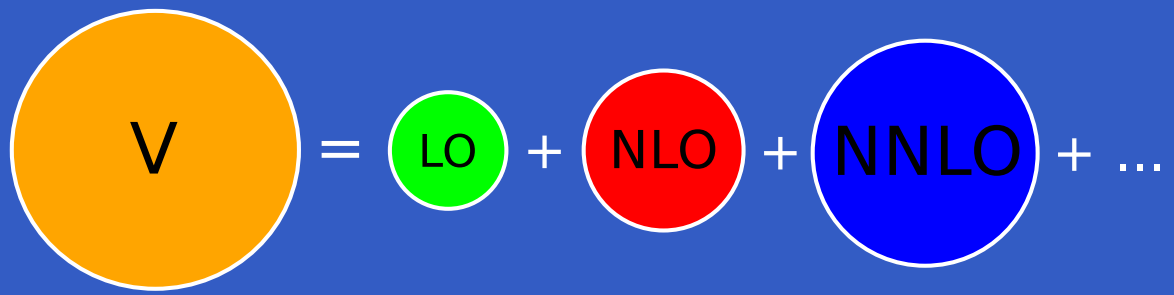


At long distances power counting implies:

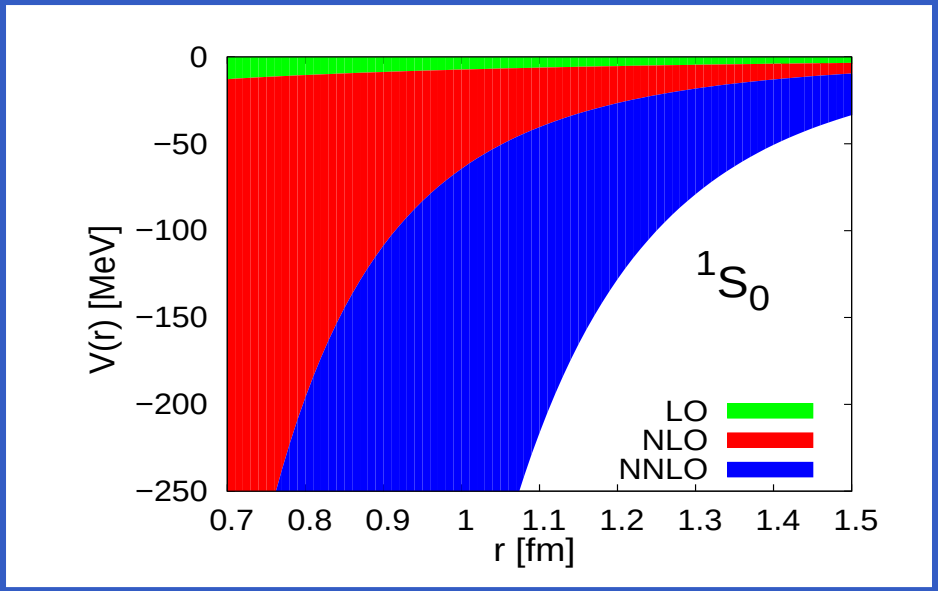


The Nucleon-Nucleon Chiral Potential (IV)

However, at short distances the situation is just the opposite:



... as can be checked in coordinate space:



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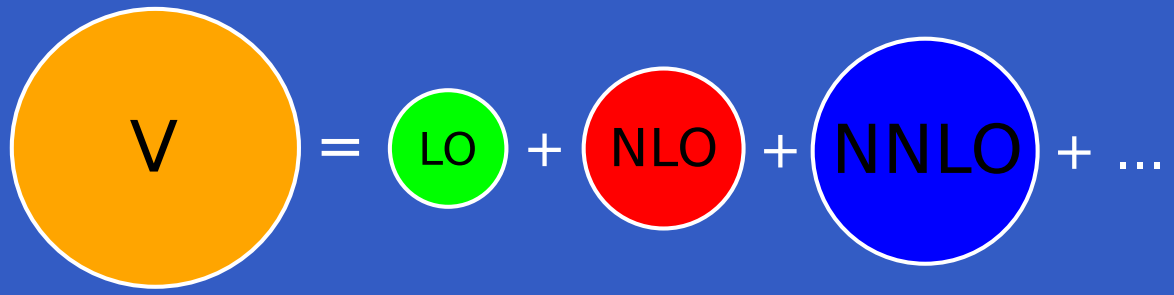


In fact, on dimensional grounds we expect the following behaviour:

$$V^{(\nu)}_{\chi,\mathrm{pions}}(\vec{q}) \sim \frac{|\vec{q}|^{\nu}}{\Lambda_0^{\nu}} f(\frac{|\vec{q}|}{m_\pi}) \quad \text{or} \quad V^{(\nu)}_{\chi,\mathrm{pions}}(\vec{r}) \sim \frac{1}{\Lambda_0^{\nu}\, r^{3+\nu}}$$

This problem is usually dealt with by a renormalization procedure:

- including a cut-off r_c or Λ ($\simeq \pi/2r_c$) in the computations
- the counterterms, which partly absorb the bad behaviour of the potential at scales of the order of the cut-off

Weinberg Counting: Description

- Potential expanded according to counting:

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \mathcal{O}(Q^4/\Lambda_0^4)$$

- The potential is conveniently regularized and iterated:

$$\begin{aligned} V &\rightarrow V_\Lambda^R \\ T &= V_\Lambda^R + V_\Lambda^R\, G_0\, T \end{aligned}$$

- Counterterms are fitted to reproduce scattering observables.
- Great phenomenological success at $\mathrm{N^3LO}$! ($\chi^2/d.o.f. \simeq 1$)

  Entem, Machleidt (03); Epelbaum, Glöckle, Meißner (05)

But there are problems, like the cut-off issue, the power counting issue or the sistematicity issue (Nogga, Timmermans, van Kolck (05); Birse (05); Epelbaum, Meißner (06); Epelbaum, Gegelia (09); Entem, Machleidt (10); etc.).

Weinberg Counting: Problems (I)

However...

Do observables follow a power counting?

- The Weinberg prescription prodives a counting for the potential, which is not an observable.
- There has not been any systematic effort to determine whether the resulting scattering observables follow the power counting.
- Without this ingredient, the Weinberg prescription would merely be a (useful) recipe for constructing nuclear potentials.
- Iteration can play very ugly tricks with us.

Weinberg Counting: Problems (II)

The interesting question is whether power counting is preserved in observables:

$$T = T^{(0)} + T^{(2)} + T^{(3)} + \mathcal{O}(Q^3/\Lambda_0^3)\,?$$

So what can fail? The contribution of subleading pieces can eventually grow larger than the leading ones, spoiling the counting.

Why? Chiral potentials are increasingly singular!

(a) Λ small enough: $T^{(0)} > T^{(2)} > T^{(3)} > \ldots$

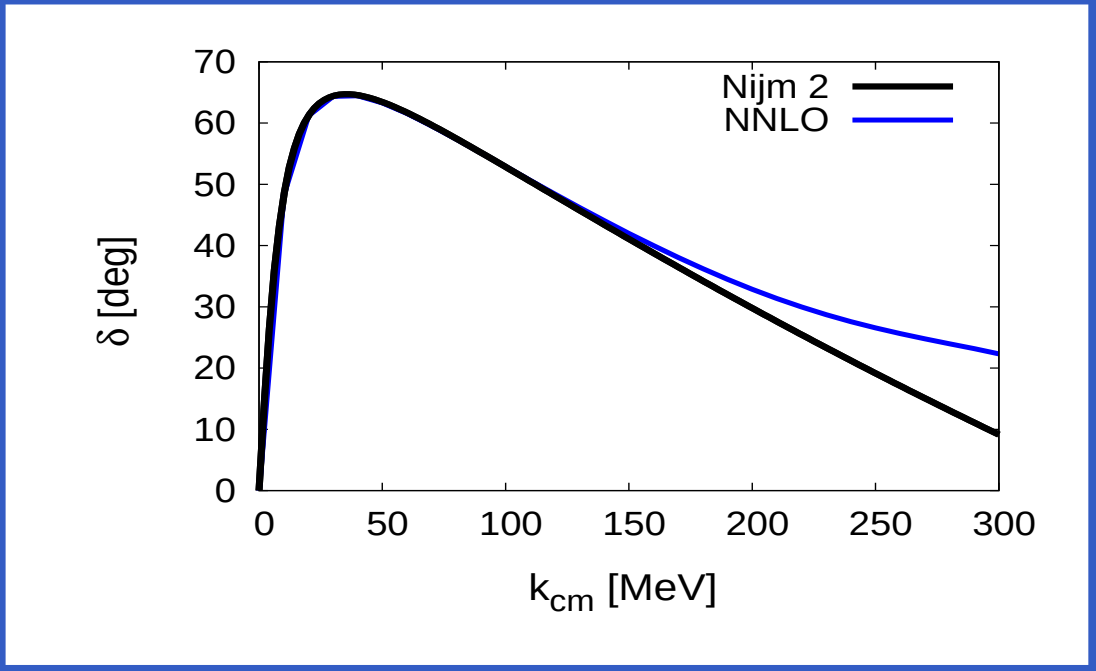
(b) Λ large enough: $T^{(0)} < T^{(2)} < T^{(3)} < \ldots$ (or whatever)

In Weinberg $\Lambda \sim 0.5\,\mathrm{GeV}$: is that within (a) or (b)?

Not everyone agrees on this view: see Epelbaum, Meißner (06) for an example.

Weinberg Counting: an Example (I)

The previous question can be answered by doing some computations:
Weinberg at $\mathrm{N^2LO}$ with a gaussian cut-off $\Lambda = 400\,\mathrm{MeV}$

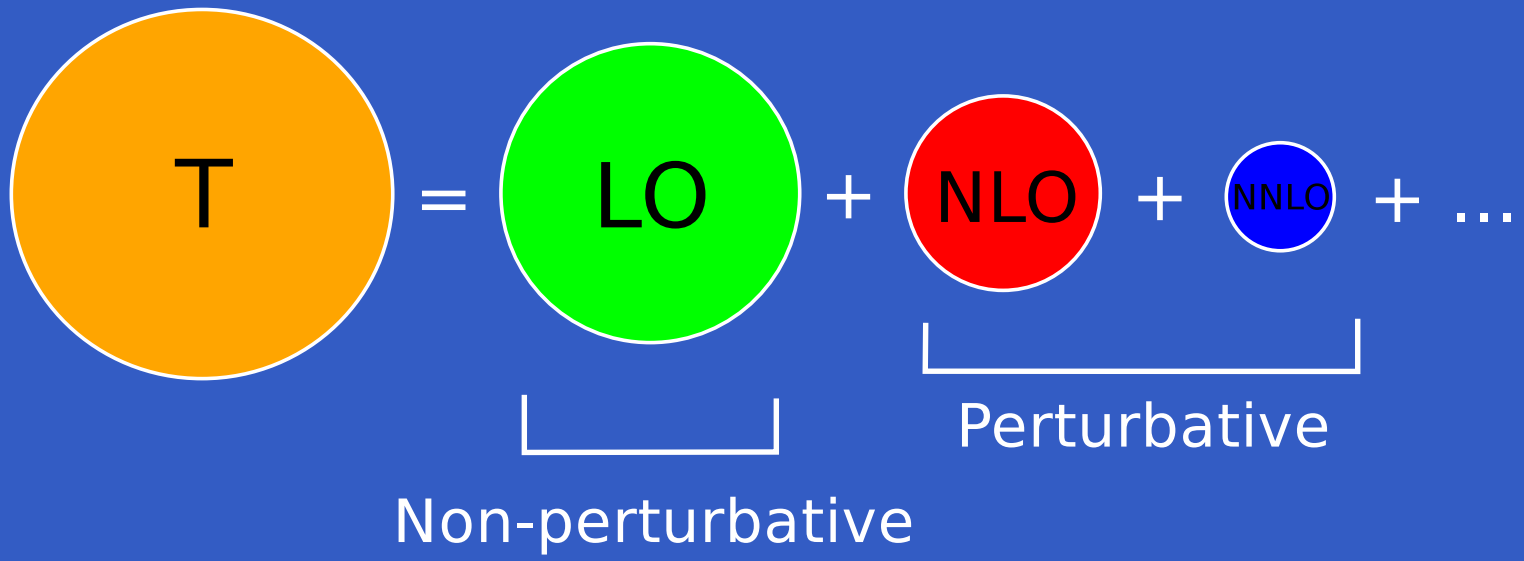


Which piece of the chiral long range interaction dominates?

Weinberg Counting: an Example (II)

Answer: if the subleading contributions to the scattering amplitude are small, we should be able to approximate them in perturbation theory.

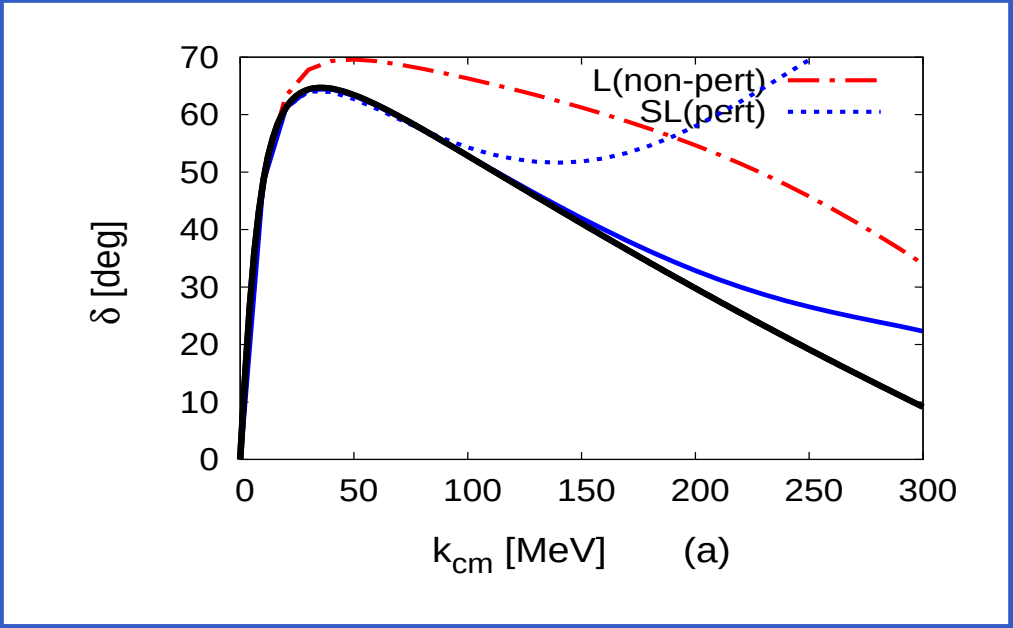
The scattering amplitude should behave as:



Weinberg Counting: an Example (II)

Answer: if the subleading contributions to the scattering amplitude are small, we should be able to approximate them in perturbation theory.

The previous scheme leads to the following approximations:

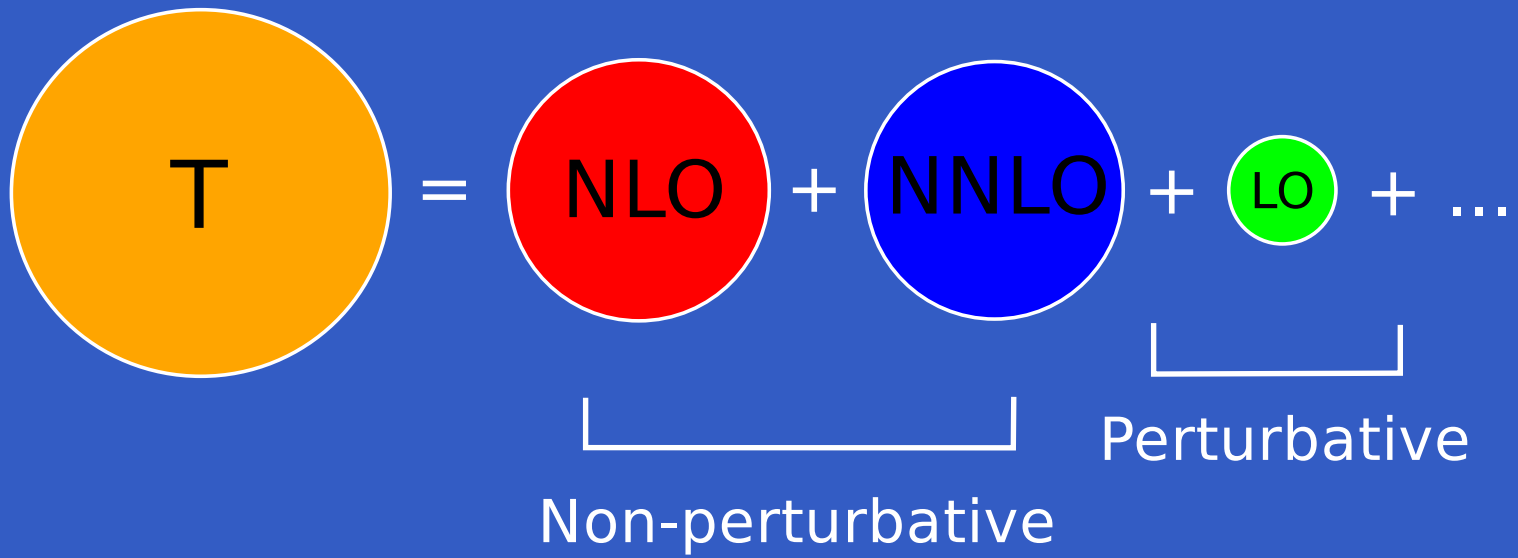


Power counting is already lost at $k \sim 100\,\mathrm{MeV}$!!!.

Weinberg Counting: an Example (III)

However, the situation is even more paradoxical than we can expect.

We can try a different approximation...

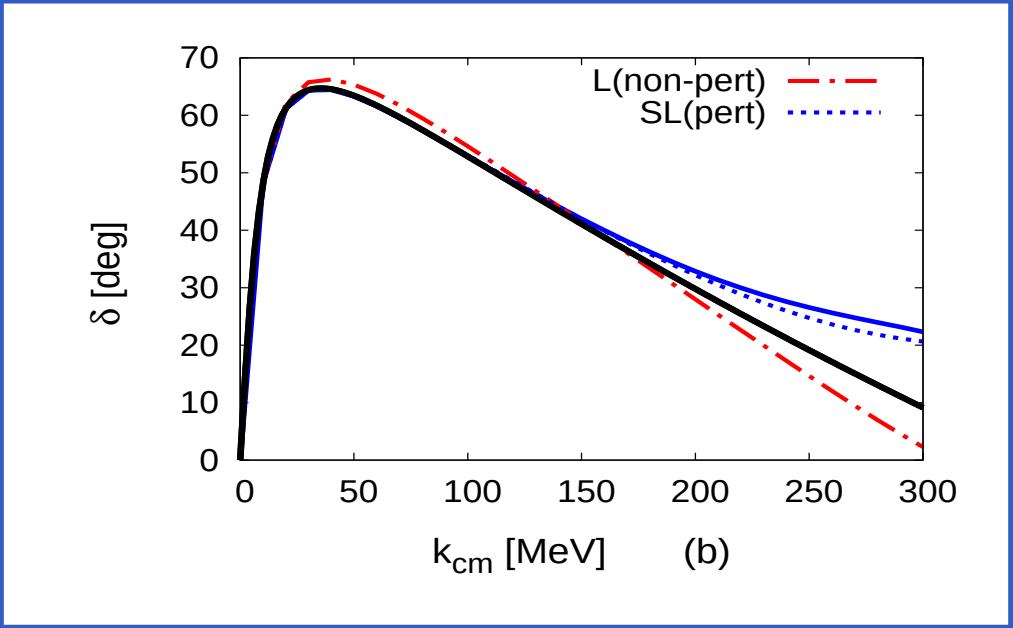


(different choices are possible depending on the regulator, the cut-off, the value of the chiral couplings, etc.)

Weinberg Counting: an Example (III)

However, the situation is even more paradoxical than we can expect.

... which gives us the following phase shifts



The original assumptions made by the power counting are completely broken by the results, which obey a different counting instead.

See related comments in Lepage (97).

Overcoming the Inconsistencies

Lesson: don't iterate unless you are sure what you are doing!

Power counting inconsistencies avoided by enforcing the counting, that is, treating the subleading pieces of the potential as perturbations:

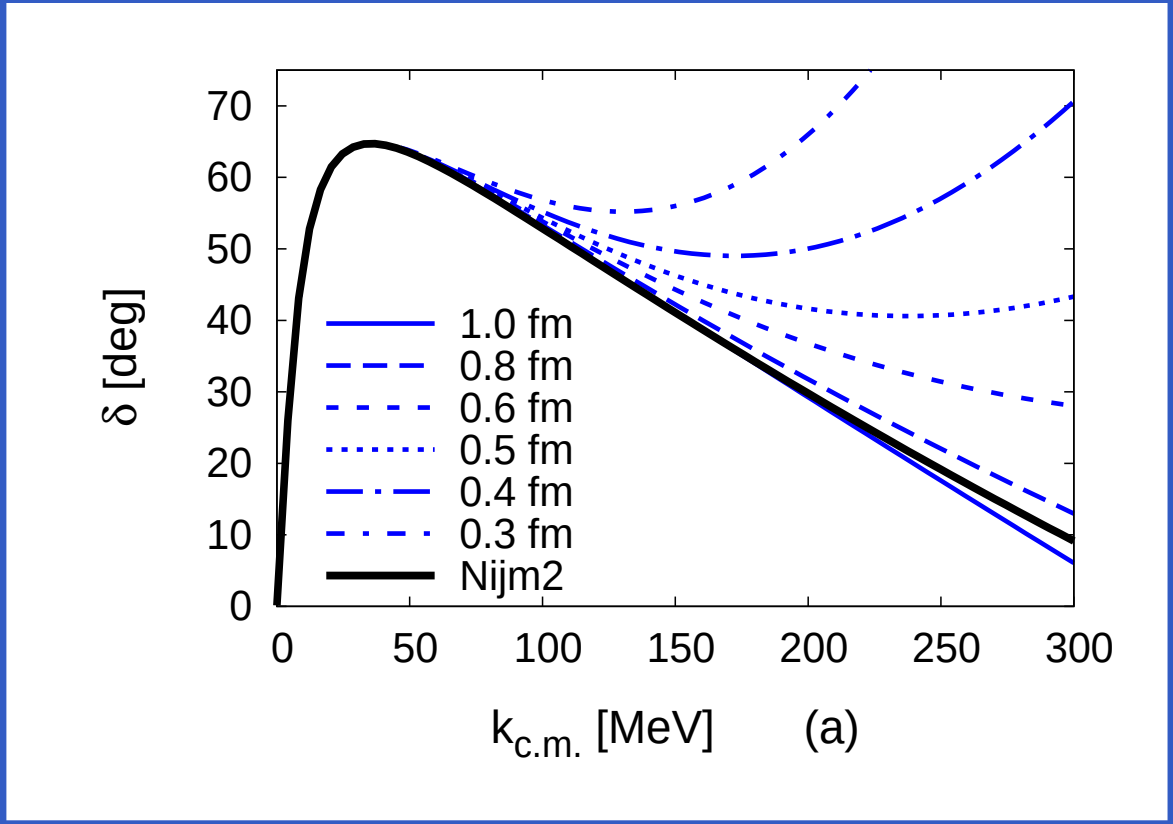
$$\begin{aligned} T^{(0)} &= V^{(0)} + V^{(0)}\, G_0\, T^{(0)} \\ T^{(2)} &= V^{(2)} + T^{(0)}\, G_0\, V^{(2)} + V^{(2)}\, G_0\, T^{(0)} \\ \ldots &= \ldots \end{aligned}$$

and now (i) $T^{(2)} \propto V^{(2)}$, (ii) $T = T^{(0)} + T^{(2)} + \mathcal{O}(Q^3/\Lambda_0^3)$.

Recent examples are given by Shukla, Phillips, Mortenson (07) and the EFT lattice computations by Epelbaum, Krebs, Lee, Meißner.

Perturbative Weinberg (I)

However, there is still a problem with cut-off dependence:



Perturbative Weinberg (II)

By analyzing the cut-off dependence of the T-matrix in the singlet channel we find the following

$$T(\Lambda) = T^{(0)}(\Lambda) + \underbrace{T^{(2)}(\Lambda)}_{\sim \log \Lambda} + \underbrace{T^{(3)}(\Lambda)}_{\sim \Lambda} + \mathcal{O}(Q^4/\Lambda_0^4)$$

- Problem: the Weinberg counting counterterms

$$V_{\chi,\mathrm{contact}}^{(2,3)} = C_0 + C_2\,(p^2 + p'^2) + \mathcal{O}(Q^4/\Lambda_0^4)$$

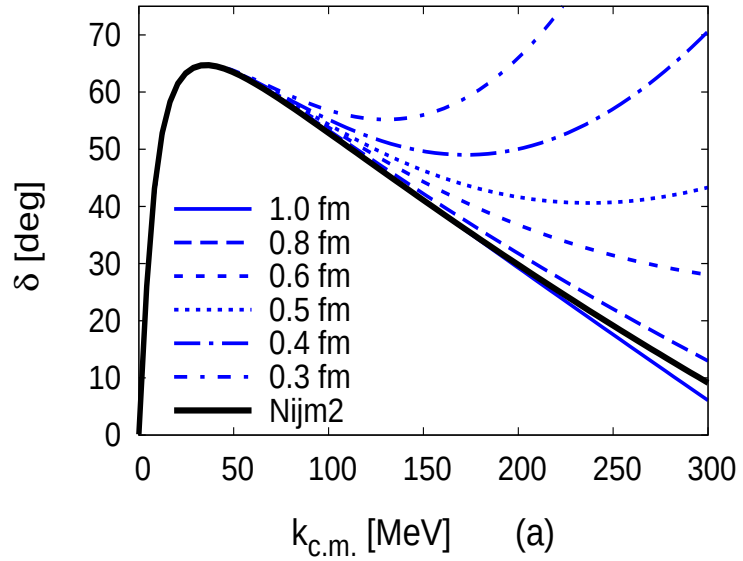
  are not enough to render the amplitudes cut-off independent.

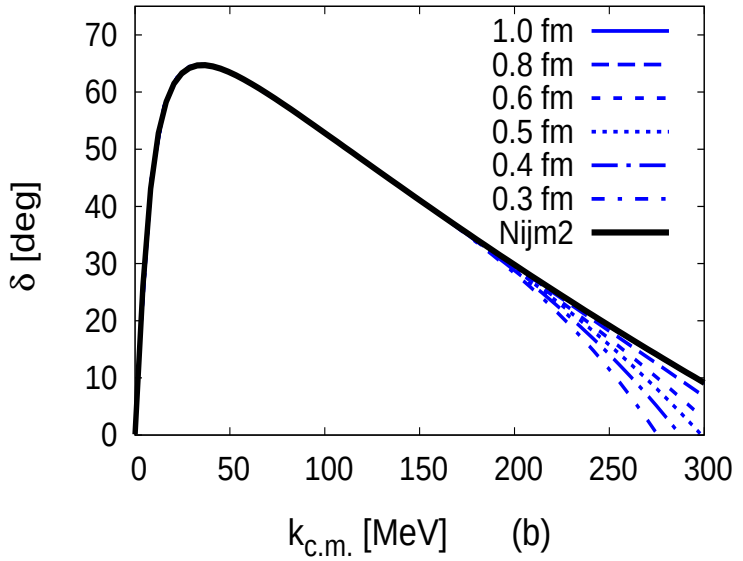
- Solution: promote the C_4 counterterm (which is Q^4 in Weinberg) to order Q^2 to achieve cut-off independence (Birse 05/10).

$$V_{\chi,\mathrm{contact}}^{(2,3)} = C_0 + C_2\,(p^2 + p'^2) + C_4\,(p^4 + p'^4) + \mathcal{O}(Q^4/\Lambda_0^4)$$

Perturbative Weinberg (III)

Can be illustrated by the following $\mathrm{N^2LO}$ results in the singlet:





(a) with the Weinberg counterterms C_0 and C_2 ($\Delta\delta \sim k^4/r_c$)

(b) with the additional counterterm C_4 ($\Delta\delta \sim k^6\, r_c$)

Modified Perturbative Weinberg

(a) Modify the counting to allow renormalizability at leading order.

Nogga, Timmermans and van Kolck (05)

(b) Only fully iterate OPE if necessary: s- and p-waves (generally).

- Minimally ${}^1S_0$, ${}^3S_1$, ${}^3P_0$ and additionally ${}^3P_2$, ${}^3D_2$.
- d-waves (and beyond) are already perturbative (Kaiser, Brockmann, Weise (97)); however, the subleading iterations of OPE can make the calculations cumbersome.

(c) Subleading corrections (TPE) treated pertubatively: counting rules determined by perturbative renormalizability.

(a), (b) and (c) corresponds to the Nogga et al. proposal.

(b) and (c) guarantee, by construction, the power counting.

Perturbation Theory: Power Counting (I)

The power counting resulting from the previous scheme:

- ${}^1S_0$: 3 CT's at NLO and $\mathrm{N^2LO}$ (4 at $\mathrm{N^3LO}$).
- ${}^3S_1 - {}^3D_1$: 6 CT's at NLO / $\mathrm{N^2LO}$ / $\mathrm{N^3LO}$
  (could be reduced by treating d-wave perturbatively).
- ${}^1P_1$: 1 CT at NLO and $\mathrm{N^2LO}$ (2 at $\mathrm{N^3LO}$).
- ${}^3P_1$: 1 CT at NLO and $\mathrm{N^2LO}$ (2 at $\mathrm{N^3LO}$)..
- ${}^3P_0$: 2 CT at NLO / $\mathrm{N^2LO}$ / $\mathrm{N^3LO}$
- ${}^3P_2 - {}^3F_2$: 6 CT's at NLO / $\mathrm{N^2LO}$ / $\mathrm{N^3LO}$ if OPE was iterated at LO (otherwise 1 CT at NLO / $\mathrm{N^2LO}$, 3 at $\mathrm{N^3LO}$).

Perturbation Theory: Power Counting (II)

- Partly equivalent to Birse's proposal for a power counting.
  - Minor departures in particular waves.
  - Less counterterms at higher orders in triplets.
  - The interesting point is what happens with D-wave triplets. (same counting as P-waves according to Birse)
- The number of counterterms (free parameters) at LO, NLO and $\mathrm{N^2LO}$ is larger than in original Weinberg.
  - In principle, less predictive power. However...
  - ...the perturbative counting catches up Weinberg's at $\mathrm{N^3LO}$. (that is, there are only more CT's at intermediate orders)
- Is there a merging with standard Weinberg counting at $\mathrm{N^3LO}$?

Perturbation Theory: Results

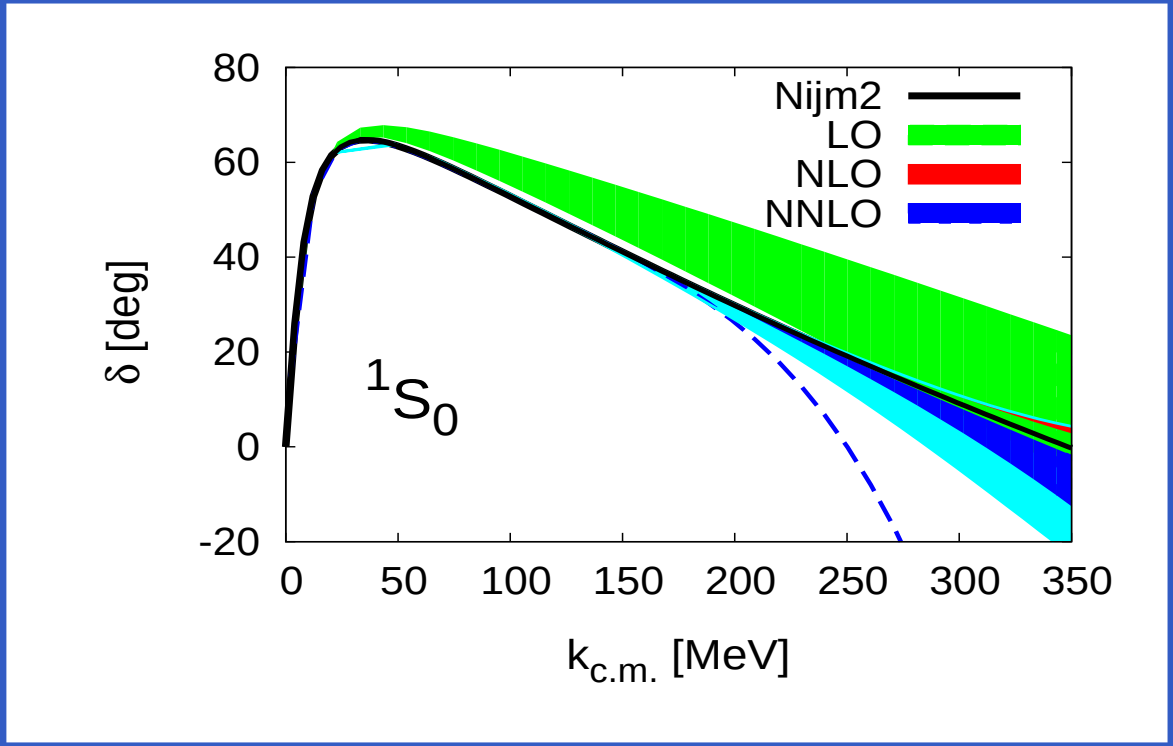
Central Waves

The following values have been taken:

$f_\pi = 92.4\,\mathrm{MeV}$, $m_\pi = 138.04\,\mathrm{MeV}$, $d_{18} = -0.97\,\mathrm{GeV}^2$
$c_1 = -0.81\,\mathrm{GeV}^{-1}$, $c_3 = -3.4\,\mathrm{GeV}^{-1}$, $c_4 = 3.4\,\mathrm{GeV}^{-1}$

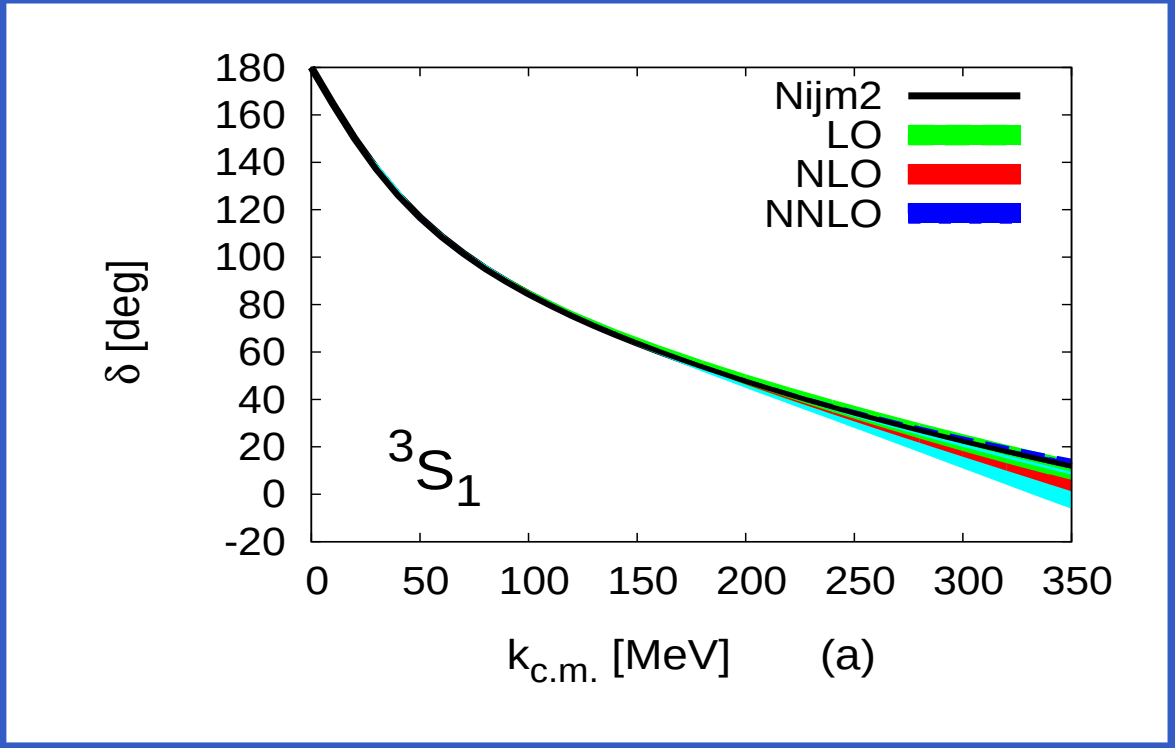
$1/M_N$ corrections included at $\mathrm{N^2LO}$

Perturbation Theory: $^1S_0$



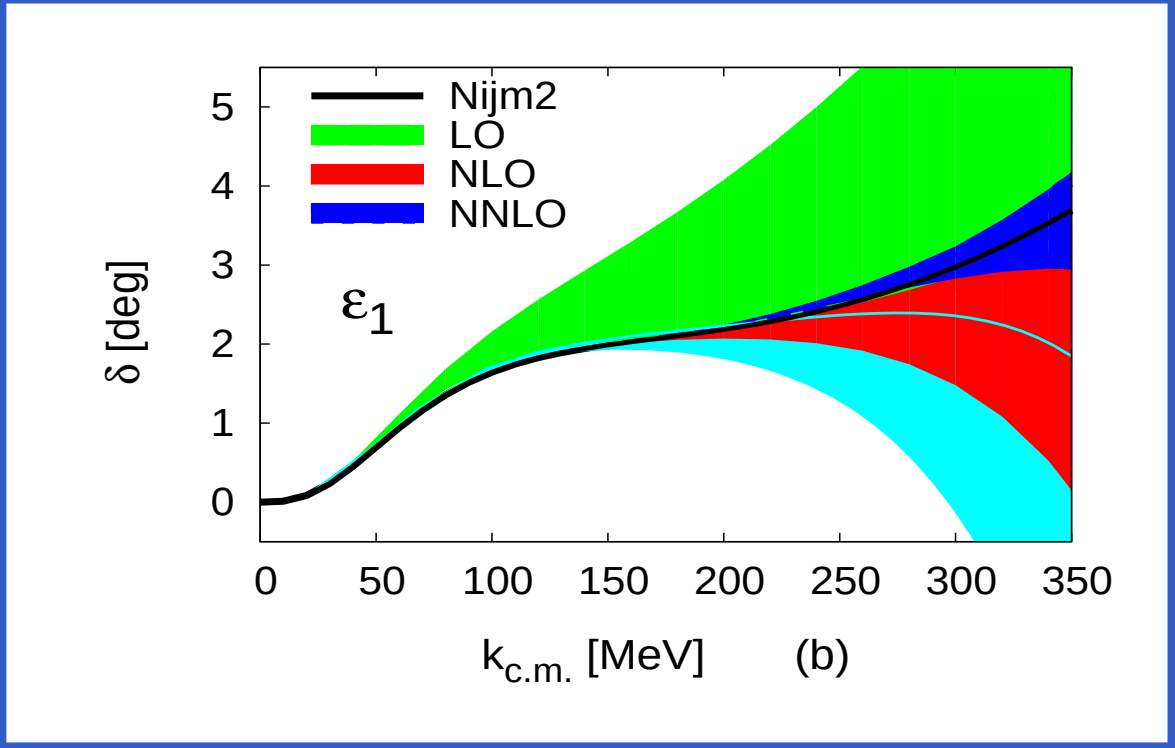
$r_c = 0.6 - 0.9\,\mathrm{fm}$ ($\sim 350 - 500\,\mathrm{MeV}$), 3 CT's, fit between $k_{\mathrm{cm}} = 40 - 160\,\mathrm{MeV}$, dashed blue: $r_c = 0.1\,\mathrm{fm}$, light blue: $\mathrm{N^2LO}$ results from Epelbaum et al. (Weinberg counting)

Perturbation Theory: $^3S_1$



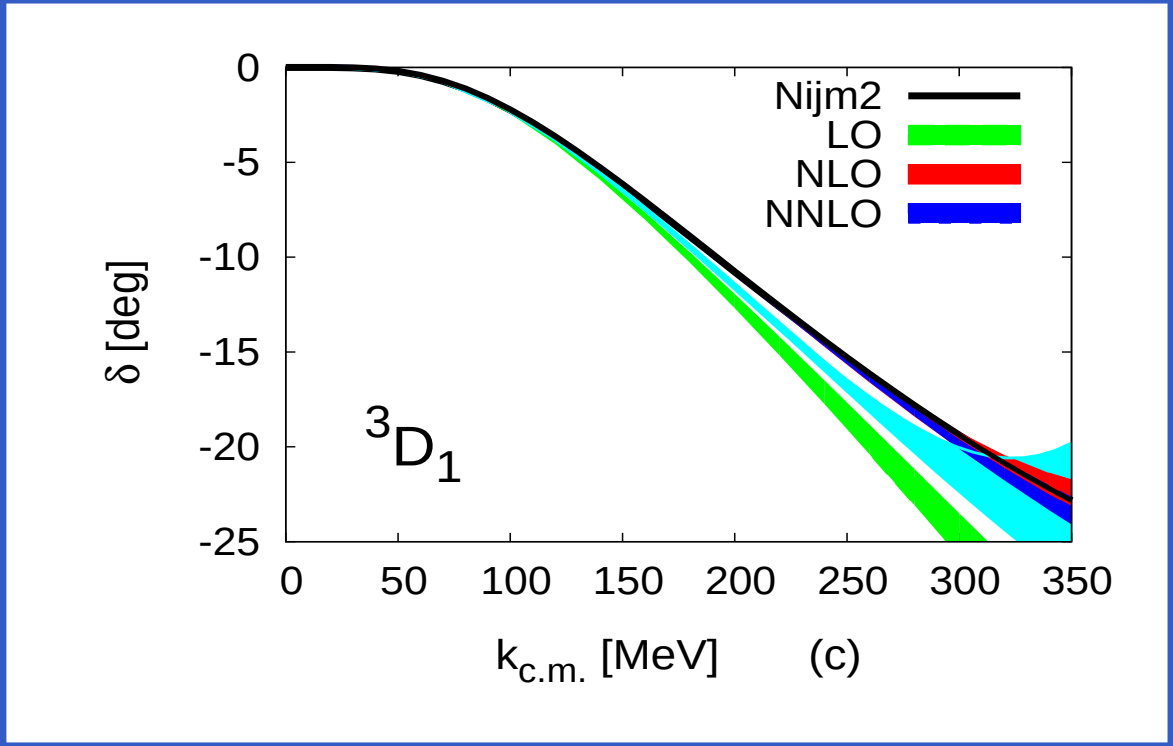
$r_c = 0.6 - 0.9\,\mathrm{fm}$ ($\sim 350 - 500\,\mathrm{MeV}$), 2 CT's, fit between $k_{\mathrm{cm}} = 40 - 160\,\mathrm{MeV}$, dashed blue: $r_c = 0.3\,\mathrm{fm}$

Perturbation Theory: E_1



$r_c = 0.6 - 0.9\,\mathrm{fm}$ ($\sim 350 - 500\,\mathrm{MeV}$), 2 CT's, fit between $k_{\mathrm{cm}} = 40 - 160\,\mathrm{MeV}$.

Perturbation Theory: $^3D_1$



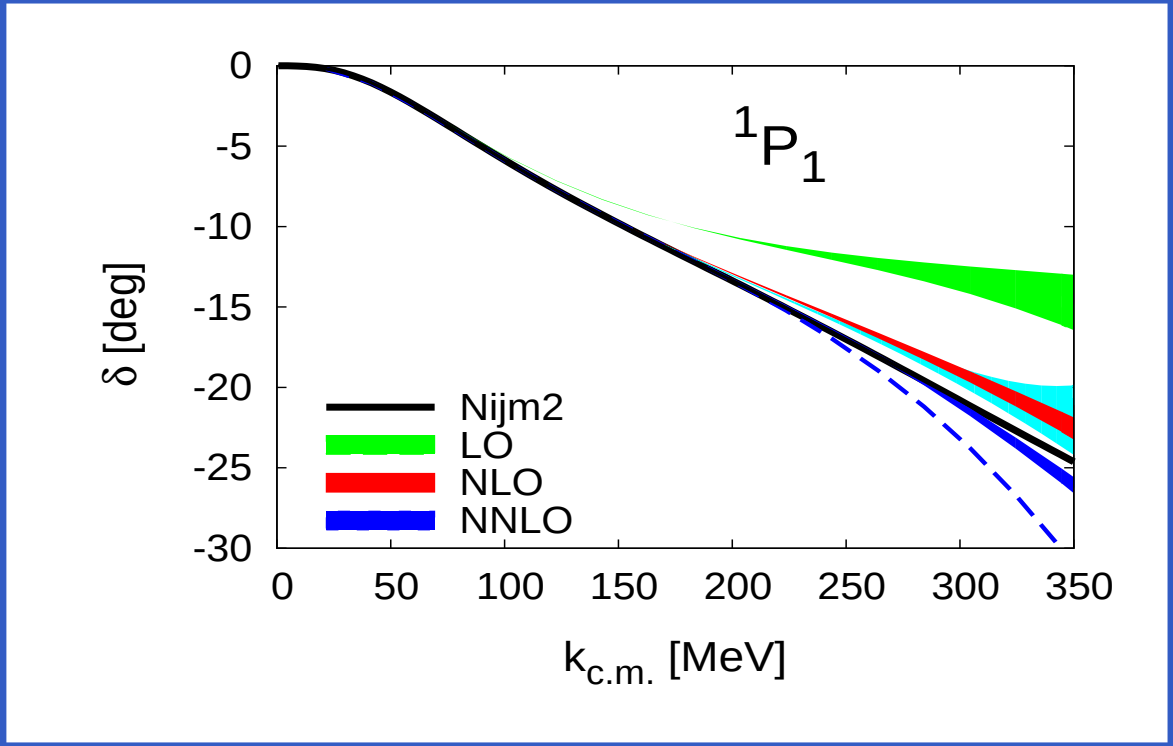
$r_c = 0.6 - 0.9\,\mathrm{fm}$ ($\sim 350 - 500\,\mathrm{MeV}$), 2 CT's, fit between $k_{\mathrm{cm}} = 40 - 160\,\mathrm{MeV}$.

Perturbation Theory: Results

P-Waves

Caution: Preliminary Results

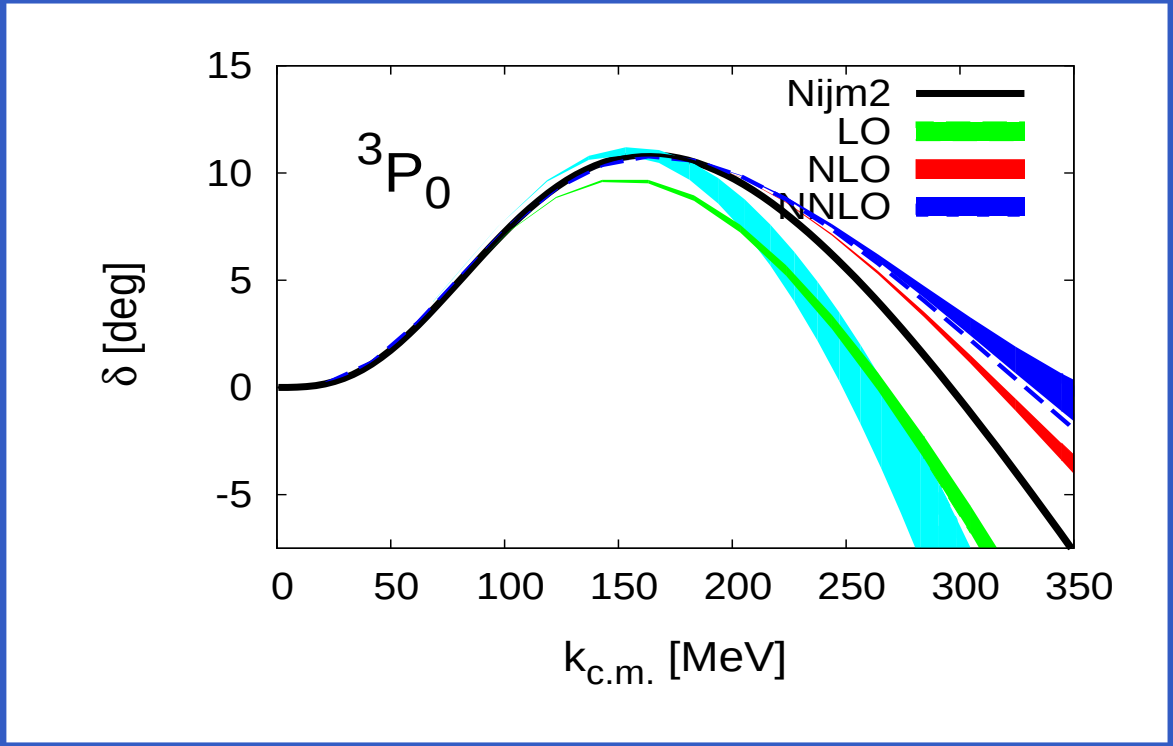
Perturbation Theory: ${}^1P_1$



$r_c = 0.6 - 0.9\,\mathrm{fm}$ ($\sim 500 - 800\,\mathrm{MeV}$), 1 CT, fit between $k_{\mathrm{cm}} = 100 - 200\,\mathrm{MeV}$.

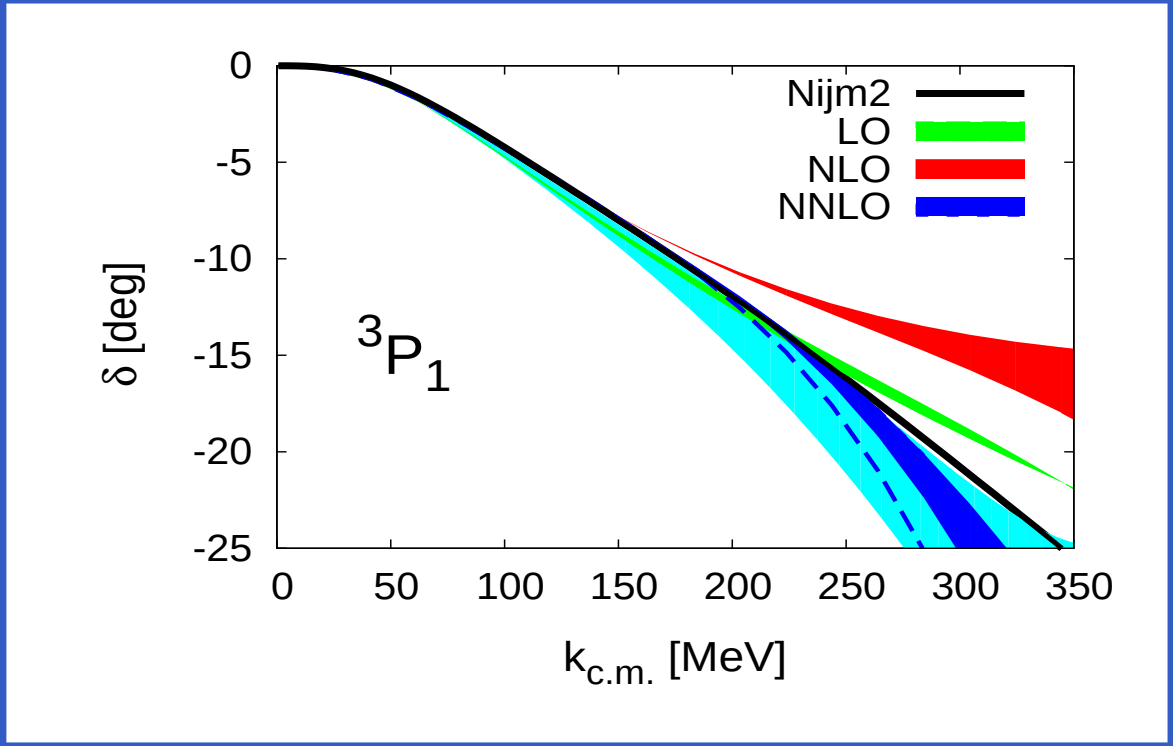
${}^1P_1$ very sensitive to the choice of chiral couplings!

Perturbation Theory: $^3P_0$



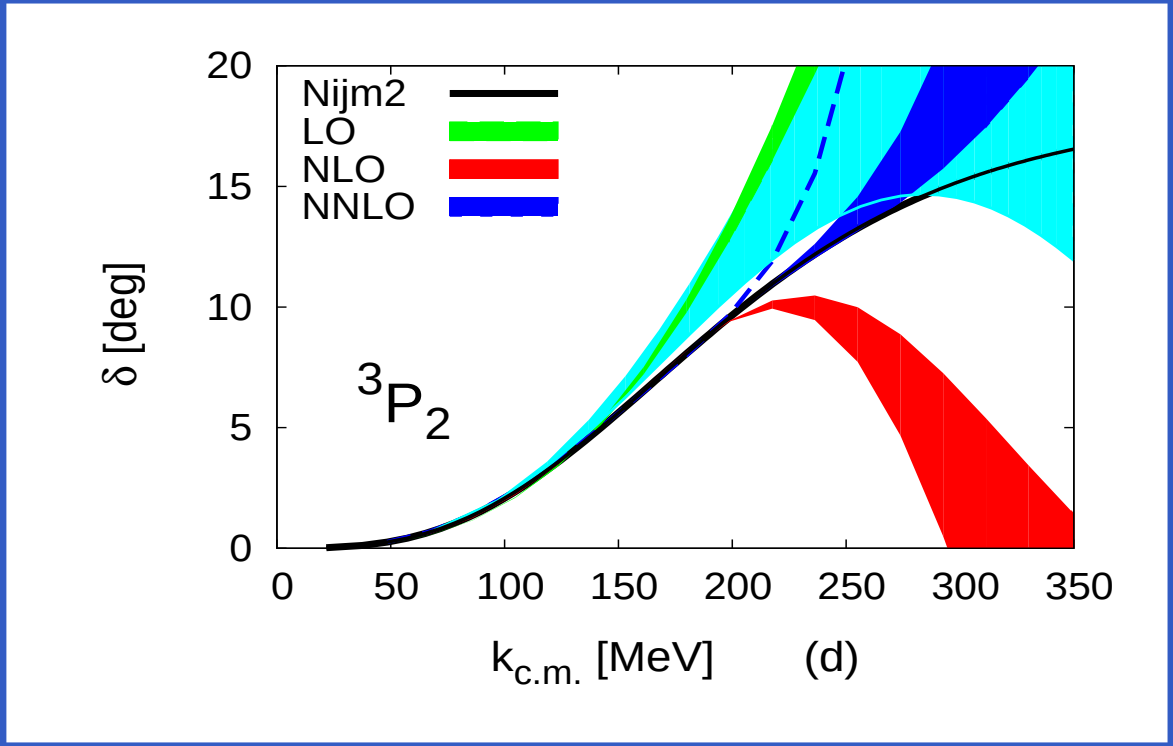
$r_c = 0.6 - 0.9\,\mathrm{fm}$ ($\sim 500 - 800\,\mathrm{MeV}$), 2 CT's, fit between $k_{\mathrm{cm}} = 100 - 200\,\mathrm{MeV}$.

Perturbation Theory: $^3P_1$



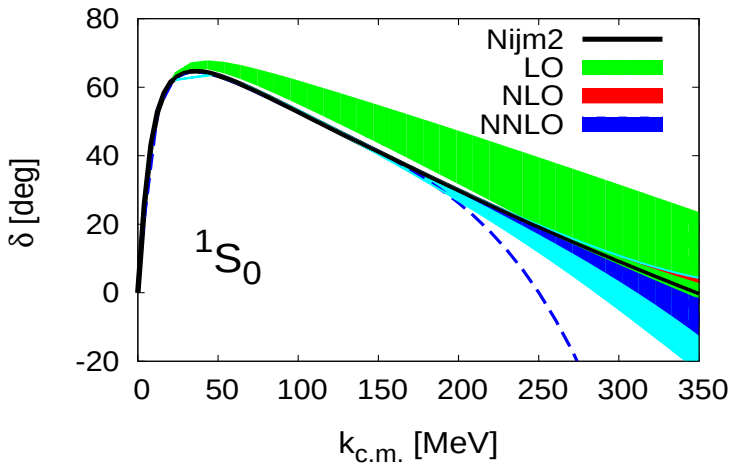
$r_c = 0.6 - 0.9\,\mathrm{fm}$ ($\sim 500 - 800\,\mathrm{MeV}$), 1 CT, fit between $k_{\mathrm{cm}} = 100 - 200\,\mathrm{MeV}$.

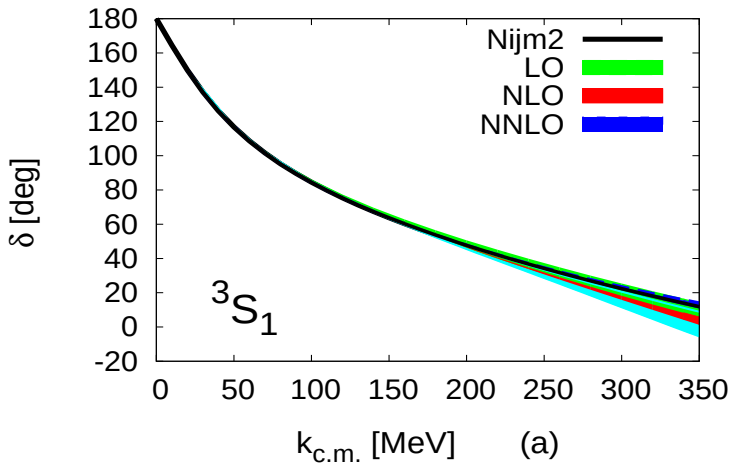
Perturbation Theory: $^3P_2$

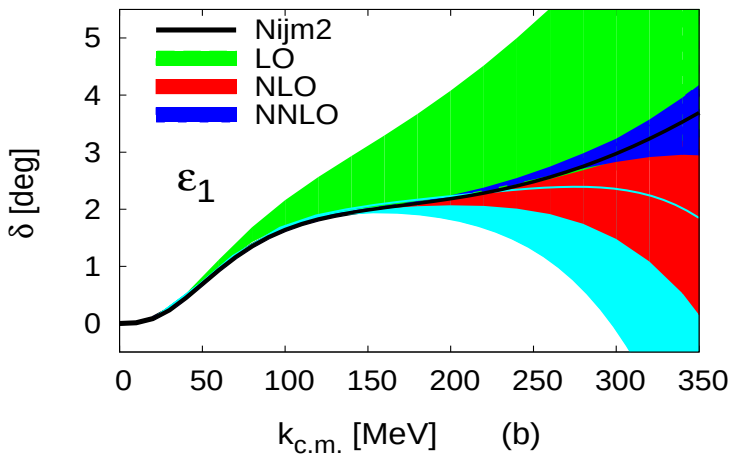


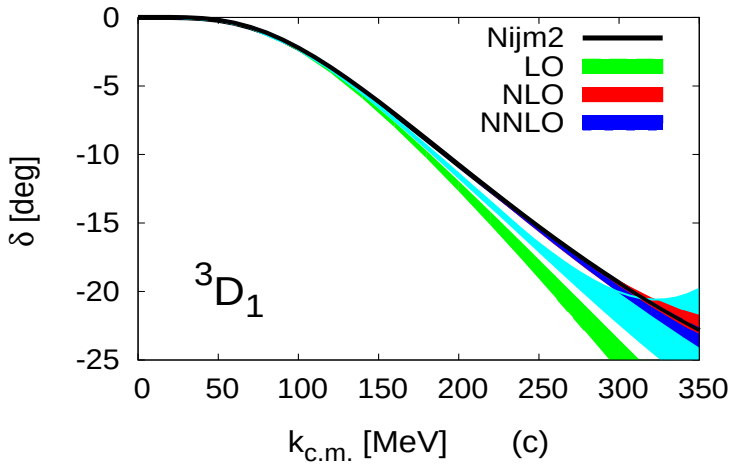
$r_c = 0.6 - 0.9\,\mathrm{fm}$ ($\sim 500 - 800\,\mathrm{MeV}$), 2 CT's, fit between $k_{\mathrm{cm}} = 100 - 200\,\mathrm{MeV}$.

Perturbation Theory: Overview (s-waves)

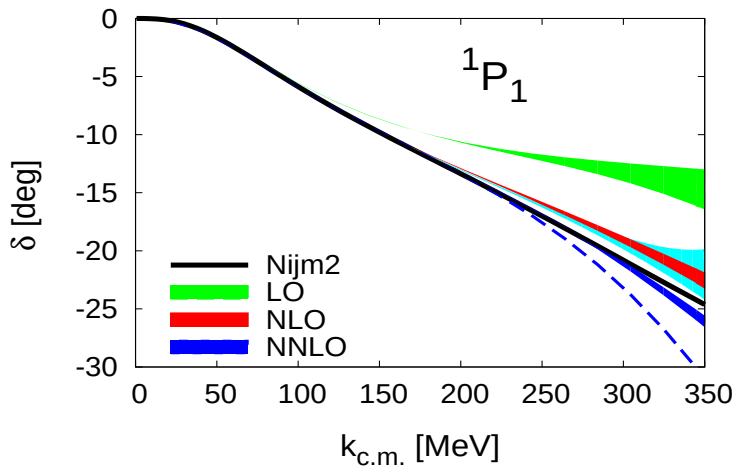


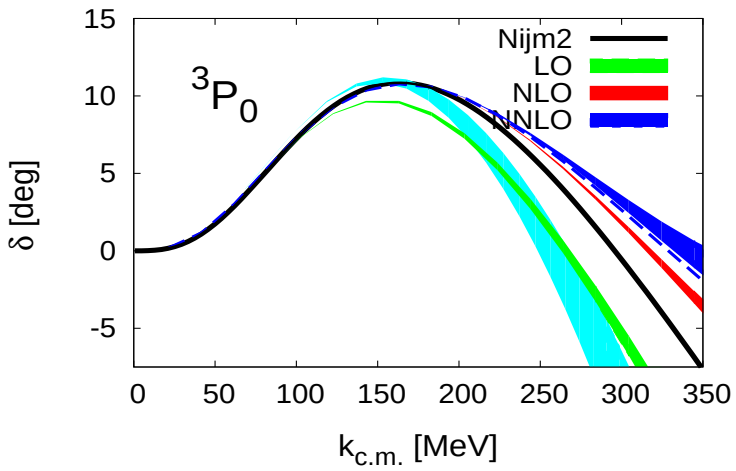


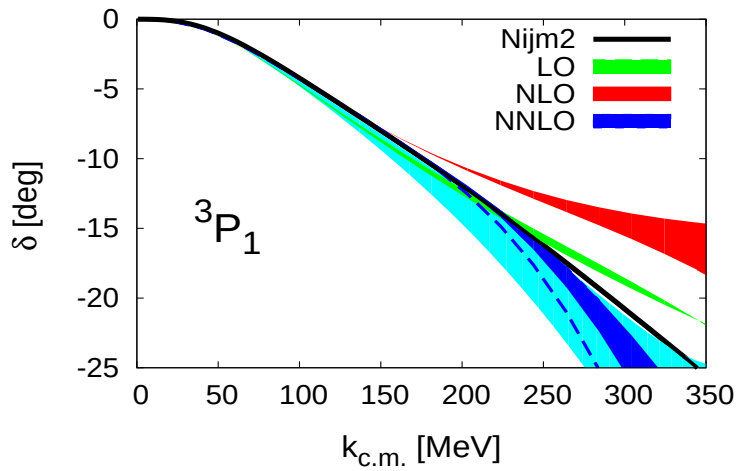


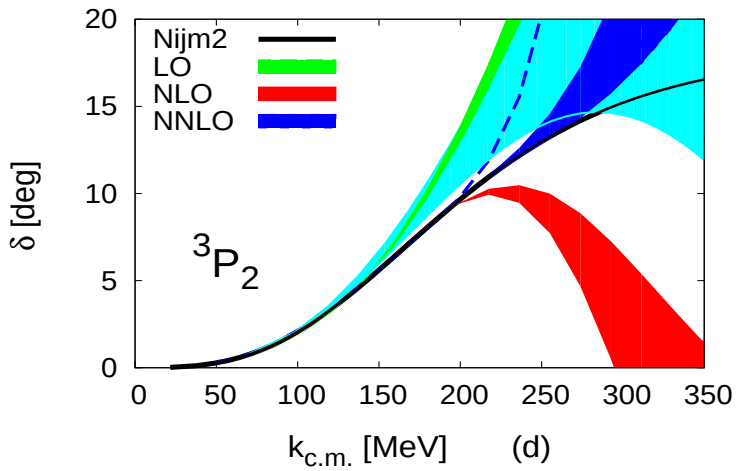


Perturbation Theory: Overview (p-waves)









Final Remarks and Conclusions (I)

- Chiral Two Pion Exchange is perturbatively renormalizable.
  - A consistent power counting emerges from renormalizability.
  - Some problematic issues of Weinberg counting are avoided.
- S- and P-waves are well-reproduced up to $k \sim 300 - 350\,\mathrm{MeV}$.
  - There is a well.defined convergence pattern.
- The residual cut-off dependence is nominally a higher order effect:
  - Consistent interpretation requires the cut-off to be a separation scale: $m_\pi < \Lambda (\sim 1/r_c) < \Lambda_0$.
  - Error estimations based on variations of the cut-off around the purported hard ($r_c \sim 0.5\,\mathrm{fm}$) and light scale ($r_c \sim 1.0\,\mathrm{fm}$).
  - Convergence of the EFT expansion also requires $r_c > 0.5\,\mathrm{fm}$ (the chiral potentials may diverge at shorter distances).

Final Remarks and Conclusions (II)

- Convergence rate and expansion parameter can be determined:
  - Scaling of the residual short range interaction at a given order: this is the deconstruction method by Birse, which yields
    - Singlets: $\Lambda_{0,s} \simeq 270\,\mathrm{MeV}$, giving $x \simeq 0.5$
    - Triplets: $\Lambda_{0,t} \simeq 340\,\mathrm{MeV}$, giving $x \simeq 0.4$

    Birse (07, 10); Ipson, Helmke, Birse (10)

  (i) This may look slow, however $\delta^{(\nu)} \propto (Q/\Lambda_0)^{(\nu+1)}$, meaning that the relative error for the $\mathrm{N^2LO}$ calculation at $k = m_\pi$ is 3% in the singlet (1% in the triplets).

  (ii) The breakdown scale could have been anticipated on sigma and rho exchange, yielding $\Lambda_{0,s} = m_\sigma/2$ and $\Lambda_{0,t} = m_\rho/2$.