Structure of Topological Strings

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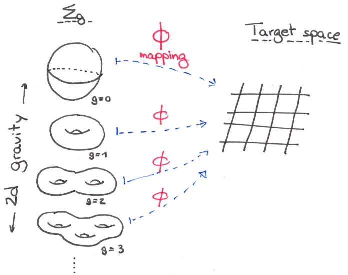
Outline

String Theories are 2d Conformal Field Theories



How to describe gravitational and gauge interactions in one theory?

- Possible solution: use 2 dimensional conformal field theory= 2d field theory with special features, very powerful, integrate over all 2d metrics

Partition functions contain are rich in information

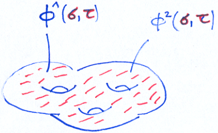
What one wants to know

- Correlation functions, $\langle \phi_i \phi_j \phi_k \rangle$
- Possible deformations of the theory
- Critical exponents or scaling dimensions of operators, especially in statistical physics

Partition function Z

- containts a lot of information
- remember statistical physics $Z \sim \sum e^{-\beta H}$
- Field Theory $Z \sim \int_X \mathcal{D}\phi e^{-S(\phi)}$, path integral

Spacetime metric is a coupling for 2d CFT



2 fields ϕ^1 & ϕ^2

$$\mathcal{L} = \underline{h^{\alpha\beta}}\,\partial_\alpha \phi^1\, \partial_\beta \phi^1 + \underline{h^{\alpha\beta}}\,\partial_\alpha \phi^2\, \partial_\beta \phi^2$$

2d metric

could also consider

$$\mathcal{L} = \underbrace{g_{ij}(\phi^1,\phi^2)}\, h^{\alpha\beta}\, \partial_\alpha \phi^i\, \partial_\beta \phi^j$$

coupling from 2d point of view $\Rightarrow$ metric in "Target" space

String and Field theories depend on the background

Background is chosen in the moment we write down the action

- Target space characteristics, metric, topology, dimension
- Boundary conditions
- Couplings of the theory

Background dependence in String Theory

- Although string theory has spin 2 excitation, the graviton, current formulation just considers fixing a background metric and looking at fluctuations of it, described by gravitons
- It is a problem of *formulation* of the theory, we'd want to understand the background as a coherent state of closed strings for example

How can we understand a theory at all couplings?

What we usually do

- Identify relevant degrees of freedom at given energy
- Write down a theory of these degrees of freedom
- Adjust the couplings to experiments, e.g. α for Q.E.D.

Ideally

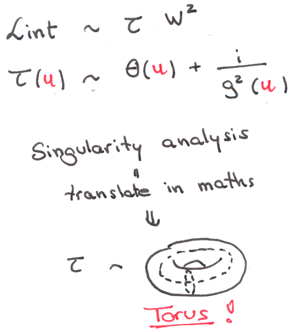
- Specify interaction pattern, symmetries
- Understand theory for all couplings
- Example exists: Seiberg-Witten understanding of supersymmetric theories

Seiberg and Witten solved a class of supersymmetric theories

Seiberg Witten's solution for all couplings

- Knowing the theory for all couplings becomes a geometric problem, coupling parameter $\Rightarrow$ geometry of an auxiliary space *Moduli Space*
- Knowing the singularities of the space = knowing where the effective description breaks down

Seiberg & Witten (1994)



Is it possible to find moduli spaces for all backgrounds?

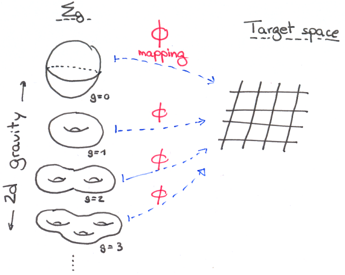
Moduli space for backgrounds?

- It's not just about couplings...
- How to specify the difference between one metric and another? topology? curvature?
- All possible boundary conditions? (In String theories this is the D-brane content)
- Sum over boundary condition? theories?
- difficult, approach through String Field Theory

Topological String Theories have close ties to maths

Topological String Theories have special features

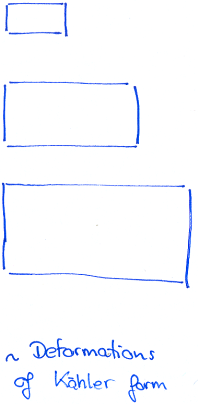
- Symmetry $Q^2 = 0$, BRST-like symmetry $Q_B^2 = 0$ for fixing gauge symmetries
- Mappings (Fields on Σ_g) capture only the *topology* of the target space
- Path integral computable
- Partition function $Z_{\text{top}} = \exp(\frac{1}{2} \sum_g \lambda^{2g-2} F_g)$
- $\Rightarrow F_g$ central objects!



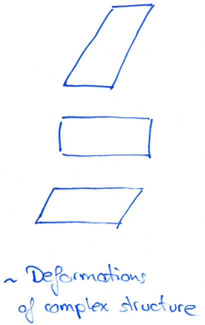
Topological String Theory captures Topology change of target space: A-Model

Size change

- Technically, changing the Kähler form which enters the definition of Volume form of the space
- There are families of spaces that only differ in that way
- Topological A-Model is a theory that captures these deformations



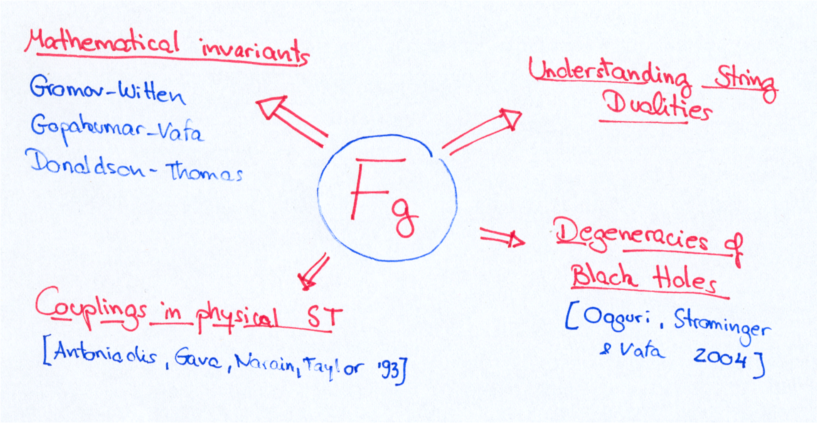
Topological String Theory captures Topology change of target space: B-Model



Shape change

- Actually: change of complex structure
- Related to the freedom in choosing what we call z and what we call $\bar{z}$
- Topological B-Model captures these changes

Topological String Theory establishes interesting connections

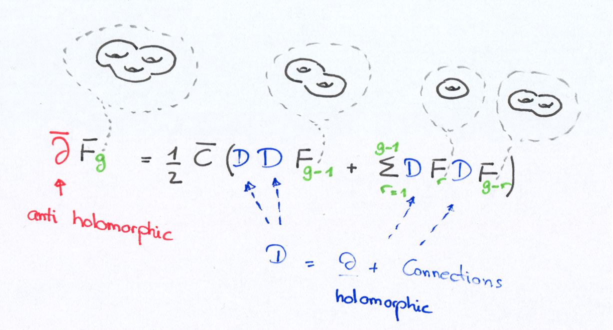


OK, then compute F_g

Strategies

- From the definition... cumbersome
- You know enough mathematics so that you can prove closed formulas... rare
- indirectly... the most common way, use features of the theory
- We'll have a closer look at a feature of topological string theory, holomorphic anomaly

Holomorphic anomaly equation gives recursive information



BCOV anomaly equation, Bershadsky, Cecotti, Ooguri & Vafa (1993)

- Relates recursively the partition function at genus g, F_g to partition functions at lower genus
- Theory is sensitive to choice of complex structure, i.e. what is holomorphic and what antiholomorphic⇒ background dependence!

Anomaly can be written for all genera

$$Z = \exp\left[\frac{1}{2}\sum_{g=0}^{\infty} \lambda^{2g-2} F_g\right]$$

$$\sum_g \Downarrow \text{Anomaly}$$

$$\left(\bar\partial - \lambda^2 \bar C D D\right) Z = 0$$

What kind of equation is this? looks familiar?

- We should look at geometric quantization!

Geometric quantization generalizes quantum mechanics

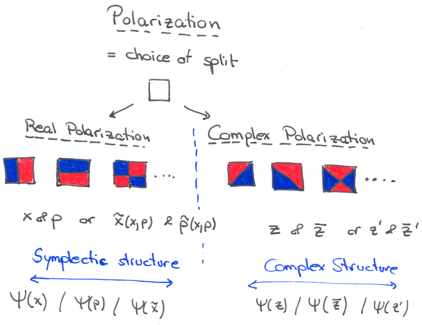
Give a geometric interpretation to quantization

- QM has phase space parameterized by x and p with Poisson bracket $\{.,.\}$
- Promote $f \to \hat{f}$, $\{.,.\} \to [.,.]$
- Throw away dependence on half of the coordinates $\psi(x)$ or $\psi(p)$, *Polarization*

Generalization

- Start with a space of dimension $2n$, that has a *symplectic* product
- Promote functions to operators (respecting the symplectic product)
- Throw away the dependence on half of the coordinates $\Rightarrow$ *Polarization*

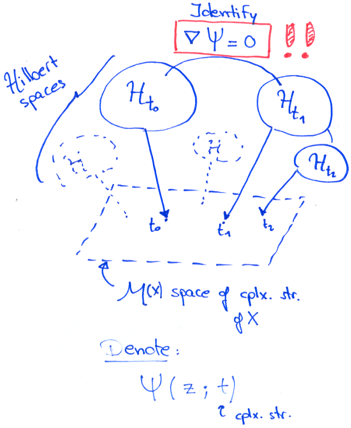
Wavefunctions only depend on half of the coordinates



Choosing a polarization means throwing away the dependence on half of the moduli

- We are familiar with this from quantum mechanics
- e.g. $\psi(x)$ or $\psi(p)$ or $\psi(a^\dagger)$

Identify all Hilbert spaces



Require that wavefunctions are covariantly constant

- Write out $\nabla\Psi = 0$

Anomaly equation is a wave equation

$$\nabla \Psi = 0$$

$$\left(\bar{\partial} - \frac{1}{4}(\bar{\partial} J\, \omega^{-1})\, D D\right) \boxed{\Psi} = 0$$

complex str.

symplectic product

now compare to

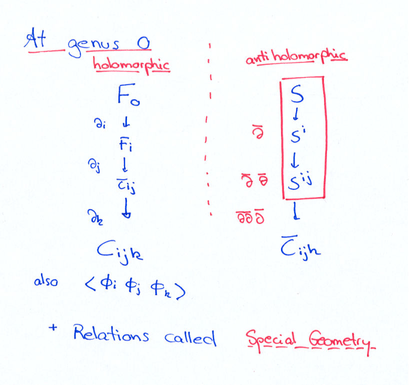
$$\left(\bar{\partial} - \lambda^2 \bar{C} D D\right) \boxed{Z_{top}} = 0$$

$\Rightarrow$ Z_{top} can be interpreted as a wavefunction in a $\mathcal{H}$ obtained by geometric quantization

We need a different interpretation for Z_{top}, Witten (1993)

- Z_{top} is a wavefunction in a Hilbert space obtained by quantizing $H^3(X)$

Genus zero is governed by special geometry

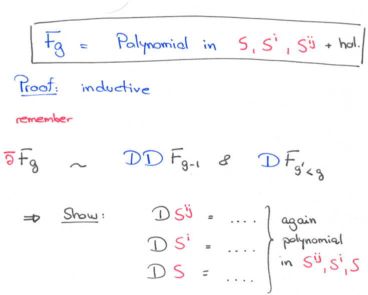


Constraints of special geometry translate into existence of potentials

- The prepotential F_0, the potential S for the antiholomorphic sector and their derivatives are the ingredients

F_g are polynomials in genus zero data

Our work Alim & Länge (2007) generalizing earlier work on the polynomial structure of the topological string for a special target space Yamaguchi & Yau (2004)



The polynomial structure facilitates computation

- however...

Still need further input

But:

$\bar{\partial} F_g \longrightarrow F_g$

needs further information
(boundary conditions)

$\Rightarrow$ Understand structure of
holomorphic part!

Still need to understand the holomorphic structure

- ongoing work...

Summary & Outlook

Summary

- Background dependence in String Theory
- Background independence in Topological String theory
- Polynomial structure of Topological Strings

Outlook

- Full understanding of polynomial structure, applications
- Connections to partition functions of Black Holes
- Understanding the Hilbert Space and the other states therein

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