Confining gauge theories
with adjoint scalars on $\mathbb{R}^3 \times S^1$

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Outline

• Brief introduction of confining gauge theories on $\mathbb{R}^3 \times S^1$

• Confining SU(2) theory on $\mathbb{R}^3 \times S^1$ with adjoint scalars
  - Perturbative calculation: The confined and Higgs phases are not compatible.
  - String tension and mass gap from a gas of monopoles.
  - Connection between the Minkowski and Euclidean monopoles.

• Conclusions
• **The Polyakov loop as an order parameter:**

\[ \langle Tr P(\vec{x}) \rangle = e^{-L F_q} \]

**Center Symmetry:**
\[ z \in \mathbb{Z}(N): P(\vec{x}) \rightarrow zP(\vec{x}) \]

\[
\begin{cases}
\langle Tr P(\vec{x}) \rangle = 0 & \rightarrow F_q = \infty \quad \text{Confined} \\
\langle Tr P(\vec{x}) \rangle \neq 0 & \rightarrow F_q = \text{Finite} \quad \text{Deconfined}
\end{cases}
\]

• **Confining gauge theories**

- **Deformation**

\[ S \rightarrow S - \int \frac{d^3 x}{L^3} H_A |Tr P|^2 \]

\(<\text{J. Myers and M. Ogilvie, PRD77, 2008}>\>
\(<\text{M. Ünsal and L. Yaffe, PRD78, 2008}>\>
\(<\text{M. Shifman and M. Ünsal, PRD78, 2008}>\>
\(<\text{E. Poppitz and M. Ünsal, JHEP0909, 2009}>\quad \text{many more...}\>

- **Adjoint fermions**

\(<\text{P. Kovtun, M. Ünsal and L. Yaffe, JHEP0706, 2007}>\>
\(<\text{M. Ünsal, PRL100, 2008}>\>
\(<\text{J. Myers and M. Ogilvie, JHEP0907, 2009}>\>
\(<\text{P. Meisinger and M. Ogilvie, PRD81, 20010}>\quad \text{many more...}\>

There is region of “semiclassical confinement.”
Semiclassical evaluations at small $L$

Because of asymptotic freedom, the coupling constant is small at small $L$.

1. Perturbative calculation of the effective potential for the Polyakov loop.
2. Non-perturbative evaluation of instanton / monopole effects (Dual Meissner effect).

Can study confinement at small $L$ both perturbatively and non-perturbatively.

Some applications of confining gauge theories on $R^3 \times S^1$

- Eguchi-Kawai reduction at large-$N$
- Conformality vs confinement
- Connection to pure gauge or supersymmetric theories
- Confinement vs. the Higgs mechanism (this talk)
More Deformations for SU(2)

\[ V_d = \left[ h_1 (trP)^2 + h_2 (trP)^4 \right] / L^4 \]

- The deformations can induce a 1st- or 2nd-order phase transition.
- Will work in 2nd-order region to simplify analysis of the phase diagram.

\[ V_d = \frac{2mLN_fN_A^2}{\pi L^4} \sum_{n=1}^{\infty} \frac{K_1 (nmL) Tr_A P^n}{n} \]

\[ m \rightarrow 0 \quad \frac{4N_fN_A^2}{\pi L^4} (\theta - \pi/2)^2 \]
Deformed SU(2) with adjoint scalars on $R^3 \times S^1$

- **Euclidean Lagrangian:**

$$ L = \left[ \frac{1}{4} (F_{\mu\nu})^2 + \frac{1}{2} (D_{\mu}\phi)^T \cdot D_{\mu}\phi + V(\phi) + V_d \right] $$

The Lagrangian has $Z(2)_C \times Z(2)_H$ symmetry.

- **Center Symmetry $Z(2)_C$:** $P \rightarrow -P$
- **Reflection Symmetry $Z(2)_H$:** $\phi \rightarrow -\phi$

**Gauge-Invariant Order Parameters**

- $<\text{Tr } P>: \text{ Transforms under } Z(2)_C$
- $<\text{Tr } P^2\phi>: \text{ Transforms under } Z(2)_H$
- $<\text{Tr } P\phi>: \text{ Transforms under } Z(2)_C \times Z(2)_H$

- **Effective potential using the background field method**

$$ U = \frac{1}{2} m^2(L) v^2 + \frac{1}{4} \lambda(L) v^4 + \frac{2}{\pi^2 L^4} \left[ \left( \theta - \frac{\pi}{2} \right)^4 + \frac{\pi a}{2} \left( \theta - \frac{\pi}{2} \right)^2 \right] + O(L^{-2}) $$

**Background fields:**

$$ \bar{A}_4 = \begin{pmatrix} 2\theta/gL & 0 \\ 0 & -2\theta/gL \end{pmatrix}, \quad \bar{\phi} = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix} $$

0-loop | 1-loop
- In this theory, the Higgs and confined phases are not compatible. Agreement with 't Hooft's argument.

- We find a new symmetry breaking pattern, $Z(2)_C \times Z(2)_H \rightarrow Z(2)$: mixed confined phase.
Topological effects

- Two monopole solutions: “mixed” BPS and Kaluza-Kline (KK) monopoles

\[ L_E = \frac{1}{2} (B_i)^2 + \frac{1}{2} (D_i A_4)^2 + \frac{1}{2} (D_i \phi)^2 \]

Two Higgs (A_4 and \( \phi \)) in the Lagrangian

\[ S_{BPS} = \frac{4\pi}{g^2} \sqrt{4\theta^2 + g^2 L^2 v^2} \]
\[ S_{KK} = \frac{4\pi}{g^2} \sqrt{(2\pi - 2\theta)^2 + g^2 L^2 v^2} \]

Non-perturbative physics supports mixed phase interpretation.

- Reproduce previous models

  - Unsal and Yaffe: no scalars (set \( v=0 \))
    <M. Ünsal and L. Yaffe, PRD78, 2008>
    \[ S_{BPS} = S_{KK} = \frac{4\pi^2}{g^2} \]

  - BPS and KK are constituents of instantons.
    <T. Kraan and P. van Baal, NPB533, 1998>
    <K. Lee and C.-h Lu, PRD58, 1998>

BPS and KK are constituents of instantons.
**Abelian duality**

- Mass gap and 3D string tension from the Abelian duality.  
  <A. Polyakov, NPB 120, 1977>

\[
\int_{R^3} \left( \frac{1}{2Lq_M^2} (\partial_i \sigma)^2 - \sum_a \xi_a e^{i q_a \sigma} \right)
\]

\[\ast d\sigma = \frac{Lq^2}{2\pi} F\]

**Dual Scalar Theory**

**Effective U(1) Theory**

\[
\int_{R^3} \frac{1}{4} (F_{ij})^2 + \text{Monopole gas}
\]

(Distance >> M_w⁻¹)

- Mass Gap: \( L \frac{16\pi^2}{g^2} \sum_a \xi_a \)
- 3D String Tension: \( \frac{2g}{\pi} \sqrt{\frac{1}{L} \sum_a \xi_a} \)
- Fugacity: \( \xi_a \sim e^{-S_a} \)

- Validity of gas approximation
  - Dilute monopole-gas approximation is valid in the orange region where \( e^{-S} \ll 1 \).
  - String tension and mass gap can be computed analytically in portions of all phases.

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Poisson duality

- Poppitz and Ünsal used the Poisson resummation to show the equivalence of an infinite sum over Euclidean monopoles to an infinite sum of Minkowski dyons in the Seiberg-Witten model.

\[ \sum_a \xi_a (\theta) = 2 \xi_{BPS} (\theta) + 2 \xi_{KK} (\theta) \propto K_1 (LM_n) \]

\[ \simeq \sum_{n \in \mathbb{Z}} \xi_{BPS} (0) \frac{\sqrt{2\pi} (Lv)^2}{(LM_n)^{3/2}} e^{-LM_n + i2n\theta} \]

where \( M_n = v \sqrt{q_M^2 + n^2 q_E^2} \)

- \( M_n \) is the mass of the Julia-Zee dyon, which also appears in Minkowski space of the same theory.

- Finite sum over BPS and KK, which are constituents of instantons, is equivalent to a gas of Julia-Zee dyons, each carrying a Polyakov loop factor appropriate to its charge.

- The interpretation is valid most of the Higgs and mixed confined phases, except in the region near \( m^2 = 0 \), where \( M_0 \) is small.

“Poisson duality” is valid in this colored region.
Conclusions

- We extended the confining gauge theories with adjoint scalar fields and showed:
  - The Higgs and confined phases are not compatible. Instead, we found a new phase called the mixed confined phase.
  - Mixed BPS and KK monopoles are generalization of other monopoles in the previous models.
  - Because the phase transition is second order, mass gap and string tension can be computed using the Abelian duality in portion of all phases.
  - A sum of BPS and KK monopoles in Euclidean space are equivalent to a gas of Julia-Zee dyons in Minkowski space by the Poisson duality.

- One future direction is to apply to the finite-temperature QCD phase transitions.
Appendix
Phase diagram of deformed SU(2) with adjoin scalars

<table>
<thead>
<tr>
<th>Residual Symmetry</th>
<th>Phases</th>
<th>$&lt;\text{TrP}&gt;$</th>
<th>$&lt;\text{TrP}^2\phi&gt;$</th>
<th>$&lt;\text{TrP}\phi&gt;$</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\checkmark$ Z(2)$_C$ X Z(2)$_H$</td>
<td>Confined</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$a &gt; a_c, m^2 &gt; 0$</td>
</tr>
<tr>
<td>$\checkmark$ Z(2)$_H$</td>
<td>Deconfined</td>
<td>$\neq 0$</td>
<td>0</td>
<td>0</td>
<td>$a &lt; a_c, m^2 &gt; 0$</td>
</tr>
<tr>
<td>$\checkmark$ $\varnothing$</td>
<td>Higgs</td>
<td>$\neq 0$</td>
<td>$\neq 0$</td>
<td>$\neq 0$</td>
<td>$a &lt; a_c, m^2 &lt; 0$</td>
</tr>
<tr>
<td>$\checkmark$ Z(2)</td>
<td>Mixed Confined</td>
<td>0</td>
<td>0</td>
<td>$\neq 0$</td>
<td>$a &gt; a_c, m^2 &lt; 0$</td>
</tr>
<tr>
<td>$\times$ Z(2)$_C$</td>
<td>Higgs &amp; Confined</td>
<td>0</td>
<td>$\neq 0$</td>
<td>0</td>
<td>N/A</td>
</tr>
</tbody>
</table>

\[ a > a_c, m^2 > 0 \]
\[ a < a_c, m^2 > 0 \]
\[ a < a_c, m^2 < 0 \]
\[ a > a_c, m^2 < 0 \]

**Deconfined:** Z(2)$_H$

**Confined:** Z(2)$_C$ X Z(2)$_H$

**Higgs:** $\varnothing$

**Mixed Confined:** Z(2)

At small L