Direct determination of strange and light quark condensates from full lattice QCD

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HPQCD collaboration

Quark Confinement, Oct 2012

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Quark condensates are an important feature of low energy QCD, broken chiral symmetry.

Indirect determination in $m \to 0$ limit from Gell-Mann, Oakes, Renner relation:

$$\frac{f_\pi^2 M_\pi^2}{4} = -\frac{m_u + m_d}{2} \left\langle 0\left| \bar{u}u + \bar{d}d \right| 0 \right\rangle$$

How to determine for $m \neq 0$? e.g. for s quark

Unphysical mass dependence appears from mixing with the unit operator - careful definition required.

Sum rules: $\frac{\left\langle \bar{s}s \right\rangle}{\left\langle \bar{u}u \right\rangle} |_{\overline{MS},2\text{GeV}} = 0.75(12)$ Narison, 0202200

$= 1.2(3)$ Maltman, 0811.1590, updating Jamin 0201174
Direct determination from lattice QCD

Basic calculation is \( \langle \text{Tr} M^{-1} \rangle \) where \( M = \gamma_\mu \Delta^\mu + m \)

BUT there is a perturbative u.v divergent contribution, linear in bare quark mass, \( a m_0 \):

\[-a^3 \langle \overline{\psi} \psi \rangle_{PT} = a m_0 \left[ c_0 (a m_0) + \alpha_s c_1 (a m_0) + \ldots \right]\]

c_0 and c_1 in lattice P.Th.

for our quark formalism (HISQ)

THEN:

\[ \langle m \overline{\psi} \psi \rangle_{NP, \overline{MS}}(\mu) = a^{-4} (a^4 \langle m \overline{\psi} \psi \rangle_0 - \Delta_{PT}) \]

difference of lattice and \( \overline{MS} \) pert. th. known thru \( \alpha_s \)
Nonperturbative calculation of $\langle \text{Tr} M^{-1} \rangle$  

Use Highly Improved Staggered Quark (HISQ) action on MILC gluon configurations inc. u,d,s,c sea quarks

$n_f = 2+1+1$

High statistical accuracy, small discretisation errors and sea u, d masses close to (at) physical value. Excellent control over decay constants and meson masses to tune quark masses.
Raw results for condensate

Plot:

\[ R_s = -\frac{4m_s \langle \bar{\psi}\psi_s \rangle}{(f_{\eta_s}^2 M_{\eta_s}^2)} \]
\[ R_l = -\frac{4m_l \langle \bar{\psi}\psi_l \rangle}{(f_\pi^2 M_\pi^2)} \]

=1 from GMOR relation - lots of systematics cancel.

Note: quadratic power divergence, now quadratic in m. Still there, much smaller, after PT subtcn through \( \alpha_s \)
Physical results for condensate

Fit $R_s, R_l$ after subtracting known pert. th., allowing for unknown higher orders, disc. errors and sea quark masses.

Friday, 5 October 2012
Conclusions

PRELIMINARY RESULTS:

\[ R_s = 0.641(89) \quad R_l = 0.993(15) \quad \frac{R_s}{R_l} = 0.645(83) \]

This gives:

\[ \langle \bar{s}s \rangle_{\overline{MS}}(2\text{GeV}) = -(300(15)\text{MeV})^3 \]
\[ \langle \bar{l}l \rangle_{\overline{MS}}(2\text{GeV}) = -(284(2)\text{MeV})^3 \]

\[ \frac{\langle \bar{s}s \rangle_{\overline{MS}}(2\text{GeV})}{\langle \bar{l}l \rangle_{\overline{MS}}(2\text{GeV})} = 1.20(17) \]

eff error dominated by pert. corrrns to power divergent terms.