LATTICE RESULTS AND THE $\tau V_{us}$ PUZZLE

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OUTLINE

• Background/context: $V_{us}$ from FB $\tau$, $\tau$-EM sum rules

• OPE c.f. lattice results for the relevant correlators

• Lattice-lesson-motivated preliminary $V_{us}$ updates
CONTEXT

• 3-family unitarity, HT10 \( |V_{ud}| \Rightarrow |V_{us}| = 0.2255(10) \)

• Compatible with \( K_{\ell 3} + \) lattice \( f_+(0) \) input \((0.2255(14))\), \( \Gamma[K_{\mu 2}]/\Gamma[\pi\mu 2] + \) lattice \( f_K/f_\pi \) input \((0.2252(10))\) results

• Contrast: “conventional” inclusive FB \( \tau \) sum rule determination, Winter 2012 HFAG \( \tau \) BFs (Gamiz, CKM12): \( |V_{us}| = 0.2173 (20)_{exp} (10??)_{th} \)

• Interesting if real, but issues with theoretical systematics, not easily quantified with continuum methods

• Goal: use lattice data to shed light on some of these issues
THE V,A CORRELATORS

- Objects of interest: \( ij = ud, us, J = 0, 1 \) \( \Pi_{ij;V/A}^{(J)}(Q^2) \)

- Minkowski space:
  \[
  \Pi_{V/A}^{\mu\nu}(q^2) \equiv i \int d^4x e^{iq \cdot x} \langle 0 | T \left( J_{V/A}^\mu(x) J_{V/A}^{\dagger \nu}(0) \right) | 0 \rangle = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_{V/A}^{(1)}(q^2) + q^\mu q^\nu \Pi_{V/A}^{(0)}(q^2)
  \]

- Euclidean space:
  \[
  \Pi_{V/A}^{\mu\nu}(Q^2) = (Q^2 \delta^{\mu\nu} - Q^\mu Q^\nu) \Pi_{V/A}^{(1)}(Q^2) - Q^\mu Q^\nu \Pi_{V/A}^{(0)}(Q^2)
  \]
• EM spectral function, $\rho_{EM}$, from $\sigma[e^+e^- \to hadrons]$

• $ij = ud, us$ V,A spectral functions, $\rho_{ij;V/A}^{(J)}$, from hadronic $
\tau$ decay data ratios $R_{ij;V/A} \equiv \frac{\Gamma[\tau \to \nu_\tau hadrons_{ij;V/A}(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]}$

• Explicitly:

\[
\sigma_{bare}(s) = \frac{16\pi^3 \alpha_{EM}(0)^2}{s} \rho_{EM}(s)
\]

\[
\frac{dR_{V/A;ij}}{ds} = \frac{12\pi^2 |V_{ij}|^2 S_{EW}}{m_\tau^2} \left[ w_\tau \left( \frac{s}{m_\tau^2} \right) \rho_{ij;V/A}^{(0+1)}(s) \right. + \left. w_L \left( \frac{s}{m_\tau^2} \right) \rho_{ij;V/A}^{(0)}(s) \right]
\]
The $\tau$ FB $V_{us}$ sum rule:

* $\delta R_\tau \equiv \frac{R_{ud;V+A}}{|V_{ud}|^2} - \frac{R_{us;V+A}}{|V_{us}|^2}$

* Basic FESR relation (Cauchy’s Theorem), valid for any $s_0 > 0$, analytic $w(s)$, $\Pi = \Pi_{ij;V/A}^{(0+1)}$ or $s \Pi^{(0)}$:

$$\int_{0}^{s_0} ds \, w(s) \, \rho(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds \, w(s) \, \Pi(s)$$

* OPE representation of $[\delta R_\tau]^{OPE}$ ($s_0 = m_\tau^2$; begins at $D = 2$, few % of separate $ud$, $us$ terms) $\Rightarrow$

$$|V_{us}| = \sqrt{\frac{R_{us;V+A}}{R_{ud;V+A}} - \frac{|V_{us}|^2}{|V_{ud}|^2} - [\delta R_\tau]^{OPE}}.$$
• Details/complications in the $\tau$ FB approach:

  * Bad convergence, spectral positivity constraint violation for truncated $D = 2$, $J = 0$ $[\delta R_\tau]^{OPE}$ series
  
  $\Rightarrow$ need $[dR/ds]^{J=0}$ subtraction

  $\chi$'lly unsuppressed $\pi$, $K$ dominate subtraction

  phenomenology for small residual $u$s scalar, PS (mildly model-dependent, “highly constrained”)

  Further support for residual subtraction: compatibility of PS phenomenological output with new lattice constraints (backup slides only)
* Advantage: $J = 0$ subtraction yields $\rho_{ij;V/A}(s)\,(0+1)$ from experimental $dR_{ij;V/A}/ds$ distributions

- $\Rightarrow V_{us}$ from FESRs based on the FB $J = 0+1$ correlator difference

$$\Delta \Pi_{\tau} \equiv \Pi_{ud;V+A}^{(0+1)} - \Pi_{us;V+A}^{(0+1)}$$

with arbitrary $w(s), s_0$

- $V_{us}$ independent of $w(s), s_0$ if theory, experimental errors under control

- Stability of $V_{us}$ wrt variation of $w(s), s_0$ crucial to control/understanding of theoretical systematics
* Key questions/issues for the $\Delta \Pi_T$ approach:

- Ongoing work on experimental $u_s$ distribution

- Currently, significant $w(s)$-, $s_0$-dependence of $V_{us}$ from the conventional kinematic weight $\Delta \Pi_T$ FESR (more below)

- Key theory issue: Impact of the slowly converging $J = 0 + 1$ $D = 2$ series? (More below)

- NOTE: $V_{us}$ obtained with the 4-loop-truncated alternate FOPT (fixed-scale) and CIPT ("local-scale") $D = 2$ treatments differ by 0.0015 (already $>$ than the 0.0010 total theory error quoted by a number of previous analyses)
• The mixed $\tau$-EM FB $V_{us}$ sum rule:

* FESRs involving $\rho_{ud,us;V/A}^{(0+1)}$, $\rho_{EM}$ and the FB correlator combination

$$\Delta \Pi_{\tau-EM} \equiv 9\Pi_{EM} - 5\Pi_{ud;V}^{(0+1)} + \Pi_{ud;A}^{(0+1)} - \Pi_{us;V+A}^{(0+1)}$$

constructed to kill LO $D = 2$ OPE series coefficient

* Strong $D = 2$ OPE suppression (by construction); $D = 4$ suppression for free

* Suppression not due to hidden symmetry as VSA version of $D = 6$ not suppressed
\* D = 2, 4 contributions, \( \bar{a} = \frac{\alpha_s(Q^2)}{\pi} \), \( \bar{m}_s = m_s(Q^2) \)

\[
\left[ \Delta \Pi_{\tau}(Q^2) \right]_{D=2}^{OPE} = \frac{3}{2\pi^2} \frac{\bar{m}_s^2}{Q^2} \left[ 1 + 2.333\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + \cdots \right]
\]

\[
\left[ \Delta \Pi_{\tau-EM}(Q^2) \right]_{D=2}^{OPE} = -\frac{3}{2\pi^2} \frac{\bar{m}_s^2}{Q^2} \left[ 0 + \frac{1}{3}\bar{a} + 4.3839\bar{a}^2 + 44.943\bar{a}^3 + \cdots \right]
\]

\[
\left[ \Delta \Pi_{\tau}(Q^2) \right]_{D=4}^{OPE} = \left[ \langle m_\ell \bar{\ell} \ell \rangle - \langle m_s \bar{s}s \rangle \right] \frac{Q^4}{2 - 2\bar{a} - \frac{26}{3}\bar{a}^2}
\]

\[
\left[ \Delta \Pi_{\tau-EM}(Q^2) \right]_{D=4}^{OPE} = \left[ \langle m_\ell \bar{\ell} \ell \rangle - \langle m_s \bar{s}s \rangle \right] \frac{Q^4}{0 - \frac{8}{3}\bar{a} - \frac{59}{3}\bar{a}^2}
\]

\* The \( \Delta \Pi_{\tau} \) Determination: \( a(m_\tau^2) \sim 0.1 \Rightarrow D = 2 \)
$\Delta \Pi_\tau$ convergence question (series looks asymptotic at spacelike point for all kinematically accessible $s_0$)

- Running of $\alpha_s \Rightarrow$ improved convergence for the CIPT $D = 2$ treatment away from the spacelike point, BUT level of improvement, in fact, minor

- Cancellations from integration on the contour at 4- and 5-loop order create misleading impression

- Even larger difference in $|V_{us}|$ from FOPT and CIPT $D = 2$ treatments if include estimated 5-loop $D = 2$ coefficient
* For the $\Delta \Pi_{\tau-EM} |V_{us}|$ determination:

- Vanishing of LO $D = 2, 4$ coefficients, suppression of remaining $D = 2$ ones for $\Delta \Pi_{\tau-EM}$ c.f. $\Delta \Pi_{\tau} \Rightarrow$ expect much reduced role for analogue of $\tau$-EM $[\delta R_{\tau}]^{OPE}$ in $\Delta \Pi_{\tau-EM}$-based FESRs for $|V_{us}|$

- Price to pay for expected reduction in theory error is enhancement of experimental errors (no cancellation of $\tau$, EM normalization uncertainties; impact of $ud \ V$, EM cancellation)

- **Issue for the $\tau$-EM FESRs:** Strong apparent suppression of $\Delta \Pi_{\tau-EM}$ c.f. $\Delta \Pi_{\tau}$ real or an artifact of the few known low-order expansion terms left after deliberate suppression of the LO coefficient?
LATTICE DATA c.f. $[\Delta \Pi_{\tau}]^{OPE}, [\Delta \Pi_{\tau-EM}]^{OPE}$

- RBC/UKQCD data for $\Delta \Pi_{\tau}(Q^2), \Delta \Pi_{\tau-EM}(Q^2)$

- Generated using new $32^3 \times 64 \times 32_5$ Iwasaki+DSDR DWF configurations with near-physical $m_\pi$ [248 and 171 MeV]

- $1/a = 1.37$ GeV, $m_\pi L \sim 5.8$ ($m_\pi = 248$ MeV), $\sim 4.0$ ($m_\pi = 171$ MeV)

- *Simulation details: arXiv:1208.4412, hep-lat*

- Allows exploration of issues/questions above for spacelike-$Q^2$ (lattice c.f. OPE with lattice $m_q, f_\pi, m_\pi$)
$\Delta \Pi_\tau(Q^2)$: OPE vs lattice data, various $D = 2$ truncations

Lattice data vs the OPE for the $\tau$ FB correlator

![Graph showing lattice data vs OPE for the $\tau$ FB correlator.]
OPE errors: lattice vs 3-loop $D = 2$ OPE for $\Delta \Pi_\tau(Q^2)$
CIPT or FOPT? Local- vs fixed-scale $D = 2$ for $\Delta \Pi^\tau(Q^2)^{OPE}$

Lattice vs OPE $\tau$ FB, fixed vs local scale $D=2$

![Graph showing lattice vs OPE for $\tau$ FB with fixed vs local scale $D=2$. The graph plots $\Pi(0+1)_{ud;\bar{s}s,\bar{u}u;V+A}$ against $Q^2$ [GeV$^2$] with data points and curves for OPE, 4-loop $D=2$, local scale, central and fixed scale, central.]
Actual vs nominal-OPE suppression of $\Delta \Pi_{\tau-EM}(Q^2)$ c.f. $\Delta \Pi_{\tau}(Q^2)$

$\Delta \Pi_\tau(Q^2)$, $\Delta \Pi_{\tau-EM}(Q^2)$ from lattice, OPE

![Graph showing $\Delta \Pi_\tau(Q^2)$ and $\Delta \Pi_{\tau-EM}(Q^2)$]
Lattice data vs $[\Delta \Pi_{\tau-EM}(Q^2)]^{OPE}$

OPE vs Lattice data, $\tau$-EM combination

- Lattice data, $m_\pi = 171$ MeV
- OPE, 4-loop $D=2$, central
- OPE, 3-loop $D=2$, central
- OPE, 3-loop $D=2 + 1\sigma$
- OPE, 3-loop $D=2 - 1\sigma$
- OPE, 2-loop $D=2$, 4, central
TENTATIVE LESSONS FROM THE LATTICE DATA

• OPE correlator series behaving “asymptotically”, despite slow convergence to minimum term (4-loop, estimated 5-loop $D = 2$ terms worsen agreement with lattice $\Delta \Pi_\tau(Q^2)$)

• 3-loop $D = 2$ provides good $\Delta \Pi_\tau(Q^2)$ OPE representation for $Q^2$ between $\sim 2.3$ GeV$^2$ and $m_\tau^2$

• $D = 2$ truncation: fixed-scale favored over local-scale ($\Rightarrow$ FOPT over CIPT for FESR integrals)

• Strong suppression of $\Delta \Pi_{\tau-EM}$ c.f. $\Delta \Pi_\tau$ confirmed (even stronger than central OPE version)
PRELIMINARY $|V_{us}|$ FOR PRESCRIPTIONS FAVORED BY LATTICE DATA

- Results for $\Delta \Pi_\tau$-based FESRs using both FOPT and CIPT $D = 2$ prescriptions, CIPT in either correlator or Adler function form, 3-loop truncation for both

- Results for $\Delta \Pi_{\tau-EM}$-based FESRs using either 2-loop-truncated $D = 2, 4$ (best match to lattice data, though still too large) or ignoring OPE contributions entirely

- In both cases, show results for a range of $w(s)$ and over the range $2 \ GeV^2 < s_0 < m^2_\tau$

- Details on updating of $\tau \ ud \ V, A$ data, treatment of $us \ V+A$ data, EM cross-sections elsewhere
$|V_{us}| \text{ vs } s_0 \text{ from the } \Delta \Pi_{\tau}(Q^2) \text{ FESRs}$

$V_{us} \text{ vs } s_0 \text{ for the } \tau \text{ ud-us FB FESRs}$
$|V_{us}|$ vs $s_0$ from the $\Delta \Pi_{\tau-EM}(Q^2)$ FESRs

$|V_{us}|$ vs $s_0$, $\tau$-EM FESRs, 2-loop $D=2$, 4 OPE
SUMMARY/CONCLUSIONS

• The $\Delta \Pi^\tau_V$ $V_{us}$ Determination:

  * $D = 2$ convergence problem confirmed for $\Delta \Pi^\tau_V$; 3-loop truncation favored for $Q^2 \sim 2 \text{ GeV}^2 \rightarrow m^2_\tau$

  * Conventional $\Delta \Pi^\tau_V |V_{us}|$ determination significantly $s_0$-dependent, no stability plateau below $s_0 = m^2_\tau$ (FOPT less so than CIPT)

  * Reduced ($\sim 0.0010$) FOPT-CIPT difference for $s_0 = m^2_\tau |V_{us}|$ with 3-loop $D = 2$ truncation

  * Significant $w(s)$-dependence remains in $|V_{us}|$ results

  * Conclusion: previous theory errors significantly underestimated (and very hard to bring down)
• **The $\Delta \Pi_{\tau-EM} V_{us}$ Determination:**

  * Strong suppression of $\Delta \Pi_{\tau-EM}$ c.f. $\Delta \Pi_\tau$ suggested by OPE confirmed (in fact, even stronger than implied by the OPE with current central NP input)

  * Improved $s_0$- and $w(s)$-choice-stability for $V_{us}$

  * Central $s_0 \sim m_\tau^2$ $V_{us}$ results in good agreement with expectations from 3-family unitarity, $K$ physics

  * Residual $s_0$-instability still to be understood (4$\pi$ experimental sector a possible candidate)

• **The $J = 0$ Subtraction:** Results from SR approach for input to $J = 0$ PS subtraction fully compatible with lattice constraint tests (backup slides only)
• **Future Prospects:**

  * Improvements to exclusive mode $e^+e^- \rightarrow \text{hadrons}$ cross-sections in progress, especially VEPP-2000 for $E_{CM} \simeq 1.4 \rightarrow 2.0$ GeV

  * $u\bar{s}$ spectral integrals still based on rescalings of (now-ancient) ALEPH distribution; some exclusive mode $u\bar{s}$ distributions already available, others in progress: More exclusive, and, even better, summed inclusive $u\bar{s}$ distribution from Belle, BaBar highly desirable!

  * Updated $u\bar{d}$ V and A distributions from Belle and BaBar also most welcome, including $4\pi$ contributions (where unexpectedly large CVC violations still not resolved)
BACKUP SLIDES

• Some relevant references

• Importance of ongoing experimental work

• Lattice constraints as tests of the phenomenological approach used in the PS part of the $J = 0$ subtraction

• Reliability of the lattice data: comparison to continuum “data” for the $ij = ud$, V-A case

• Details of CIPT improvement on the contour for $w_\tau(y)$
References

- The “conventional” inclusive, \( s_0 = m_\tau^2 \), kinematic-weight-based FB \( \tau \) \( V_{us} \) determination:
  * Current status: E. Gamiz, I. Nugent CKM12 talks

- Problems with the \( J = 0 \) OPE representation and the phenomenological fix(es):


Relevance of ongoing experimental work

• Central result from conventional $s_0 = m_T^2$ FB approach (Gamiz CKM 2012): $|V_{us}| = 0.2173$

• Tension between $\tau \rightarrow K\pi\nu_\tau$ BFs and fits of $K\pi$ form factor using $K_{\ell3}$, Belle $\tau \rightarrow K\pi\nu_\tau$ distribution data (E. Passemar, Tau 2012): larger $K\pi \tau$ BFs implied by theory-constrained fits $\Rightarrow$ central $|V_{us}| \rightarrow 0.2203$

• BaBar (Adametz July 2011 thesis) $B[\tau \rightarrow K n\pi^0\nu_\tau]$ results (N.B. esp. $B[K^-\pi^0\nu_\tau]$) $\Rightarrow$ central $|V_{us}| \rightarrow 0.2196$

• Recent higher-multiplicity strange mode BF upper bounds (R. Sobie, Tau 2012) $\Rightarrow$ missing $\rho_{us};V_{+A}$ contributions as source of lower $V_{us}$ highly unlikely
CONSTRAINTS ON THE $\pi'$, $\pi''$ DECAY CONSTANTS

- **Basic idea:** Rearranged, once-subtracted dispersion relation for $P(Q^2) \equiv Q^2 \Pi_{V-A}^{(0)}(Q^2) = -Q^2 \Pi_A^{(0)}(Q^2)$ (constraints on \(\chi\)'ly suppressed excited PS state decay constants from quantities measurable on the lattice):

\[
P(Q^2) - P(Q_0^2) + \frac{(Q^2 - Q_0^2) 2 f_{\pi}^2 m_{\pi}^2}{(s + Q^2)(s + Q_0^2)} =
\]

\[
- (Q^2 - Q_0^2) \int_{9 m_{\pi}^2}^{\infty} ds \frac{s \rho_A^{(0)}(s)}{(s + Q^2)(s + Q_0^2)}
\]

- $\rho_A^{(0)} > 0 \Rightarrow$ individual resonance constraints

- Scaling to physical $m_q$: linearity of $f_{\pi'}$, $f_{\pi''}$ with $m_u + m_d$
$m_\pi = 289$ MeV, $1/a = 2.28$ GeV lattice constraints scaled down to physical $m_q$
**Lattice data c.f. continuum** \( \Pi^{(0+1)}_{V-A}(Q^2) \)

Lattice vs continuum \( \Pi^{(0+1)}_{V-A}, m_\pi = 171 \) MeV

- **OPAL data (+ DV model)**
- lattice \((m_\pi=171 \) MeV, \(1/a=1.37 \) GeV)
- lattice \((\pi \text{ pole corrected to physical } f_\pi, m_\pi)\)

Lattice vs continuum \( \Pi^{(0+1)}_{V-A}, m_\pi = 248 \) MeV

- **OPAL data (+ DV model)**
- lattice \((m_\pi=248 \) MeV, \(1/a=1.37 \) GeV)
- lattice \((\pi \text{ pole corrected to physical } f_\pi, m_\pi)\)
“Continuum” vs $\pi$-pole-corrected, $m_\pi = 289, 345, 394$ MeV

Fine lattice vs continuum $\Pi_{\nu-A}^{(0+1)}$, physical $f_\pi$, $m_\pi$
Re CIPT improvement for the $w_\tau(y)$ FESR

$|\alpha_s(Q^2)/\alpha_s(m_\tau^2)|, |w_\tau(Q^2)/w_\tau(m_\tau^2)|$ vs $\phi$, $Q^2 = m_\tau^2 e^{i\phi}$