EFFECTS OF COULOMB INTERACTION IN SUSPENDED GRAHENE

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- Problem of Dynamical Gap Generation
- Reshape of Graphene’s Linear Energy Spectrum due to Fermi Velocity renormalization
- Graphene with a chemical potential
Basics of Graphene

- Single-atom thick layer of graphite

- **Noninteracting** (tight binding) model:
  2D massless fermions (low energy approximation)
  - linear dispersion around Dirac points:

  \[ E(\vec{k}) = \hbar v_F |k| \]  
  [Wallace, 1947; Semenoff, 1984]

- Fermi velocity \( v_F \sim 10^6 \text{ m/s} \)

- include **electron-electron interactions** \( \rightarrow \) nonlinear spectrum!

- **Experiment:** micromechanical cleavage of bulk graphite (adhesive tape exfoliation)  
  [Geim, Novoselov et al, 2004]

**energy bands touch at the Dirac points** \( (K \text{ and } K') \) (zero gap semiconductor)
QED$_3$ vs Graphene vs QED$_4$

<table>
<thead>
<tr>
<th>Number of dimensions</th>
<th>fermions</th>
<th>gauge bosons</th>
</tr>
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<tbody>
<tr>
<td>QED$_3$</td>
<td>3D</td>
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<tr>
<td>Graphene</td>
<td>3D</td>
<td>4D</td>
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</tbody>
</table>

Perturbation theory:

- **QED$_4$**: primitively divergent;
  - renormalization of $\alpha_{\text{QED}}$ due to renormalization of photon self-energy

- **QED$_3$**: coupling constant has dimension of mass ➔ “superrenormalizable”

- **Graphene**: gauge propagator: $\frac{1}{|k|}$ instead of $\frac{1}{k^2}$ ➔ only divergent

- Coulomb interaction plays a distinct role...
Coulomb Interaction

- Strength of the interaction
  - Bare "graphene fine structure constant" $\alpha_G = \frac{1}{4\pi v_F \varepsilon}$

$\alpha_G \sim 2.19$ for suspended graphene $\rightarrow$ strong coupling!
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- Interesting proposal: opening of an energy gap
  - [Gamayun et al, 2007, 2010; Drut, Lähde, 2009; Son, 2007...]
  - no experimental evidence so far!
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- Renormalization of Fermi velocity
  - Experiment: spectrum of suspended graphene nonlinear near the neutrality point  
    [Elias et al, 2011]
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- Hamiltonian of 2D massless fermions interacting through Coulomb potential

\[
H = H_{\text{free}} + H_{\text{int}}
\]

\[
H_{\text{free}} = v_F \int d^2 \vec{r} \bar{\psi}(\vec{r}) \gamma^i \partial_i \psi(\vec{r})
\]

\[
H_{\text{int}} = e^2 \int d^2 \vec{r} d^2 \vec{r}' \frac{\psi^+(\vec{r}) \psi(\vec{r}) \psi^+(\vec{r}') \psi(\vec{r}')}{|\vec{r} - \vec{r}'|}
\]
\[ S^{-1}(p_0, \vec{p}) = p_0 \gamma^0 - A \vec{p} \cdot \vec{\gamma} - \Delta \]
Gap Equation

\[ S^{-1}(p_0, \vec{p}) = p_0 \gamma^0 - \theta \vec{p} \cdot \vec{\gamma} - \Delta \]

A(p) renormalizes the Fermi velocity

\[ v_F(p) = v_F A(p) \]

[Gonzalez, Vozmediano et al, 1994]
Gap Equation

\[ S^{-1}(p_0, \vec{p}) = p_0 \gamma^0 - \vec{A} \cdot \vec{n} - \Delta \]

\[ v_F(p) = v_F A(p) \]

\[ \text{dynamical gap generation at some critical coupling?} \]

\[ \text{(semimetal-insulator phase transition?)} \]

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\[ S^{-1}(p_0, \vec{p}) = p_0 \gamma^0 - A \vec{p} \cdot \vec{\gamma} - \Delta \]

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((semimetal-insulator phase transition ?)

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\[ \nu_F(p) = \nu_F A(p) \]  

[Gonzalez, Vozmediano et al, 1994]

Coupled integral eqn’s for A(p), \Delta(p)

Approximations: bare vertex

one-loop photon propagator:

\[ D(\omega, q) = \frac{1}{|q| + \Pi(\omega, q)} \]

\[ \Pi(\omega = 0, \vec{q}) = \frac{\pi e^2 N_f}{4 \epsilon \hbar v_f |\vec{q}|} \]
Search for the critical coupling by applying bifurcation theory

\[
\Delta(p) = \frac{\alpha}{\pi^2} \int_0^\infty dk \Delta(k) K(p, k) \quad [\text{kernel } K \text{ includes the renormalized Fermi velocity}]
\]
Gap Generation?

Search for the critical coupling by applying bifurcation theory

\[ \Delta(p) = \frac{\alpha}{\pi^2} \int_0^\infty dk \Delta(k) K(p, k) \quad [\text{kernel } K \text{ includes the renormalized Fermi velocity}] \]

- Critical coupling \(2.72 < \alpha_C < 3.19\) higher than the bare coupling constant \(\alpha_0 = 2.19\)
- Running Fermi velocity weakens the Coulomb interaction
  \[ \rightarrow \text{suppression of dynamical gap generation in suspended graphene!} \]
  
  \[ [\text{scale invariant ansatz } \Delta(p) = p^{-\gamma}] \]

- Confirms the (lack of) experimental evidence

- *Previous studies*: Vafek, Case (2008): \(\alpha_C = 0.83\) [two loop calculation];
  Gamayun, Gorbar, Gusynin (2010): \(\alpha_C = 0.92\) \([A(k)=1]\);
  Khveshchenko et al (2009): \(\alpha_C = 1.13\) [RPA calculation]...
Gap Generation?

- Consider a “gapped phase”
- Numerical evaluation of the dynamical fermion mass
- Valid if graphene is placed on a substrate (higher dielectric constant)
Running of Fermi Velocity

- Assume a gapless phase:

\[
A(\vec{p}) = 1 + \frac{e^2}{\varepsilon p^2} \int_{-\infty}^{\infty} d\omega \int \frac{d^2k}{(2\pi)^2} D(\omega, \vec{p} - \vec{k}) \frac{\vec{p} \cdot \vec{k} A(\vec{k})}{\omega^2 + v_F^2 k^2 A^2(\vec{k})}
\]

- Analytic result (confirmed by numerical investigations):

\[
v_F(p) = v_F \left[ 1 + f_1(\alpha) \ln \frac{\Lambda}{p} + f_2(\alpha) \right]
\]

\[f_1, f_2\text{ functions of the coupling}
\]

\[\Lambda \sim \text{inverse of the lattice spacing}\]

- Agreement with (perturbative) RG studies [Gonzalez, Vozmediano et al, 1994, 2002...]

Graphene with a chemical potential

Fermi velocity dressing function:

\[ A(\vec{p}) = 1 + \frac{e^2}{\varepsilon p^2} \int_{-\infty}^{\infty} d\omega \int \frac{d^2k}{(2\pi)^2} D(\omega, \vec{p} - \vec{k}) \frac{\vec{p} \cdot \vec{k} A(\vec{k})}{(\omega + i\mu)^2 + v_F^2 \vec{k}^2 A^2(\vec{k})} \]
Graphene with a chemical potential

- Fermi velocity dressing function: $A(\vec{p}) = 1 + \frac{e^2}{\varepsilon_p^2} \int_{-\infty}^{\infty} d\omega \int \frac{d^2k}{(2\pi)^2} D(\omega, \vec{p} - \vec{k}) \frac{\vec{p} \cdot \vec{k} A(\vec{k})}{(\omega + i\mu)^2 + v_F^2 k^2 A^2(\vec{k})}$

- **Aim:** describe the available experimental data that measure Fermi velocity as function of carrier concentration

[Elia, Gorbachev, Mayorov et al, Nature Physics, 2011]
Summary

- Coulomb interaction in graphene:
  - Dynamical gap generation suppressed by the running of Fermi velocity
  - Energy dispersion relation logarithmic instead of linear near the neutrality point
  - Agreement with experiment

- Future work
  - solve consistently the DS equation for the photon propagator
  - include four fermion interactions [Mesterhazy, Berges, von Smekal, 2012]
  - ....