The sigma meson and its components in a functional approach

Xth Quark Confinement and the Hadron Spectrum, Munich

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Introduction

★ QCD complicated (but interesting)
  • Dynamical Chiral Symmetry Breaking
  • Confinement of quarks/glues

★ Understanding important
  • Hadronic spectrum
  • Nucleon/Delta form factors
    see G. Eichmann, H. Sanchis-Alepuz
  • Electroweak Observables (muon g-2)
    see C. S. Fischer, T. Goecke
**Introduction**

★ Non-perturbative effects - need tools!
  - EFTs, ChPT, NJL, CQM ... functional methods

★ Lattice QCD
  - *ab initio*, systematic
  - expensive, finite volume/size

★ DSEs, FRG + BSE
  - *ab initio*, continuum, infinite volume
  - truncated (improvements in progress)

*see V. Mader, M. Hopfer*
Introduction

★ Scalar sector of interest
  • Proposed by Teller/Dürr for nuclear int.
  • Linear sigma-model / NJL predict state

★ Long and complicated history e.g. Amsler & Törnqvist
  • Large widths (strong pi-pi, K-K decay)
  • Inverted mass hierarchy
  • non-qqbar behaviour of decays
  • Too many states for QM, glueball mixing?
Introduction

★ Composition of the sigma

• conventional \( q\bar{q} \)
• tetraquark \( q\bar{q}q\bar{q} \ q\bar{q}q\bar{q} \)
• molecule \( q\bar{q} \ q\bar{q} \)
• ...

Dynamical mixture of all of above?

★ Existence of scalar states - Classification?

• Here: investigate conventional composition and dress with dominant \( \pi \pi \) decay using BSE. Steps towards coupled channel.
Introduction

★ T-Matrix pole

\[ f_0(600) \]

\begin{align*}
M \ (\text{MeV}) & \quad 400-1200 \\
\Gamma \ (\text{MeV}) & \quad 250-500
\end{align*}

PDG 1996-2010
Introduction

★ T-Matrix pole

\[ M \text{ (MeV)} \quad 440-550 \]
\[ \Gamma \text{ (MeV)} \quad 200-350 \]

uChPT reliable

PDG 2012-present

\[ f_0(500) \]
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Quark Gap Equation

★ DSE

\[
S^{-1} = Z_f^{-1}(p^2) \left( -i \not{p} + M(p^2) \right)
\]

★ Solution requires:

- Gluon
- QG-Vertex

→ Truncation

★ Mass function
Truncation: Rainbow

★ Gluon propagator

$$D_{\mu \nu}(q) = \left( \delta_{\mu \nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) \frac{Z(q^2)}{q^2}$$

★ Quark-Gluon vertex

$$\Gamma_\mu(k, p) = \gamma_\mu$$

• Scalar dressing “Z” chosen to give DCSB
• Interaction corresponds to OGE

$q\bar{q}$ interaction for BSE must be chosen consistent with chiral symmetry breaking.
Truncation: Ladder

★ axial-vector Ward-Takahashi identity

★ Gluon ladder

- Here, easy to write down ansatz that satisfies axWTI
- Hard to go beyond!

C. S. Fischer and RW
L. Chang and C. D. Roberts
Truncation: Decay channels?

- Need corresponding truncations for quark DSE
  
  \[ \Sigma \sim K \]


★ Measuring width from pole mass requires dynamical modeling of the decay channel.

- Use a simpler approach here
Bethe-Salpeter Equation

- Ladder approximation - iterated OGE
- Satisfies axial-vector WTI
- Dynamically generates s-channel poles
- No-decay channels or flavour mixing

★ meson with $J^{PC} = 0^{++}$, $M \sim 647$ MeV
Sigma-Pi-Pi Form Factor

- Form-factor is a scalar

\[ F(P, Q) = \frac{g_{\sigma\pi\pi}}{M_{\sigma}} \]

- All particles on-shell, determines \( g_{\sigma\pi\pi} \sim 2.2 \)

- Breit-Wigner width given by

\[ \Gamma_{\sigma \rightarrow \pi\pi} = 3g_{\sigma\pi\pi}^2 \frac{\sqrt{1 - 4M_{\pi}^2/M_{\sigma}^2}}{16\pi M_{\sigma}} \sim 0.4 \text{ GeV} \]
Breit-Wigner?

- Only valid for narrow widths
- BW extracted influenced by background/process
- Pole mass more robust

$$\Gamma \approx 2 \text{Im} \sqrt{s_{\text{pole}}}$$

Can estimate shift in pole position due to decay channel via semi-perturbative correction to the sigma
Diamond Diagram

★ Self-Energy correction

\[ \Delta P^2 \sim \sigma \]

- Extend FF to off-shell momenta
Diamond Diagram

- Phase space information contained in integrand.

★ integrate over pion propagator poles when

\[ M_\sigma > 2M_\pi \]

- Tune masses with quark mass as parameter
- open/close sigma-pi-pi channel
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Self-energy correction

- At physical pion mass, \( \Delta P^2 \sim 0.13 - 0.33i \)
Shift in T-matrix pole

- At physical pion mass, \( s^{1/2} \sim 0.6 - 0.274i \)
Breit-Wigner Mass/Width

At physical pion mass, \( M \sim 0.54, \Gamma \sim 0.61 \)
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Conclusion

★ Iterated one-gluon exchange

- $q\bar{q}$ core, BS mass 0.647 GeV
- No decay width

★ Sigma to pi-pi decay

- on mass shell: $g_{\sigma\pi\pi} \sim 2.3$
- Breit Wigner: $\Gamma \sim 0.4$ GeV
Conclusion

★ Semi-perturbative pi-pi decay

• phase space/threshold
• sizable width
• negative mass shift

★ T-matrix pole
\[ s^{1/2} \sim 0.6 - 0.274i \]
★ Breit-Wigner
\[ M \sim 0.54, \quad \Gamma \sim 0.61 \]

• Compare with uChPT
\[ s^{1/2} \sim 0.457^{+0.014}_{-0.013} - 0.279^{+0.011}_{-0.007}i \]

J. Pelaez et al, arXiv:1209.1241
Outlook

• Dynamical inclusion of decay channel in BSE
  
  
  P. Watson, RW, C. S. Fischer, Work In Progress

• Coupling to tetraquarks & glueballs needed.

  
  see S. Strauss

• Still question of explaining decay channels / rates