The Quark-Gluon Vertex in Landau Gauge QCD
Xth Quark Confinement and the Hadron Spectrum, 7-12 October 2012

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(A. Windisch, R. Alkofer)
1 Motivation

2 Dyson-Schwinger Equations
   - The Coupled System
   - Solution Strategies

3 Preliminary Results

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Physical Motivation

The Phase Diagram of QCD

- **No free quarks observed in nature → Confinement**
- **How can three light quarks constitute a heavy hadron? → Dynamical Breaking of Chiral Symmetry \(D_XSB\)**
- **Quarks behave massless for high \(T\) → chiral symmetry restored**
- **Possible relation between the two phenomena?**

Why studying the Quark-Gluon Vertex?

- **Quark-gluon vertex ⇔ interaction between quarks/gluons**
- **No full self-consistent solution of this object at \(T = 0\)**
- **Only models available for \(T \neq 0\) and/or \(\mu \neq 0\)**
Quark Condensate

- Order parameter for **chiral symmetry breaking**
  - No crossover because of **quenched gluon input**!
  - **Quark-Gluon vertex models work remarkably well**!
    - see e.g. [C. Fischer, J. Luecker, 2012], poster J. Luecker

![Graph showing chiral symmetry broken and restored with parameter d1 = 4.6](image-url)

[Fischer, Maas and Mueller, 2010]
Quark Condensate

- Order parameter for *chiral symmetry breaking*
  - No crossover because of *quenched gluon input*!
  - *Quark-Gluon vertex models* work remarkably well!
    - But: introduce new parameters \( \rightarrow \) fix to physical observables (e.g. \( f_\pi \))
  - Goal: better understanding of the quark-gluon vertex function

\[
\begin{align*}
\Delta \pi (T) [\text{GeV}^3] \\
T [\text{MeV}] \\
\end{align*}
\]

\[
\begin{align*}
d_1 = 5.5 \\
d_1 = 4.6 \\
d_1 = 4.0 \\
d_1 = 3.5 \\
d_1 = 3.3 \\
d_1 = 3.0 \\
\end{align*}
\]
The quark–gluon vertex in Landau gauge QCD: Its role in dynamical chiral symmetry breaking and quark confinement

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\textbf{ABSTRACT}

The infrared behavior of the quark–gluon vertex of quenched Landau gauge QCD is studied by analyzing its Dyson–Schwinger equation. Building on previously obtained results for Green functions in the Yang–Mills sector, we analytically derive the existence of power-law infrared singularities for this vertex. We establish that dynamical chiral symmetry breaking leads to the self-consistent separation of
The Infrared Behavior of the System

Yang-Mills Sector of QCD

Kugo-Ojima Confinement Scenario (and similar in Gribov-Zwanziger scenario)

Gluon Propagator

\[ p^2 \langle A(p)A(-p) \rangle \xrightarrow{p^2 \to 0} 0 \]

- vanishes for small momenta

Ghost Propagator

\[ p^2 \langle c(p)\bar{c}(-p) \rangle \xrightarrow{p^2 \to 0} \infty \]

- diverges for small momenta

The Role of the Quark-Gluon Vertex?

- Linear rising quark potential?
- \( D_\chi SB \Leftrightarrow \) effective quark interaction strength
  \( \rightsquigarrow \) enhanced quark-gluon vertex?
- Running coupling \( \rightsquigarrow \) infrared fixed point?
Some Comments

- **Dyson-Schwinger equations (DSEs)**
  - non-perturbative approach
  - (coupled) integral equations
  - $\mu - T$-plane accessible
  - infinite tower $\rightsquigarrow$ truncations 😞

- **Quark-Gluon Vertex**: rainbow truncation, BC/CP-type vertex constructions, ...

- **Goal**: full self-consistent solution
Dyson-Schwinger Equations

Dyson-Schwinger Equation for the Quark Propagator

\[ S^{-1}(p) = S_0^{-1}(p) + g^2 C_N \epsilon \int \text{d}k \, \gamma^\mu S(k) \gamma^\nu D^{\mu\nu}_\gamma(p - k) \]

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Consider: \( T = \mu = 0 \)
Dyson-Schwinger Equations

Dyson-Schwinger Equation for the Quark Propagator

\[ S^{-1}(p) = S_0^{-1}(p) + g^2 C_{Nc} \int \overline{d}k \, \gamma^\mu S(k) \, \Gamma^{\nu}(p, k) \, D^{\mu\nu}_{\gamma}(p - k) \]

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Consider: \( T = \mu = 0 \)

Basis (if \( T = \mu = 0 \))

\[ \Gamma^{\nu} \sim \left\{ \begin{array}{c} 1 \\ k \\ \overline{k} \end{array} \right\} \otimes \left\{ \begin{array}{c} \gamma^{\nu} \\ k^{\nu} \\ p^{\nu} \end{array} \right\} \]
Dyson-Schwinger Equation for the Quark Propagator

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Consider: \( T = \mu = 0 \)

Basis (if \( T = \mu = 0 \))

\[ \Gamma^{\nu} \rightsquigarrow \left\{ \begin{array}{c} 1 \\ k \\ \frac{k}{p} \end{array} \right\} \otimes \left\{ \begin{array}{c} \gamma^{\nu} \\ k^{\nu} \\ p^{\nu} \end{array} \right\} \]
The Coupled System

Dyson-Schwinger Equation for the Quark Propagator

\[ \begin{array}{c}
-1 = -1 + \\
\end{array} \]  

Dyson-Schwinger Equation for the Quark-Gluon Vertex

\[ \begin{array}{c}
\begin{array}{c}
\text{Gluon propagator} \\
\text{taken from lattice/DSE calculations}
\end{array} = \begin{array}{c}
\text{3-gluon vertex} \\
\text{only models available}
\end{array} \end{array} \]  

Remarks/Ingredients

- **Gluon propagator** → taken from lattice/DSE calculations 😊
- **3-gluon vertex** → only models available 😞
The Coupled System

Dyson-Schwinger Equation for the Quark Propagator

\[ -1 = -1 + \]

Dyson-Schwinger Equation for the Quark-Gluon Vertex

\[ - \frac{1}{N_c} + N_c \]

Remarks/Ingredients

- **Dress all vertices** ⇔ include higher order corrections
- **Correspondence to DSE-like equations in 3PI formalism** [J. Berges, 2004]
- **Strategy adopted** from: The quark-gluon vertex in Landau gauge QCD [Alkofer, Fischer, Llanes-Estrada, Schwenzier, 2008]
The Coupled System

Dyson-Schwinger Equation for the Quark Propagator

\[ -1 = -1 + \]  

\[ -1 + \]  

Dyson-Schwinger Equation for the Quark-Gluon Vertex

\[ = -\frac{1}{N_c} + N_c \]  

Simplifications/Tools

- **Investigate scalar theory** ⇔ fundamentally charged scalars
  - No Dirac structure → simplified playground [MH, diploma thesis, 2011]
- **Calculations performed on GPUs** → see appendix
The Coupled System

**Dyson-Schwinger Equation for the Quark Propagator**

\[ -1 = -1 + \]

**Abelian Diagram**  
**non-Abelian Diagram**

**Dyson-Schwinger Equation for the Quark-Gluon Vertex**

\[ = \frac{-1}{N_c} + N_c \]

**Simplifications/Tools**

- **Investigate scalar theory** ⇔ fundamentally charged scalars
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Contribution of the Abelian/non-Abelian Diagram

\[ \Gamma_{\mu}(p_1, p_2; p_2 - p_1) = \sum_{i=1}^{12} \lambda_i(p_1^2, p_2^2, p_1 \cdot p_2) L_{i\mu} \]

Pick out an arbitrary vertex dressing function:
- here: \( \lambda_1 \) at \( p_1^2 = p_2^2 = p^2 \) and \( p_1 \cdot p_2 = 0 \)

Abelian diagram is sub-leading - non-Abelian diagram is important.

Leading Order Skeleton Expansion generates no dynamical mass!!
The Coupled System in a Leading Order Skeleton Expansion

Contribution of the Abelian/non-Abelian Diagram

\[ -1 = -1 + \]

\[ \sum_{i=1}^{12} \lambda_i(p_1, p_2, p_1 \cdot p_2) L_i^\mu \]

Pick out an arbitrary vertex dressing function
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Leading Order Skeleton Expansion generates no dynamical mass !!
Solution Strategy - Step 1

- Include **only parts of the non-Abelian diagram**
- Dress all vertices in the non-Abelian diagram
  - System is now **much** more complicated
  - Preliminary calculations indicate: only small corrections (similar as in scalar theory)
    - But: a lot more things to do/check
- Include the Abelian diagram (suppressed!?)
Solution Strategies

The Quark-Gluon Vertex - Step-by-Step

Solution Strategy - Step 2

- Include only parts of the non-Abelian diagram ✓
- Dress all vertices in the non-Abelian diagram ✓
  - System is now much more complicated
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- Include the Abelian diagram (suppressed!?)
Solution Strategies

The Quark-Gluon Vertex - Step-by-Step

\[ \frac{-1}{N_c} + N_c \]

Solution Strategy - Step 3

- Include only parts of the non-Abelian diagram ✓
- Dress all vertices in the non-Abelian diagram ✓
  - System is now much more complicated
  - Preliminary calculations indicate: only small corrections (similar as in scalar theory)
    - But: a lot more things to do/check
- Include the Abelian diagram (suppressed!?)

non-Abelian Diagram
The Coupled System

Dyson-Schwinger Equation for the Quark Propagator

\[ -1 = -1 + \]

Dyson-Schwinger Equation for the Quark-Gluon Vertex

\[ +N_c \]

Setup

- Include only the (partially dressed) non-Abelian diagram
- Solve the system self-consistently using all tensor structures
  - Gluon taken from [Alkofer, Fischer, Llanes-Estrada, Schwenzer, 2008]
Preliminary Results

**Quark Propagator**

1. $\frac{1}{A(x)}$
2. $B(x)$
3. $M(x) = \frac{B(x)}{A(x)}$

**Dressing Functions**

- **Quark propagator:** $S^{-1}(p) = i p A(p^2) + 1 B(p^2)$
- **Dynamical chiral symmetry breaking**
3-Gluon Vertex Dependence

\[ x = p^2 \ [\text{GeV}^2] \]

IR Exponent: -0 \( \kappa \)

\[ M(x) \]

3-Gluon-Vertex \( \rightarrow \) bare structure + model

\[ \Gamma_{3g}^{\mu \nu \sigma} = \left( \frac{x}{x + d_1} \right)^{-3\kappa} \left( d_3 \frac{d_1}{d_1 + x} + d_2 \log \left[ \frac{x}{d_1} + 1 \right] \right)^{17/44} \Gamma_{3g,0}^{\mu \nu \sigma} \]

\[ x = p_1^2 + p_2^2 + p_3^2 \ \land \ d_1, d_2, d_3 \ldots \text{ free parameters} \]
3-Gluon Vertex Dependence

3-Gluon-Vertex \rightarrow \text{bare structure + model}

- \Gamma_{3g}^{\mu\nu\sigma} = \left( \frac{x}{x + d_1} \right)^{-3\kappa} \left( d_3 \frac{d_1}{d_1 + x} + d_2 \log \left[ \frac{x}{d_1} + 1 \right] \right)^{\frac{17}{44}} \Gamma_{3g,0}^{\mu\nu\sigma}

- x = p_1^2 + p_2^2 + p_3^2 \land d_1, d_2, d_3 \ldots \text{free parameters}
3-Gluon Vertex Dependence

$$\Gamma^{\mu\nu\sigma}_{3g} = \left( \frac{x}{x + d_1} \right)^{-0\kappa} \left( d_3 \frac{d_1}{d_1 + x} + d_2 \log \left[ \frac{x}{d_1} + 1 \right] \right)^{17/44} \Gamma^{\mu\nu\sigma}_{3g,0}$$

- $$x = p_1^2 + p_2^2 + p_3^2 \land d_1, d_2, d_3 \ldots$$ free parameters
Preliminary Results

3-Gluon Vertex Dependence

$\Gamma_{3g}^{\mu\nu\sigma} = \left( \frac{x}{x + d_1} \right)^{-1\kappa} \left( d_3 \frac{d_1}{d_1 + x} + d_2 \log \left[ \frac{x}{d_1} + 1 \right] \right)^{17/44} \Gamma_{3g,0}^{\mu\nu\sigma}$

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u\sigma} \]

- \( x = p_1^2 + p_2^2 + p_3^2 \wedge d_1, d_2, d_3 \ldots \text{ free parameters} \)
3-Gluon Vertex Dependence

\[ \Gamma_{3g}^{\mu\nu\sigma} = \left( \frac{x}{x + d_1} \right)^{-3\kappa} \left( d_3 \frac{d_1}{d_1 + x} + d_2 \log \left[ \frac{x}{d_1} + 1 \right] \right)^{17/44} \Gamma_{3g,0}^{\mu\nu\sigma} \]

\[ x = p_1^2 + p_2^2 + p_3^2 \land d_1, d_2, d_3 \ldots \text{free parameters} \]
Running Coupling

\[ \alpha_{qg}(x) = \frac{g^2 \lambda_1^2(p^2) Z(p^2)}{4\pi A^2(p^2)} \]

- **Running Coupling:** \( \alpha_{qg}(p^2) \)
- **No scaling behavior:** \( \lambda_1 = \text{const} \) in IR region
  - Needs further investigation \( \sim \) different initialization strategy
What Can We Do Now

- Isolate important tensor structure relevant for $D\chi_{SB}$
- Crucial for $T \neq 0$ and $\mu \neq 0$ calculations

The Next Steps - To Do List

- System is rather unstable for $-3\kappa \rightarrow$ improve 3-gluon vertex model
  - (future) lattice data [Maas, 2007], [Cucchieri, Maas, Mendes, 2008]
  - Sign flip in 3-gluon vertex? [Huber, Maas, Smekal, 2012]
    - Would stabilize the system considerably
- System is rather unstable in “3PI” $\rightarrow$ improve integral kernels
- Look for scaling behavior $\rightarrow$ different initialization strategy
- Include Abelian diagram $\rightarrow$ additionally stabilize the system
- Compare with lattice data
  - [A. Kizilersu, D. Leinweber, J. Skullerud, A. Williams, 2006]
  - [J. Skullerud, P. Bowman, A. Kizilersu, D. Leinweber, A. Williams, 2005]
Summary and Outlook

Summary

- Quark confinement $\leftrightarrow D_{xSB}$
- Quark-gluon vertex $\leftrightarrow$ quark/gluon coupling
- Systematically improve on the truncation
- Solve the full vertex DSE

Quark Propagator DSE

$$\begin{align*}
-1 &= -1 + \frac{1}{N_c}
\end{align*}$$

Quark-Gluon Vertex DSE

$$\begin{align*}
\frac{1}{N_c} &= +N_c
\end{align*}$$
Summary and Outlook

Outlook

- **Include Yang-Mills system** → unquenching effects
- Extend towards $\mu - T$ → study **critical endpoint**
- Investigate **scalar theory**

### Quark Propagator DSE

\[-1 = -1 + \]

### Quark-Gluon Vertex DSE

\[-\frac{1}{N_c} + N_c\]
Thank you for your attention!

Appendix

YOU LOOK SO MUCH THINNER!
THANKS! I HAD MY APPENDIX REMOVED...

© Scott Fisk, Lomax
Numerics

GPU Programming
- Calculations on GPUs
- Mephisto GPU Cluster
  - Research Core Area 'Modeling and Simulation'

Code Development
- CUDA-C++/FORTRAN
- OpenACC
- GPU- and CPU Cluster

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Parallelization of the Code

Step 1 - Discretization

\[ \Gamma^\mu (p_1^2, p_2^2, p_1 \cdot p_2) \rightarrow \Gamma^\mu (x, y, z) \rightarrow \Gamma^\mu [x_i][y_n][z_j] \]

Step 2 - Load Distribution

\[ \Gamma^\mu [x_1][y_n][z_j] \leftarrow \text{GPU}_1 \text{ (processID = 0)} \]
\[ \Gamma^\mu [x_2][y_n][z_j] \leftarrow \text{GPU}_2 \text{ (processID = 1)} \]
\[ \vdots \]
\[ \Gamma^\mu [x_N][y_n][z_j] \leftarrow \text{GPU}_N \text{ (processID = N-1)} \]

Step 3 - Global Synchronization

Collect GPU Data and repeat until convergence

Some Technical Details

- Coarse graining with openMPI / fine graining with CUDA
  - In principle arbitrary number of GPUs possible
  - Minimal communication costs / Optimal Load Balancing
  - Very good scaling behavior of the code
Parallelization of the Code

Scaling of the CUDA Code

Some Technical Details
- Coarse graining with **openMPI** / fine graining with **CUDA**
  - In principle **arbitrary number of GPUs** possible
  - **Minimal communication costs** / **Optimal Load Balancing**
  - Very **good scaling behavior** of the code
More Technical Details
Dyson-Schwinger Equation for the Quark-Gluon Vertex

Version 1

\[
\begin{align*}
\text{Version 1} & = \text{triangle} - \text{triangle} + \text{triangle} - \frac{1}{2} \\
& - \frac{1}{2} + \text{triangle} + \text{triangle} + \frac{1}{2} + \frac{1}{6}
\end{align*}
\]

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The Quark-Gluon Vertex in Landau Gauge QCD
Dyson-Schwinger Equation for the Quark-Gluon Vertex II

Analogy to 3PI Formalism [J. Berges, 2004]

The Resulting “Simplified” System
Dyson-Schwinger Equation for the Quark-Gluon Vertex II

Analogy to 3PI Formalism [J. Berges, 2004]

The Resulting "Simplified" System