Vortex liquid in electromagnetically superconducting vacuum due to strong magnetic field: numerical lattice results

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I. Any superconductor has zero electrical DC resistance

II. Any superconductor is an enemy of the magnetic field:
   1) weak magnetic fields are expelled by all superconductors (the Meissner effect)
   2) strong enough magnetic field always kills superconductivity
The claim:

In a background of strong enough magnetic field the vacuum becomes a superconductor.

The superconductivity emerges in empty space. Literally, “nothing becomes a superconductor”.


The claim seemingly contradicts textbooks which state that:
1. Superconductor is a material (= a form of matter, not an empty space)
2. Weak magnetic fields are suppressed by superconductivity
3. Strong magnetic fields destroy superconductivity
### General features

Some features of the superconducting state of vacuum:

1. spontaneously emerges above the critical magnetic field

   \[ B_c \approx 10^{16} \text{ Tesla} = 10^{20} \text{ Gauss} \]

   or

   \[ eB_c \approx m^2_\rho \approx 31 \ m^2_\pi \approx 0.6 \ \text{GeV}^2 \]

2. usual Meissner effect does not exist

3. perfect conductor (= zero DC resistance) in one spatial dimension (along the axis of the magnetic field)

4. insulator in other (perpendicular) directions
Too strong critical magnetic field?

\[ eB_c \approx m_{\rho}^2 \approx 31 \, m_{\pi}^2 \approx 0.6 \, \text{GeV}^2 \]

Over-critical magnetic fields (of the strength \( B \approx 2\ldots3 \, B_c \)) may be generated in ultraperipheral heavy-ion collisions (duration is short, however – detailed calculations are required)

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+ Vladimir Skokov, private communication.
Approaches to the problem:

0. General arguments;

1. Effective bosonic model for electrodynamics of $\rho$ mesons based on vector meson dominance
   [M.Ch., arXiv:1008.1055]

2. Effective fermionic model (the Nambu-Jona-Lasinio model)
   [M.Ch., arXiv:1101.0117]

3. Nonperturbative effective models based on gauge/gravity duality (AdS/CFT)
   [Erdmenger, Kerner, Strydom (Munich, Germany), arXiv:1106.4551]
   [Callebaut, Dudal, Verschelde (Gent U., Belgium), arXiv:1105.2217]

5. Numerical simulation of vacuum
   ITEP Lattice Group, Moscow, arXiv:1104.3767 + (this talk)
Pairing of quarks in strong magnetic field

Well-known “magnetic catalysis”:

S.P. Klevansky and R. H. Lemmer (’89); H. Sugaumna and T. Tatsumi (’91); V. P. Gusynin, V. A. Miransky and I. A. Shovkovy (’94, ’95, ’96,...)

attractive channel: spin-0 flavor-diagonal states

 enhanced chiral symmetry breaking

This talk:

attractive channel: spin-1 flavor-offdiagonal states (quantum numbers of $\rho^{\pm}$ mesons)

electrically charged condensates: leads to electromagnetic superconductivity
Key players: $\rho$ mesons and vacuum

- $\rho$ mesons:
  - electrically charged ($q=\pm e$) and neutral ($q=0$) particles
  - spin: $s=1$, vector particles
  - quark contents: $\rho^+ = ud$, $\rho^- = du$, $\rho^0 = (uu-dd)/2^{1/2}$
  - mass: $m_\rho = 775.5$ MeV
  - lifetime: $\tau_\rho = 1.35$ fm/$c$ (very short: size of the $\rho$ meson is 0.5 fm)

- vacuum: QED+QCD, zero temperature and density
Naïve qualitative picture of quark pairing in the electrically charged vector channel: $\rho$ mesons

- Energy of a relativistic particle in the external magnetic field $B_{\text{ext}}$:

$$\varepsilon_{n,s_z}^2(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\text{ext}} + m^2$$

momentum along the magnetic field axis
nonnegative integer number
projection of spin on the magnetic field axis

(the external magnetic field is directed along the $z$-axis)

- Masses of $\rho$ mesons and pions in the external magnetic field

$$m_{\pi^\pm}^2(B_{\text{ext}}) = m_{\pi^\pm}^2 + eB_{\text{ext}} \quad \text{becomes heavier}$$
$$m_{\rho^\pm}^2(B_{\text{ext}}) = m_{\rho^\pm}^2 - eB_{\text{ext}} \quad \text{becomes lighter}$$

- Masses of $\rho$ mesons and pions:

$$m_\pi = 139.6 \text{ MeV}, \quad m_\rho = 775.5 \text{ MeV}$$
Condensation of \( \rho \) mesons

The \( \rho^\pm \) mesons become massless and condense at the critical value of the external magnetic field

\[
B_c = \frac{m^2}{e} \approx 10^{16} \text{ Tesla}
\]

masses in the external magnetic field

Kinematical impossibility of dominant decay modes

The pion becomes heavier while the rho meson becomes lighter

- The decay \( \rho^\pm \rightarrow \pi^\pm \pi^0 \)
  stops at certain value of the magnetic field

\[
m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}
\]

- A similar statement is true for \( \rho^0 \rightarrow \pi^+ \pi^- \)
Structure of the condensates

In terms of quarks, the state $\rho_1 = -i\rho_2 = \rho$ implies

$$\langle \bar{u}\gamma_1 d \rangle = \rho(x_{\perp}), \quad \langle \bar{u}\gamma_2 d \rangle = i\rho(x_{\perp})$$

Depend on transverse coordinates only

(\text{the same structure of the condensates in the Nambu-Jona-Lasinio model})

- The absolute value of the condensate:

$$|\rho_0| = \begin{cases} \sqrt{\frac{e(B_{\text{ext}} - B_c)}{2g_s^2}}, & B_{\text{ext}} \geq B_c \\ 0, & B_{\text{ext}} < B_c \end{cases}$$

Second order (quantum) phase transition, critical exponent = 1/2
Condensates of $\rho$ mesons, solutions

Superconducting condensate
(charged rho mesons)

Superfluid condensate
(neutral rho mesons)

$B = 1.01 B_c$

New objects, topological vortices, made of the rho-condensates

The phases of the rho-meson fields wind around vortex centers, at which the condensates vanish.

Anisotropic superconductivity
(via an analogue of the London equations)

- Apply a weak electric field $E$ to an ordinary superconductor
- Then one gets accelerating electric current along the electric field:

$$\frac{\partial \vec{J}_{GL}}{\partial t} = m_A^2 \vec{E}$$

[London equation]

- In the QCDxQED vacuum, we get an accelerating electric current along the magnetic field $B$:

$$\frac{\partial}{\partial t} \langle J_3 \rangle = -\frac{2e^3}{g_s^2} (B_{\text{ext}} - B_c) E_3$$

$$\frac{\partial}{\partial t} \langle J_1 \rangle = \frac{\partial}{\partial t} \langle J_2 \rangle = 0$$

(for $B \geq B_c$)

Written for an electric current averaged over one elementary (unit) rho-vortex cell

(similar results in NJL)
Numerical simulations in the magnetic field background


\[ \eta(B) = C_\rho \cdot (eB - eB_c) \]

[qualitatively realistic vacuum, quantitative results may receive corrections (20%-50% typically)]

Theory:
\[ \eta \sim \sqrt{B - B_c} \]
for \( B \geq B_c \)

The $\rho$ meson condensate is difficult to observe in lattice simulations due to strong inhomogeneities of the ground state

1) The phase of the $\rho$ meson condensate is a lively function of the transverse coordinates ($x,y$): its average is zero (analytically). But the condensate itself is nonzero.

2) The phase of the condensate jumps by $2\pi$ around the $\rho$ vortices.

3) The $\rho$ vortices move even within the same configuration (= neither straight nor static)
Can we visualise the $\rho$ meson condensate?


Observable:

$$\langle \Box \rangle_W = \frac{\langle W \Box \rangle}{\langle W \rangle} - \langle \Box \rangle$$

The color field strength tensor

$$\Box = U_{\mu\nu}(n, \tau)$$

in the presence of the Wilson loop (quark source) “$W$”
Observables:

The $\rho$ meson field $\rho(x) = \bar{u}(x)\gamma_+d(x)$; $\gamma_+ = \gamma_1 + i\gamma_2$ in the background of magnetic field $B$ and SU(2) gauge configuration $A^{SU(2)}$

$$\phi(x) = \langle \rho(0)\rho(x) \rangle_{A^{SU(2)}, B}$$

transforms as a charged field under the electromagnetic $U(1)$:

$$U(1)_{\text{em}} : \quad \phi(x) \rightarrow e^{i\omega(x)}\phi(x)$$

Normalized energy:

$$E(x) = \frac{|D_\mu \phi(x)|^2}{|\phi(x)|^2}$$

Normalized electric current:

$$j_\mu(x) = \frac{\phi^*(x)\overrightarrow{D_\mu}\phi(x) - \phi^*(x)\overleftarrow{D_\mu}\phi(x)}{2i|\phi(x)|^2}$$

Vortex density ($2\pi$ singularities in the phase of the $\rho$ meson field):

$$\nu(x) = \text{sing arg } \phi(x) = \frac{\varepsilon^{ab}}{2\pi} \partial_a \partial_b \text{ arg } \phi(x)$$
**Expectations:**

[J. Van Doorsselaere, H. Verschelde, M. Ch, arXiv:1111.4401]

Ground state: hexagonal lattice arrangement of the vortices along the magnetic field $B$

However: other lattice arrangements (square, rhombic, etc) are very close (0.1%) in energy the hexagonal order may be destroyed by fluctuations.

Expected reality: quantum fluctuations move and disturb vortices. They are no more straight parallel and static.
Normalized energy of the $\rho$ meson condensate in the transverse plane.

Check $x$-$y$ slice at fixed time $t$ and distance $z$.

Instead of a regular lattice structure we see an irregular vortex pattern (vortex liquid?) The vortices move as we move the slice.
Normalized energy of the $\rho$ meson condensate along the magnetic field.

Check $x$-$z$ slice at fixed time $t$ and coordinate $y$.

Vortices are not straight and static: they are curvy moving one-dimensional (in 3d) structures.
Electric currents around vortices:

Theory:
It is indeed a vortex liquid!

Vortex density-density correlators:
(similar pictures for other values of the magnetic field strength)

Real superconductors possess a vortex phase:

Vortex-matter phase diagram in YBa$_2$Cu$_3$O$_y$

Conclusions

- In a sufficiently strong magnetic field, condensates with $\rho^{\pm}$ meson quantum numbers are formed spontaneously.

- The vacuum (= no matter present, = empty space, = nothing) becomes electromagnetically superconducting.

- The superconductivity is anisotropic: the vacuum behaves as a perfect conductor only along the axis of the magnetic field.

- New type of topological defects, "$\rho$ vortices", emerge.

- The ground state of $\rho$ vortices is the Abrikosov-type lattice in transverse (w.r.t. the axis of magnetic field) directions.

- The liquid of the vortices is seen in lattice simulations.

- (Signatures of) the superconducting phase can be checked at LHC?
Backup
Topological structure of the $\rho$ mesons condensates
Simplest approach: Electrodynamics of $\rho$ mesons

- Lagrangian (based on vector dominance models):

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \rho^{\dagger}_{\mu\nu} \rho^{\mu\nu} + m^2_{\rho} \rho^{\dagger}_{\mu} \rho^\mu$$

$$- \frac{1}{4} \rho^{(0)}_{\mu\nu} \rho^{(0)}_{\mu\nu} + \frac{m^2_{\rho}}{2} \rho^{(0)}_{\mu} \rho^{(0)}_{\mu} + \frac{e}{2g_s} F^{\mu\nu} \rho^{(0)}_{\mu\nu}$$

- Tensor quantities

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$f^{(0)}_{\mu\nu} = \partial_\mu \rho^{(0)}_\nu - \partial_\nu \rho^{(0)}_\mu,$$

$$\rho^{(0)}_{\mu\nu} = f^{(0)}_{\mu\nu} - ig_s (\rho^{\dagger}_{\mu} \rho_\nu - \rho_{\mu} \rho^{\dagger}_\nu)$$

$$\rho_{\mu\nu} = D_\mu \rho_\nu - D_\nu \rho_\mu,$$

- Covariant derivative

$$D_\mu = \partial_\mu + ig_s \rho^{(0)}_\mu - ieA_\mu$$

- Kawarabayashi-Suzuki-Riadzuddin-Fayyazuddin relation

$$g_s \equiv g_{\rho \pi \pi} = \frac{m_{\rho}}{\sqrt{2} f_{\pi}} = 5.88$$

$$g_s \gg e \equiv \sqrt{4\pi \alpha_{\text{e.m.}}} \approx 0.303$$

- Gauge invariance

$$U(1):
\left\{
\begin{align*}
\rho^{(0)}_\mu(x) &\to \rho^{(0)}_\mu(x), \\
\rho_\mu(x) &\to e^{i\omega(x)} \rho_\mu(x), \\
A_\mu(x) &\to A_\mu(x) + \partial_\mu \omega(x)
\end{align*}
\right.$$
Too quick for the condensate to be developed, but signatures may be seen due to instability of the vacuum state.

\( \rho \)-meson vacuum state between the ions in ultraperipheral collisions.

\[
B = 0 \quad \rightarrow \quad B > B_c \quad \rightarrow \quad B > B_c \quad \rightarrow \quad B = 0
\]

“no condensate” vacuum state

instability due to magnetic field

rolling towards new vacuum state

rolling back to “no condensate” vacuum state

Emission of \( \rho \) mesons
Anisotropic superconductivity
(Lorentz-covariant form of the London equations)

We are working in the vacuum, thus the transport equations may be rewritten in a Lorentz-covariant form:

\[ \partial_{[\mu} j_{\nu]} = \kappa \frac{(F \cdot \tilde{F})}{(F \cdot F)} \tilde{F}_{\mu \nu} \]

Electric current averaged over one elementary rho-vortex cell

A scalar function of Lorentz invariants. In this particular model:

\[ \kappa = (e^3/g_s^2) \sqrt{(F \cdot F)/2 - B_c} \]

(slightly different form of \( \kappa \) function in NJL)

Lorentz invariants:

\[ (F \cdot \tilde{F}) = F_{\mu \nu} \tilde{F}_{\mu \nu} \equiv 4(\vec{B} \cdot \vec{E}) \]
\[ (F \cdot F) = F_{\mu \nu} F_{\mu \nu} \equiv 2(\vec{B}^2 - \vec{E}^2) \]
\[ \tilde{F}_{\mu \nu} = \frac{1}{2} \varepsilon_{\mu \nu \alpha \beta} F^{\alpha \beta} \]

If \( B \) is along \( x_3 \) axis, then we come back to

\[ \frac{\partial}{\partial t} \langle J_3 \rangle = -\frac{2e^3}{g_s^2} (B_{\text{ext}} - B_c) E_3 \]

and

\[ \frac{\partial}{\partial t} \langle J_1 \rangle = \frac{\partial}{\partial t} \langle J_2 \rangle = 0 \]