Polyakov loop and QCD thermodynamics from the Landau gauge gluon and ghost propagators

Kouji Kashiwa

RIKEN BNL Research Center

hep-ph/1206.0685,

『Polyakov loop and QCD thermodynamics from the gluon and ghost propagators』
K. Fukushima, K.K.

Phys. Rev. D 85 (2012) 114029,

『Critical endpoint for deconfinement in matrix and other effective models』
K.K., R. D. Pisarski, V. V. Skokov.

In preparation,

『(tentative title) Colombia plot and QCD thermodynamics at imaginary chemical potential』
K.K., R. D. Pisarski.

In preparation,

『(tentative title) Quark back reaction to deconfinement transition via gluon propagator』
K.K., Y. Maezawa.
Introduction: QCD phase diagram

So far, several effective model of QCD are proposed...

The original NJL model can describe the spontaneous chiral symmetry breaking.

The PNJL model can describe the approximate deconfinement transition because Polyakov-loop effect is included by introducing the Polyakov-loop effective potential.
Our word: Symmetry

Chiral symmetry
(Order parameter: chiral condensate)

Exist

$m_q=0$

None

Center symmetry

(No order parameter given)

Chiral symmetry

None

$m_q=\infty$

Exist

Center symmetry
(Order parameter: Polyakov-loop)
**Numerical results:** Matrix model for deconfinement transition


Model unclearness for gluonic sector


\[ A_4 = \frac{2\pi T}{g}, \quad q = \text{diag}(\phi_a, \phi_b, \phi_c) \]

Matrix model for deconfinement


\[ \nu_{\text{pt}}^g = \frac{2\pi^2 T^4}{3} \sum_{i,j=1}^N q_{ij}^2 (1 - |q_{ij}|)^2 - (N_c^2 - 1) \frac{\pi^2 T^4}{45} \]

\[ + \nu_{\text{npt}}^g = T^2 T_c \sum_{i,j=1}^N \left[ c_1 |q_{ij}||1 - |q_{ij}|| + c_2 q_{ij}^2 (1 - |q_{ij}|)^2 + c_3 \right] \]

**2-d Clombia plot**

Model dependence is quite large!
Numerical results: Matrix model for deconfinement transition

Imaginary chemical potential

\[ \Psi = e^{i\theta} \Phi \]
\[ \theta = \mu_I / T \]

There is no sign problem

Matrix model for deconfinement

<table>
<thead>
<tr>
<th>( m_I = \infty )</th>
<th>( m_I = m_s )</th>
<th>( m_s = \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice</td>
<td>5.6</td>
<td>6.7</td>
</tr>
<tr>
<td>This model</td>
<td>5.0</td>
<td>6.4</td>
</tr>
</tbody>
</table>

\[ m_c / T_c \]  Lattice data: M. Fromm, J. Langelage, S. Lottini, and O. Philipsen, JHEP 01 (2012) 042.

The logarithmic Polyakov-loop effective potential can not reproduce the LQCD data.
From here, let consider the *gluon* and *ghost propagators*. These are fundamental quantities of QCD and confinement can be discussed from those behavior.

We can expect to accurately discuss the QCD thermodynamics and deconfinement transition from this approach.

For example, Functional renormalization group approach


Fit the LQCD data of the gluon and ghost propagators.

Effective potential is calculated by those propagators.

Minimize the potential respect to the phase of the Polyakov-loop.
Introduction: Gluon and ghost propagator in Landau gauge

We start from the Landau-gauge gluon and ghost propagators obtained by lattice QCD simulation.

\[ D_{\mu\nu}(x - y) = \langle A_{\mu}(x) A_{\nu}(y) \rangle \]

\[ D_{\mu\nu}(p^2) = T_{\mu\nu} D_T(p^2) + L_{\mu\nu} D_L(p^2) \]

At finite T,

\[ D^T_{\mu\nu} = P^T_{\mu\nu} D^{(T)}_T + P^L_{\mu\nu} D^{(L)}_T \]

Landau gauge

\[ D^{(T)}_T(p) = \frac{1}{2N_q} \left\langle A^a_i A^a_i - \frac{q^2_4}{q^2} A^a_4 A^a_4 \right\rangle \]

\[ D^{(L)}_T(p) = \frac{1}{N_q} \left( 1 + \frac{q^2_4}{q^2} \right) \langle A^a_4 A^a_4 \rangle \]
Introduction: Gluon and ghost effective potential in Landau gauge


Pure gauge result at finite T.

This lattice data indicates the decoupling solution in 4D.

In this study, we fit the lattice data by the Gribov-Stingl form:

\[
D(p) = \frac{c^2 d^2 (p^2 + d^{-2})}{(p^2 + r^2)^2 + b^2}
\]

IR non zero gluon propagator.

We use \( b = 0 \) fitting result.
Numerical results:  Gluon and ghost potential in Landau gauge


Propagators in Landau gauge

Gluon propagator

Ghost dressing function

\[ D_G = \frac{p^2 + g^{-1}}{(p^2)^2} \]
Numerical results: Gluon and ghost potential in Landau gauge


There is the balance between the gluon and ghost. The non-perturbative effect is introduced through the LQCD gluon and ghost propagators.

\[ \beta \Omega_{\text{glue}} \simeq -\frac{1}{2} \text{tr} \ln D_A^{-1} + \text{tr} \ln D_C^{-1} \]

There is the \textbf{first-order} transition in SU(3).

Transition happens between \( T = 270 \) – \( 300 \) MeV .

(Actual value is 286 MeV)
Numerical results: Gluon and ghost potential in Landau gauge

Thermodynamics

Introduction: Symmetry

Chiral symmetry
(Order parameter: chiral condensate)

Exist

Center symmetry
(Order parameter: Polyakov-loop)

None

$m_q = 0$

Our word

$m_q = \infty$

Chiral symmetry

None

Exist
Numerical results: Gluon and ghost potential in Landau gauge


Quark contributions

We introduce the NJL part to describe the quark part.


Numerical results: Gluon and ghost potential in Landau gauge

How strong the quark back reaction to the gluonic sector?


Our effective potential is based on the gluon and ghost propagator and thus we can discuss it from the microscopic viewpoint.

We consider the un-quenched gluon propagator.

As a first step, we use same analytic function to fit lattice QCD data.
Numerical results: Gluon and ghost potential in Landau gauge

By using these propagator, we can calculate the effective potential of the gluonic sector. We can extract the quark back reaction!

Lattice: $16^3 \times 4$
configuration:
Nf=2 QCD with Wilson quark action
$m_\pi/m_\rho = 0.65$
generated by WHOT-QCD

By using these propagator, we can calculate the effective potential of the gluonic sector.

We can extract the quark back reaction!
Numerical results:  
Gluon and ghost potential in Landau gauge

We fix the parameters at certain $T$.  
Calculating the effective potential for all $T$ region.

Quark back reaction to the gluonic sector

Our result for $T_c'$

<table>
<thead>
<tr>
<th>$T_{pc}$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.18</td>
<td>0.82</td>
<td>0.341</td>
</tr>
<tr>
<td></td>
<td>0.86</td>
<td>0.302</td>
</tr>
<tr>
<td></td>
<td>0.94</td>
<td>0.257</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>0.201</td>
</tr>
</tbody>
</table>

$T/T_{pc}$ for using parameters

$\beta^4 V_{\text{glue}}[A_0]$
We investigate the QCD thermodynamics using the Landau gauge gluon and ghost propagators.

LQCD data of the gluon propagator is fitted by the Gribov-Stingl form.

Effective potentials for the gluon and ghost at finite T is calculated.

The first and second-order transition are obtained for SU(3) and SU(2), respectively.

Thermodynamical quantities are well reproduced near Tc.

In this approach, we can also discuss the quark back reaction to the gluonic sector.