Correlation lengths of non-perturbative stochastic Yang-Mills fields

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Motivation.

Vacuum correlation lengths from the gluelump Green functions.

Other non-perturbative world-line calculations: Quark condensate for various heavy flavors.

Conclusions.

Motivation

The Wilson loop expressed through the non-Abelian Stokes’ theorem and the cumulant expansion (H.G. Dosch and Yu. Simonov, ’88):

\[ \langle W(C) \rangle \simeq \operatorname{tr} \exp \left[ -\frac{1}{2!} \frac{g^2}{4} \int d\sigma_{\mu\nu}(x) \int d\sigma_{\lambda\rho}(x') \langle F^a_{\mu\nu}(x) T^a \Phi_{xx'} F^b_{\lambda\rho}(x') T^b \Phi_{x'x} \rangle \right]. \]

The \( \langle FF \rangle \) correlation function contains two mutually independent tensor structures – one yields confinement, the other is present already in massive QED.

Pisa lattice Group (A. Di Giacomo, M. D’Elia, E. Meggiolaro, H. Panagopoulos, ’92-’97): Within the errors, the corresponding two correlation lengths are equal.

Reconstructing $\langle FF \rangle$ from the lattice data on the static potential (G.S. Bali, N. Brambilla, A. Vairo, ’98) ⇒ the two correlation lengths can be different.

H.G. Dosch, M. Eidemüller, M. Jamin (’98) arrived at the same conclusion when obtaining the vacuum correlation lengths from the QCD sum rules.

N. Brambilla, A. Pineda, J. Soto, A. Vairo, (’99) obtained two different correlation lengths through the pNRQCD-analysis of the gluelumps’ spectra measured on the lattice by K.J. Juge, J. Kuti, C. Morningstar (’98) and by M. Foster, C. Michael (’99).

Yu. Simonov (’05) expressed $\langle FF \rangle$ via the Green functions of the 1- and 2-gluon gluelumps.

**Aim of this project:** to calculate analytically these Green functions and the vacuum correlation lengths.
Gluelumps’ Green functions $\Rightarrow$ vacuum correlation lengths

Figure: The one- and the two-gluon gluelumps.

Note: In the adjoint representation, two different objects exist, unlike just one heavy-light meson in the fundamental representation.
Gluelumps’ Green functions ⇒ vacuum correlation lengths

\[ \text{tr} \left\langle F^a_{\mu\nu}(x) T^a F^b_{\lambda\rho}(0) T^b \right\rangle = (\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda}) D(x) + \]
\[ + \frac{1}{2} \left[ \partial_\mu (x_\lambda \delta_{\nu\rho} - x_\rho \delta_{\nu\lambda}) + \partial_\nu (x_\rho \delta_{\mu\lambda} - x_\lambda \delta_{\mu\rho}) \right] D_1(x). \]

The confining and the non-confining formfactors are expressed through the gluelumps’ Green functions as

\[ D(x) \propto G_{2\text{gl}}(x), \quad D_1(x) \propto -\frac{dG_{1\text{gl}}(x)}{dx^2}. \]

Starting with the 1-gluon gluelump:

\[ G_{1\text{gl}}(x) = \int_0^\infty ds \int_0^x Dz_\mu \exp \left( -\int_0^s \frac{z_\mu^2}{4} d\lambda - \sigma S_{\text{min}} \right), \quad \text{where} \ x = (0, L). \]
Gluelumps’ Green functions $\Rightarrow$ vacuum correlation lengths

Majorating $S_{\text{min}}$ through the Cauchy-Schwarz inequality:

$$S_{\text{min}} = \int_{0}^{L} d\tau |z(\tau)| \leq \sqrt{L \int_{0}^{L} d\tau z^2}$$

$\Rightarrow$ the path integral is reduced to that of an harmonic oscillator of a variable frequency $\Rightarrow$

$$G_{1\text{gl}}(x) \simeq \frac{\sigma}{\sqrt{3\pi^3}}e^{-\sqrt{6}\sigma|x|} \Rightarrow$$

mass of the $1g$ gluelump $= \sqrt{6}\sigma \simeq 1.6$ GeV.

Cf. the lattice value of $1.4$ GeV (M. Foster, C. Michael, ’99).
The 2-gluon gluelump: \( G_{2\text{gl}}(x) \) is defined through the Wilson loop
\[ \sim e^{-\sigma_f S_{\text{min}}}, \]
where
\[ S_{\text{min}} = \int_0^L d\tau \left( |z| + |\bar{z}| + |z - \bar{z}| \right). \]

Yet another form of the Cauchy-Schwarz inequality is useful:
\[ |z| + |\bar{z}| + |z - \bar{z}| \leq \sqrt{3} \cdot \sqrt{z^2 + \bar{z}^2 + (z - \bar{z})^2}. \]

One more step: Simultaneous diagonalization of the kinetic and the potential energies of the two gluons ⇒ two non-interacting harmonic oscillators.
Gluelumps’ Green functions \( \Rightarrow \) vacuum correlation lengths

The final analytic expression:

\[
G_{2\text{gl}}(x) = \frac{3^{3/2} \sigma_f^2}{64\pi^{9/2}} \cdot d^{5/2} \int_0^\infty \frac{d\nu}{\nu^{3/2}} \int_0^\infty \frac{d\bar{\nu}}{\bar{\nu}^{3/2}} \left(1 + \alpha^2 \frac{\nu}{\bar{\nu}}\right)^3 \int_0^\infty \frac{d\lambda}{\lambda^2} 
\cdot \exp \left[-\lambda - \frac{(\nu + \bar{\nu})d}{2}\right] 
\cdot \left[\left(\beta^2 - \beta + 1\right)\left(\alpha^2 - \alpha + 1\right)(\nu + \bar{\nu} \beta^2)(\bar{\nu} + \nu \alpha^2)\right]^{3/4} 
\cdot \sinh^{-3/2} \left(d^{3/2} \sqrt{\frac{\beta^2 - \beta + 1}{\lambda}} \cdot \frac{\beta^2 - \beta + 1}{\nu + \bar{\nu} \beta^2}\right) 
\cdot \sinh^{-3/2} \left(d^{3/2} \sqrt{\frac{\alpha^2 - \alpha + 1}{\bar{\nu} + \nu \alpha^2}}\right),
\]

where

\[
\alpha = 1 - \frac{\bar{\nu}}{\nu} + \sqrt{\frac{\nu}{\bar{\nu}}} + \left(1 - \frac{\nu}{\bar{\nu}}\right)^2, \quad \beta = -\frac{\nu}{\bar{\nu}} \cdot \alpha, \quad d = \sqrt{\sigma_f} L.
\]
Gluelumps’ Green functions $\Rightarrow$ vacuum correlation lengths

Figure: $-\frac{\ln(G_{2gl}(x)/\sigma^2)}{d}$ in the range of $d \in [3, 25]$. 
mass of the $2g$ gluelump $= 6\sqrt{\sigma_f} \simeq 2.6 \text{ GeV}$,

Cf. the value of 2.56 GeV obtained within the Hamiltonian approach (Yu. Simonov, '05).

N.B. At the distances $\lesssim (\text{vacuum correlation length})$, $F_{\mu\nu} \simeq \text{const} \Rightarrow$ the nonperturbative part of the Wilson loop is

$$\langle W(C) \rangle = N_c \cdot e^{-\frac{\langle (gF_{\mu\nu})^2 \rangle}{48N_c}} S_{\text{min}}^2.$$ 

On the lattice, no $R^2$-potential at $R \gtrsim 0.2 \text{ fm}$ (G.S. Bali, '99) $\Rightarrow$ this is an upper bound for the vacuum correlation lengths in the Yang-Mills theory.
Quark condensate for various heavy flavors

The “area-squared” law for the Wilson loop yields the heavy-quark condensate of QCD sum rules (D.A., J.E.F.T. Ribeiro, ’10):

\[
\langle \bar{\psi} \psi \rangle_{SVZ} = -\frac{\langle (gF_{\mu\nu}^a)^2 \rangle}{48\pi^2 M}.
\]

The problem “How good is this formula for various heavy flavors?” requires the calculation of the effective action with the quark Wilson loop expressed through \(\langle FF\rangle\) whose correlation length \(\langle \propto \infty \rangle\).

Through the formalism of A. Barvinsky and G. Vilkovisky (’90), we show that the quark condensation is provided entirely by the confining interactions of gluonic fields, i.e. \(\langle \bar{\psi} \psi \rangle\) does not depend on \(D_1(x)\) (D.A., J.E.F.T. Ribeiro, ’12).

Choosing \(D(x) \propto e^{-\mu|x|}\), we obtain

\[
\frac{\langle \bar{\psi} \psi \rangle}{\langle \bar{\psi} \psi \rangle_{SVZ}} \equiv J(\lambda), \quad \text{where} \quad \lambda \equiv \frac{M}{\mu}.
\]
Figure: $J(\lambda)$ in the range of $\lambda \in [1, 298]$, where $298 \approx \frac{M_t}{\mu}$ corresponds to $\frac{1}{\mu} = 0.34\,\text{fm}$ in full QCD with light flavors (Pisa lattice Group, '97).
Quark condensate for various heavy flavors

If one keeps using the SVZ formula with the decrease of $M$, then $M$ effectively gets larger than just the current quark mass.

For $M_c \simeq 1.3\text{ GeV}$, $M_b \simeq 4.2\text{ GeV}$, and $M_t \simeq 173\text{ GeV}$, we obtain

$$J(M_c/\mu) \simeq 0.60, \quad J(M_b/\mu) \simeq 0.84, \quad J(M_t/\mu) \simeq 0.996$$

in full QCD with light flavors.

In QCD with only heavy flavors, the vacuum correlation length $1/\mu$ is smaller $\Rightarrow$ the values $J(M_{c,b,t}/\mu)$ also get smaller $\Rightarrow$ the SVZ formula acquires even larger corrections, becoming inapplicable to the $c$- and the $s$-quarks (corrections can reach 50% already for the $c$-quark).

When $M$ decreases down to $\mu$, $|\langle \bar{\psi}\psi \rangle|$ decreases by 64% compared to the value provided by the SVZ formula.
Green functions of the 1- and 2-gluon gluelumps are calculated analytically. The correlation length of confining (chromo-electric) fields, as defined by the mass of the 2-gluon gluelump, is smaller than the correlation length of non-perturbative non-confining (chromo-magnetic) fields, defined by the mass of the 1-gluon gluelump.

The formula of QCD sum rules for the heavy-quark condensate is inapplicable to the $c$- and the $s$-quarks. In the $b$-quark case, the correction amounts to 20%. The quark condensation is provided entirely by the non-perturbative chromo-electric fields.

The world-line effective action with the most general ansatz for the quark Wilson loop was calculated.
Conclusions

Open problems:

How to obtain the constituent masses of light quarks within the same, gauge-invariant, world-line formalism?

When approaching the chiral limit, how does the formula of QCD sum rules, along with the corrections thus obtained, go over to the chiral condensate of the form (N. Brambilla et al., '97)

$$\langle \bar{\psi} \psi \rangle \propto -\frac{\langle (gE^a)^2 \rangle}{\mu} \quad ?$$