CONFORMAL or CONFINING?
higher-representation gauge theories on the lattice

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SU(2,3,4) gauge theories with $N_f = 2$ fermions in the SYM$_2$ rep

1. Confining or conformal? And what lies in between
2. The running coupling at $m = 0$: Schrödinger Functional (= background field method)
3. Phase diagrams on a finite lattice ($m$, “$T$” ≠ 0)
4. Mass anomalous dimension $\gamma(g^2)$
POSSIBILITIES for IR PHYSICS

- Confinement & $\chi$SB $\rightarrow$ RUNNING [QCD]
  - or WALKING [ETC — extended technicolor]
- IRFP — conformal theory $\rightarrow$ STANDING STILL [unparticles?]

WALKING and IRFP [the *conformal window*] are HARD CASES:

- Running is slow — so strong coupling in IR is also strong coupling in UV (i.e., at lattice cutoff)
  i.e., we require $L \gg a$ for a weak-coupling continuum limit.
  *OTHERWISE* you are looking at a narrow range of scales!
- If merely $L \gg a$, then you might be far from the IRFP at your largest scale $L$ — so you miss the scale invariance.
- Scale invariance (approximate for WALKING) means all particle masses $\sim m_q^{1/y_m}$ with the same $y_m$. Hard to tell the two apart.
- Gauge coupling is irrelevant; $m_q$ and $1/L$ are *relevant* couplings.
  $m_q \rightarrow 0$: really, really BAD finite-size effects.

Schrödinger functional turns finite volume from a *hindrance* to a *method*. 
GAUGE GROUPS, REPs, and \( N_f \)

(Dietrich & Sannino, PRD 2007)

Our work: \( N = 2, 3, 4; \) \( \text{REP=SYM=3, 6, 10}; \) \( N_f = 2 \)

Is there an IRFP?  
Ladder approx says NO
THE $\beta$ FUNCTION in the MASSLESS THEORY: the Schrödinger Functional

Continuum SF definition of $g(L)$:  

(Lüscher et al., ALPHA collaboration)

- Hypercubical Euclidean box, volume $L^4$, massless limit
- Fix the gauge field on the two time boundaries  
  $\Rightarrow$ background field — unique classical minimum of $S_{YM}^{cl} = \int d^4x F_{\mu\nu}^2$. Make sure $L$ is the only scale.
- Calculate (if you can)

$$
\Gamma \equiv - \log Z = \text{tree-level + one-loop + \cdots} \\
= \left( \frac{1}{g^2(1/\mu)} + \frac{b_1}{32\pi^2} \log(\mu L) + \cdots \right) S_{YM}^{cl} \\
\equiv \frac{1}{g^2(L)} S_{YM}^{cl} \quad \text{nonperturbatively!}
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**LATTICE THEORY:**

- Wilson fermions
  + clover term + fat links (\(nHYP = \text{normalized HYPercubic}\))
- SF: fix spatial links \(U_i\) on time boundaries \(t = 0, L\)
  + give fermions a spatial twist
A PROPOS CHIRAL SYMMETRY:

- Define $m_q$ via AWI

$$\partial_\mu A^{a\mu} = 2m_q P^a \implies m_q \equiv \frac{1}{2} \left. \frac{\partial_4 \langle A^b_4(t) \mathcal{O}_b(t' \equiv 0, \vec{p} = 0) \rangle}{P^b(t) \mathcal{O}_b(t' \equiv 0, \vec{p} = 0) \rangle} \right|_{t = L/2}$$

- Find $\kappa_c(\beta)$ by setting $m_q = 0$. Work directly at $\kappa_c$: stabilized by SF BC's!

EXTRACTING PHYSICS

1. Fix lattice size $L$, bare couplings $\beta = 6/g_0^2$, $\kappa \equiv (8 + 2m_0a)^{-1} = \kappa_c(\beta)$

2. Calculate $1/g^2(L)$ and $1/g^2(2L)$. Use common lattice spacing ($= \text{UV cutoff}$) $a$.

3. Result: Discrete Beta Function

$$B(u, 2) = \frac{1}{g^2(2L)} - \frac{1}{g^2(L)},$$

a function of $u \equiv 1/g^2(L)$. 
The DISCRETE BETA FUNCTION — SU(2)/triplet

$u = 1/g^2$ (6$^4$ or 8$^4$)

$B(u, 2)$ crosses zero near the BZ coupling

$\Rightarrow$ IRFP
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\( \Rightarrow \) IRFP
SLOW RUNNING IS ALMOST NO RUNNING

Let \( u(s) \equiv 1/g^2(s) \), and \( \tilde{\beta}(u) \equiv du/d\log s = 2\beta(g^2)/g^4 \). [We have been plotting \( B(u, 2) = u(2) - u(1) \).]  

Slow running: \( \tilde{\beta}(u(s)) \simeq \tilde{\beta}(u(1)) — \text{quasi-conformal!} \)

Then

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\frac{u(s) - u(1)}{\log s} \simeq \tilde{\beta}(u(1))
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\[\implies \text{linear fit to } 1/g^2(\log L)\]

(improved action is crucial)
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Then

$$\frac{u(s) - u(1)}{\log s} \approx \tilde{\beta}(u(1))$$

$\Rightarrow$ collapse data for different $s$.

$\Rightarrow$ Reduced DBF $R(g^2) \approx \tilde{\beta}(g^2)$
NOW FOR SU(3)/sextet

Fits from $L = 6, 8, 12, 16$

SLOW running . . .

but does it cross zero?

Why did we stop?
PHASE DIAGRAM: (SU(3)/sextet)

THE WALL

in strong coupling:

\( m_q \) discontinuous in \( \kappa \), never zero

cf. SU(3) with large \( N_f \) fund rep


[cf. SU(2)/triplet: critical point at intersection]
PHASE DIAGRAM: (SU(3)/sextet)

Cf. QCD

No critical point
PHASE DIAGRAM: (SU(3)/sextet)

MOVING THE WALL:

Change the gauge action —

\[ S_g = \frac{\beta}{2N_c} \sum \text{Tr} U_p + \frac{\beta_f}{2d_f} \sum \text{Tr} V_p \]

where \( V_p \) is made of fat links in the fermion rep (e.g. \( \beta_f = +0.5 \))
⇒ pushes the wall to stronger coupling:

An IRFP in the SU(3)/sextet theory*

*at low significance
MASS ANOMALOUS DIMENSION

Expected: $\gamma(g_*^2) \rightarrow 1$ at sill of conformal window  
(Cohen & Georgi 1988; Kaplan, Lee, Son, Stephanov 2010)

Work with correlation functions on lattice:

$$\langle P^b(t) \mathcal{O}^b(t' = 0) \rangle|_{t = L/2} = Z_P Z_O e^{-m_\pi L/2}$$

$$\langle \mathcal{O}^b(t = L) \mathcal{O}^b(t' = 0) \rangle = Z_O^2 e^{-m_\pi L}$$

Take ratio, extract $Z_P(L)$, whence

$$\frac{Z_P(L)}{Z_P(L_0)} = \left( \frac{L}{L_0} \right)^{-\gamma}$$

assuming $\gamma \simeq \text{const}$ as $L_0 \rightarrow L$,

since the running is SLOW
MASS ANOMALOUS DIMENSION — SU(2)/triplet

\[ \text{slope} = -\gamma_m(g^2) \]

Cf. one loop: \[ \gamma = \frac{6C_2(R)}{16\pi^2} g^2 \]
Mass renormalization

slope = $-\gamma_m(g^2)$

Cf. one loop: $\gamma = \frac{6C_2(R)}{16\pi^2} g^2$
FINALLY, SU(4)/decuplet — compare all 3 theories

beta function $\tilde{b} = \frac{d}{d \log s} \left( \frac{1}{g^2 N} \right)$

$\gamma \rightarrow \sim 0.45$ — new universality?
SUMMARY

1. SU(2) gauge theory with \( N_f = 2 \) fermions in the SYM\(_2\) rep has an IRFP. SU(3), SU(4) might — at least, they run very slowly.

2. In each case, the mass anomalous dimension \( \gamma \) flattens out well short of 1.

THEORETICAL POINTS

Schwinger–Dyson eqns say these theories have no IRFP.

- Our fixed point(s) contradict the Schwinger–Dyson analysis.

SDEs also predict \( \gamma \simeq 1 \) near the sill of the conformal window (walking technicolor).

- For each \( N = 2, 3, 4 \) — \( \gamma \lesssim 0.5 \) means:
  1. We are deep in the conformal phase, or
  2. S–D eqns, model calculations are inapplicable here, too.
SUMMARY

1. SU(2) gauge theory with $N_f = 2$ fermions in the $\text{SYM}_2$ rep has an IRFP. SU(3), SU(4) might — at least, they run very slowly.

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SDEs also predict $\gamma \simeq 1$ near the sill of the conformal window (walking technicolor).

- For each $N = 2, 3, 4$ — $\gamma < 0.5$ means:
  1. We are deep in the conformal phase, or
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FOR THE FUTURE

$\gamma$ is much easier to calculate than $\beta$. More anomalous dimensions are waiting . . . (⇒ “spectrum” of conformal theories)

. . . and also more gauge theories.