Nonrelativistic effective field theory for meson-loop effects in heavy quarkonia

Feng-Kun Guo

Helmholtz-Institut für Strahlen- und Kernphysik, Universität Bonn

Quark Confinement and the Hadron Spectrum X
Garching, Oct. 07-12, 2012

Based on the following papers:
- $c\bar{c} \rightarrow c\bar{c} + \pi^0(\eta)$

**S-wave charmonia:** $\eta_c^{(')}$, $J/\psi$, $\psi'$;  
**$P$-wave charmonia:** $h_c^{(')}$, $\chi^{(')}_{cJ}(J = 0, 1, 2)$

Thick (magenta) lines: enhanced loops, thin (blue) lines: suppressed loops

Masses of the $h_c'$ and $\chi^{(')}_{c0,1}$ are taken from predictions in Li&Chao, PRD79(2009)094004

- Radiative transitions with meson-loop effects and predictions for lattice
Light quark mass ratio from $\psi' \to J/\psi \pi^0(\eta)$

- The decays $\psi' \to J/\psi \pi^0(\eta)$ were widely used to extract light quark mass ratio Ioffe (1979), Ioffe, Shifman (1980), Donoghue, Wyler (1992), Leutwyler (1996),...

- QCD multipole expansion ($\lambda_{\text{gluon}} \gg r_{Q\bar{Q}}$) $\Rightarrow$

$$R_{\pi^0/\eta} \equiv \frac{\Gamma(\psi' \to J/\psi \pi^0)}{\Gamma(\psi' \to J/\psi \eta)} = \left( \frac{\langle 0 \left| G \tilde{G} \right| \pi^0 \rangle}{\langle 0 \left| G \tilde{G} \right| \eta \rangle} \right)^2 \frac{q_\pi^3}{q_\eta^3}$$

- Axial anomaly $\Rightarrow$

$$\frac{\langle 0 \left| G \tilde{G} \right| \pi^0 \rangle}{\langle 0 \left| G \tilde{G} \right| \eta \rangle} \propto \frac{3 \sqrt{3} F_\eta}{4 F_\pi} \frac{m_d - m_u}{m_s - \bar{m}}$$

$\bar{m} \equiv (m_d - m_u)/2$

<table>
<thead>
<tr>
<th></th>
<th>$R_{\pi^0/\eta}$</th>
<th>$m_u/m_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO(2008)</td>
<td>(3.88 ± 0.23 ± 0.05)%</td>
<td>0.40 ± 0.01</td>
</tr>
<tr>
<td>BES(2004)</td>
<td>(4.8 ± 0.5)%</td>
<td>0.35 ± 0.02</td>
</tr>
</tbody>
</table>

- From Goldstone boson masses

$$\frac{m_u}{m_d} = \frac{M_{K^+}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} \left(1 + \Delta\right) = 0.553 \pm 0.043$$

Large discrepancy!
Light quark mass ratio from $\psi' \rightarrow J/\psi\pi^0(\eta)$

- The decays $\psi' \rightarrow J/\psi\pi^0(\eta)$ were widely used to extract light quark mass ratio

- QCD multipole expansion ($\lambda_{\text{gluon}} \gg r_{Q\bar{Q}}$) \Rightarrow
  \[ R_{\pi^0/\eta} = \frac{\Gamma(\psi' \rightarrow J/\psi\pi^0)}{\Gamma(\psi' \rightarrow J/\psi\eta)} = \left( \frac{\langle 0 \mid G\tilde{G} \mid \pi^0 \rangle}{\langle 0 \mid G\tilde{G} \mid \eta \rangle} \right)^2 \frac{q_\pi^3}{q_\eta^3} \]

- Axial anomaly \Rightarrow
  \[ \frac{\langle 0 \mid G\tilde{G} \mid \pi^0 \rangle}{\langle 0 \mid G\tilde{G} \mid \eta \rangle} \propto \frac{3\sqrt{3}F_\eta}{4F_\pi} \frac{m_d - m_u}{m_s - \bar{m}} \]

- From Goldstone boson masses
  \[ m_u/m_d = \frac{M_{K^+}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^0}^2} (1 + \Delta) = 0.553 \pm 0.043 \]

\[ \bar{m} \equiv (m_d - m_u)/2 \]

<table>
<thead>
<tr>
<th></th>
<th>$R_{\pi^0/\eta}$</th>
<th>$m_u/m_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO(2008)</td>
<td>$(3.88 \pm 0.23 \pm 0.05)$%</td>
<td>$0.40 \pm 0.01$</td>
</tr>
<tr>
<td>BES(2004)</td>
<td>$(4.8 \pm 0.5)$%</td>
<td>$0.35 \pm 0.02$</td>
</tr>
</tbody>
</table>

Large discrepancy!
Light quark mass ratio from $\psi' \rightarrow J/\psi \pi^0(\eta)$

- The decays $\psi' \rightarrow J/\psi \pi^0(\eta)$ were widely used to extract light quark mass ratio
  

- QCD multipole expansion ($\lambda_{\text{gluon}} \gg r_{Q\bar{Q}}$) ⇒
  
  $$R_{\pi^0/\eta} \equiv \frac{\Gamma(\psi' \rightarrow J/\psi \pi^0)}{\Gamma(\psi' \rightarrow J/\psi \eta)} = \left(\frac{\langle 0 | G\tilde{G} | \pi^0 \rangle}{\langle 0 | G\tilde{G} | \eta \rangle}\right)^2 \frac{q_3}{q_\eta^3}$$

- Axial anomaly ⇒
  
  $$\frac{\langle 0 | G\tilde{G} | \pi^0 \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} \propto \frac{3\sqrt{3}F_\eta}{4F_\pi} \frac{m_d - m_u}{m_s - \bar{m}}$$

  $$\bar{m} \equiv (m_d - m_u)/2$$

- From Goldstone boson masses
  

  $$\frac{m_u}{m_d} = \frac{M_{K^+}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2} \frac{1 + \Delta}{M_{\pi^0}^2 + M_{\pi^+}^2} = 0.553 \pm 0.043$$

  **Large discrepancy!**
Intermediate loops: non-multipole effects:

Lipkin, Tuan, PLB206(1988)349
Moxhay, PRD39(1989)3497
Zhou, Kuang, PRD44(1991)756

For $\psi' \rightarrow J/\psi\pi^0(\eta)$
Intermediate loops: non-multipole effects:

Lipkin, Tuan, PLB206(1988)349
Moxhay, PRD39(1989)3497
Zhou, Kuang, PRD44(1991)756

For $\psi' \rightarrow J/\psi \pi^0(\eta)$
Power counting analysis — loops

\[ 2M_D - M_{c\bar{c}} \ll M_D \Rightarrow \text{Nonrelativistic in charmed meson velocity } v \sim \sqrt{\frac{2M_D - M_{c\bar{c}}}{M_D}} \sim 0.5 \]

**Power counting rules of NREFT:**
- Energy \( \sim v^2 \)
- Momentum \( \sim v \)

\[ \mathcal{M}^{\text{loop}} \sim \frac{v^5}{(v^2)^3} v^2 q_\pi \frac{\Delta}{v^2} = \frac{1}{v} \Delta q_\pi \]

- \( v^5 \): non-relativistic integral measure
- \( (v^2)^{-3} \): NR propagators
- \( v^2 q_\pi \): \( P \) wave couplings
- \( \Delta \): \( M_{D^+} - M_{D^0} \sim m_d - m_u \)

Recall \( \mathcal{M}^{\text{multipole}} \propto (m_d - m_u)q_\pi \)

Loops are enhanced by a factor of \( \frac{1}{v}! \)

Including scaling of the coupling constants does not spoil the picture.
Results

- Comparing with data:

\[ R_{\pi^0/\eta} = \frac{\Gamma(\psi' \rightarrow J/\psi\pi^0)}{\Gamma(\psi' \rightarrow J/\psi\eta)} = 0.11 \pm 0.06 \]

<table>
<thead>
<tr>
<th>Data</th>
<th>$R_{\pi^0/\eta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO(2008)</td>
<td>(3.88 $\pm$ 0.23 $\pm$ 0.05)%</td>
</tr>
<tr>
<td>BES(2004)</td>
<td>(4.8 $\pm$ 0.5)%</td>
</tr>
<tr>
<td>PDG(2012)</td>
<td>(4.0 $\pm$ 0.3)%</td>
</tr>
</tbody>
</table>

\( \nu \approx 0.5 \Rightarrow \text{large corrections}. \) Both tree-level and loop contributions are considered in Mehen, Yang, PRD85(2012)014002

- Solution of the puzzle:

  The value of $m_u/m_d$ extracted from the $\psi' \rightarrow J/\psi\pi^0(\eta)$ is NOT reliable since it suffers from very large meson loop contributions!

- Role of the charmed loops in other charmonia transitions needs to be analyzed case by case
Results

• Comparing with data:

\[ R_{\pi^0/\eta} = \frac{\Gamma (\psi' \to J/\psi\pi^0)}{\Gamma (\psi' \to J/\psi\eta)} = 0.11 \pm 0.06 \]

<table>
<thead>
<tr>
<th>Data</th>
<th>( R_{\pi^0/\eta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO(2008)</td>
<td>(3.88 ± 0.23 ± 0.05)%</td>
</tr>
<tr>
<td>BES(2004)</td>
<td>(4.8 ± 0.5)%</td>
</tr>
<tr>
<td>PDG(2012)</td>
<td>(4.0 ± 0.3)%</td>
</tr>
</tbody>
</table>

\( \nu \sim 0.5 \Rightarrow \text{large corrections.} \) Both tree-level and loop contributions are considered in Mehen, Yang, PRD85(2012)014002

• Solution of the puzzle:

The value of \( m_u/m_d \) extracted from the \( \psi' \to J/\psi\pi^0(\eta) \) is NOT reliable since it suffers from very large meson loop contributions!

• Role of the charmed loops in other charmonia transitions needs to be analyzed case by case
Results

• Comparing with data:

\[ R_{\pi^0/\eta} = \frac{\Gamma(\psi' \to J/\psi \pi^0)}{\Gamma(\psi' \to J/\psi \eta)} = 0.11 \pm 0.06 \]

<table>
<thead>
<tr>
<th>Data</th>
<th>( R_{\pi^0/\eta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO(2008)</td>
<td>(3.88 ± 0.23 ± 0.05)%</td>
</tr>
<tr>
<td>BES(2004)</td>
<td>(4.8 ± 0.5)%</td>
</tr>
<tr>
<td>PDG(2012)</td>
<td>(4.0 ± 0.3)%</td>
</tr>
</tbody>
</table>

\( \nu \approx 0.5 \Rightarrow \text{large corrections.} \) Both tree-level and loop contributions are considered in Mehen, Yang, PRD85(2012)014002

• Solution of the puzzle:

The value of \( m_u/m_d \) extracted from the \( \psi' \to J/\psi \pi^0(\eta) \) is NOT reliable since it suffers from very large meson loop contributions!

• Role of the charmed loops in other charmonia transitions needs to be analyzed case by case
Charmonia transitions with the emission of one pion / eta

$S$-wave charmonia: $\eta_c^{(i)}$, $J/\psi$, $\psi'$; $P$-wave charmonia: $h_c^{(i)}$, $\chi_{cJ}^{(i)} (J = 0, 1, 2)$

$\heart SS (PP)$ transitions — Enhancement of loops
$\heart SP$ transitions — Process dependent, sometimes strongly suppressed

Thick (magenta) lines: enhanced loops, thin (blue) lines: suppressed loops

Masses of the $h'_c$ and $\chi'_{c0,1}$ are taken from predictions in Li&Chao, PRD79(2009)094004
Extracting quark mass ratio from $\Upsilon(4S) \rightarrow h_b\pi^0(\eta)$

- Considering bottomonium transitions $\Upsilon(4S) \rightarrow h_b\pi^0(\eta)$, loops are suppressed:
  \[ q_{\pi(\eta)}^2 / (v^3 M_B^2) \approx 0.6(0.2) \quad [v \approx 0.3] \]
  \[ \Delta = M_{B^0} - M_{B^+} = 0.33 \pm 0.06 \text{ MeV} \ll m_d - m_u \]
  due to the destructive interference between the e.m. and strong contributions

- $\Upsilon(4S) \rightarrow h_b\pi^0(\eta)$ can be used to extract the light quark mass ratio!

\[
\frac{\Gamma(\Upsilon(4S) \rightarrow h_b\pi^0)}{\Gamma(\Upsilon(4S) \rightarrow h_b\eta)} = \left( \frac{\langle 0 | G\tilde{G} | \pi^0 \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} \right)^2 \frac{|\bar{q}_\pi|}{|\bar{q}_\eta|}
\]

\[
r = \frac{m_d - m_u}{m_d + m_u} \frac{m_s + \bar{m}}{m_s - \bar{m}} = 10.59(1 + 132.1L_{14}) \frac{\langle 0 | G\tilde{G} | \pi^0 \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} = 0.1059(1 + 132.1L_{14}) \]

\[
L_{14} = (2.3 \pm 1.1) \times 10^{-3}
\]

F.-K.G., Hanhart, Meißner, PRL105(2010)162001


Donoghue,Holstein,Wyler (1992)
Extracting quark mass ratio from $\Upsilon(4S) \rightarrow h_b \pi^0(\eta)$

- Considering bottomonium transitions $\Upsilon(4S) \rightarrow h_b \pi^0(\eta)$, loops are suppressed:
  \[ q^2_{\pi(\eta)}/(\nu^3 M_B^2) \approx 0.6(0.2) \quad [\nu \approx 0.3] \]
  \[ \Delta = M_{B^0} - M_{B^+} = 0.33 \pm 0.06 \text{ MeV} \ll m_d - m_u \]
  due to the destructive interference between the e.m. and strong contributions

- $\Upsilon(4S) \rightarrow h_b \pi^0(\eta)$ can be used to extract the light quark mass ratio!

\[
\frac{\Gamma(\Upsilon(4S) \rightarrow h_b \pi^0)}{\Gamma(\Upsilon(4S) \rightarrow h_b \eta)} = \left( \frac{\langle 0 | G\tilde{G} | \pi^0 \rangle}{\langle 0 | G\tilde{G} | \eta \rangle} \right)^2 \frac{\bar{q}_\pi}{\bar{q}_\eta}
\]

\[
r = \frac{m_d - m_u}{m_d + m_u} \frac{m_s + \bar{m}}{m_s - \bar{m}} = 10.59(1 + 132.1 L_{14}) \frac{\langle 0 | G\tilde{G} | \pi^0 \rangle}{\langle 0 | G\tilde{G} | \eta \rangle}
\]

\[
L_{14} = (2.3 \pm 1.1) \times 10^{-3}
\]

Donoghue, Holstein, Wyler (1992)
The hindered M1 transitions of heavy quarkonia occur between two states with different radial excitations.

They are highly suppressed in quark models because the overlap of nonrelativistic wave functions vanishes:

\[ \Gamma_{M1} \propto |\langle \psi_f | \psi_i \rangle|^2 E^3_\gamma \]

P-wave heavy quarkonia couple to open-flavor heavy mesons in an S wave.

Hence, the coupled-channel effects due to heavy meson loops dominate the transitions.

\[ \gamma_{M1}^{\text{loop}} \sim \frac{v^5}{(v^2)^3} \frac{E_\gamma}{m_c} = \frac{E_\gamma}{m_c v} \]
The hindered M1 transitions of heavy quarkonia occur between two states with different radial excitations.

They are highly suppressed in quark models because the overlap of nonrelativistic wave functions vanishes:

$$\Gamma_{M1} \propto |\langle \psi_f | \psi_i \rangle|^2 E_\gamma^3$$

P-wave heavy quarkonia couple to open-flavor heavy mesons in an S wave.

Hence, the coupled-channel effects due to heavy meson loops dominate the transitions:

$$\mathcal{A}_{M1}^{\text{loop}} \sim \frac{v^5}{(v^2)^3} \frac{E_\gamma}{m_c} = \frac{E_\gamma}{m_c v}$$
Hindered M1 transitions: results

Widths calculated using NREFT

- If taking model values for $g_1 = -4 \text{ GeV}^{-1/2}$ \cite{Colangelo, De Fazio, Pham (2004)}, $|g'_1| \sim 1 \text{ GeV}^{-1/2}$ (quark model)

$$\Gamma(\chi'_{c2} \to \gamma h_c) = (10.7 \pm 4.3) \frac{(g_1 g'_1)^2}{\text{GeV}^{-2}} \text{ keV} \sim 170 \text{ keV} \gg 1.3 \text{ keV} \text{ (quark model } \text{Barnes et al}(2005)\text{)}$$

- Strong and nonanalytic pion mass dependence  

\cite{Feng-Kun Guo (Uni.Bonn)}
Hindered M1 transitions: results

Widths calculated using NREFT

• If taking model values for $g_1 = -4 \text{ GeV}^{-1/2}$ (Colangelo, De Fazio, Pham (2004)), $|g'_1| \sim 1 \text{ GeV}^{-1/2}$ (quark model)

$$\Gamma(\chi'_c \rightarrow \gamma h_c) = (10.7 \pm 4.3) \frac{(g_1 g'_1)^2}{\text{GeV}^{-2}} \text{ keV} \sim 170 \text{ keV}$$

$$\gg 1.3 \text{ keV} \text{ (quark model Barnes et al(2005))}$$

• Strong and nonanalytic pion mass dependence


![Graph showing M1 transition widths](image)
Summary

- Effective theory for meson loop effects in heavy quarkonium transitions
- Effects of meson loops need to be analyzed case by case. Quark mass ratio extraction from $\psi' \rightarrow J/\psi \pi^0(\eta)$ suffers from large loop effects. $\Upsilon(4S) \rightarrow h_b \pi^0(\eta)$ are much cleaner.

- Hindered M1 transitions between $P$-wave heavy quarkonia are dominated by meson loops
  - Can be tested using both quenched and unquenched lattice calculations
  - Strong and nonanalytic pion mass dependence
Power counting analysis for $SP$ transitions — A different case

- $J^{PC}(\psi') = 1^{--}$, $J^{PC}(h_c) = 1^{+-}$, S-wave decay:

  Tree-level amplitude $\propto (m_d - m_u)$

- Charmed meson loops:

  \[
  q_\pi = 86 \text{ MeV} \ll M_D = 1870 \text{ MeV}
  \]

  \[
  \frac{v^3}{(v^2)^2} \frac{q_\pi^2}{M_D^2} \frac{\Delta}{\nu^2} = \frac{q_\pi^2}{v^3 M_D^2} \Delta \sim \frac{\Delta}{50}
  \]

  $v^5$: non-relativistic integral measure

  $(v^2)^{-3}$: NR propagators

  $q_\pi^2$: $P$ wave couplings

  [vector loop $l^i(q) = q_\pi^i l^{(1)}(q)$]

Charmed meson loops are highly suppressed here, confirmed by explicit calculation.
If taking into account the scaling of coupling constants (NOT rigorous)

<table>
<thead>
<tr>
<th></th>
<th>Tree-level</th>
<th>Loops</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SS</strong></td>
<td>$\frac{1}{m_c} q\delta$</td>
<td>$\frac{1}{4\pi v_c^3} \frac{1}{m_c} \frac{q\Delta}{v}$</td>
</tr>
<tr>
<td><strong>SP</strong></td>
<td>$\delta$</td>
<td>$\frac{1}{2\sqrt{3}\pi v_c^4} \frac{q^2}{v^3 M_D^2} \Delta$</td>
</tr>
<tr>
<td><strong>PP</strong></td>
<td>$\frac{1}{\Lambda_{QCD}} q\delta$</td>
<td>$\frac{1}{3\pi v_c^5} \frac{1}{\Lambda_{QCD}} \frac{q\Delta}{v^3}$</td>
</tr>
</tbody>
</table>

For details, see Guo et al, PRD83(2011)034013
Light quark sea in a heavy quarkonium

Pion cloud: operators
\[ \psi^\dagger \psi \langle \chi_+ \rangle, \, \psi^\dagger \psi \langle u_\mu u^\mu \rangle, \ldots \]

They give contribution of the form
\[ c_1 M^2_\pi + c_2 M^4_\pi \log \frac{M^2_\pi}{\mu^2} + c_3 M^4_\pi + \mathcal{O}(M^6_\pi) \]

Self-energy:
\[ \Sigma(P^2) = \frac{1}{4\pi(m_1+m_2)} \left( -\frac{\lambda}{\pi} + \frac{1}{2} \sqrt{\frac{2m_1m_2}{m_1+m_2} (m_1 + m_2 - M) - i\epsilon} \right) \]

Renormalized mass of a P-wave \( Q\bar{Q} \):
\[ M(M_\pi) = M_0(\lambda, M_\pi) + g^2 m_1 m_2 \Re \Sigma(M^2, \lambda, M_\pi) \]