Thermal current correlators in Two-flavour QCD

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Quark Confinement and the Hadron Spectrum X

October 07 - 12, 2012
Thermal currents in observables

- Dileptons are produced at all stages of a heavy ion collision
  - Impact of thermal effects on the experimental results?


- Thermal effects?
Connection to thermal current correlators

- The connection between theory and experiment is given via the vector current spectral function:

\[
\frac{dN_{l^+l^-}}{d\omega d^3p} = C_{em} \frac{\alpha_{em}}{6\pi^3} \frac{\rho_{ii}(\omega, \vec{p}, T)}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)}
\]

- All thermal modification of the spectral function has direct influence on the dilepton rate!

- The spectral function encodes also the information on the electrical conductivity of the medium in the limit of vanishing frequency via the Kubo-formula:

\[
\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{\omega T}
\]
SPF via Euclidean correlators

- SPF's can be shown to be connected to Minkowski and Euclidean correlation functions at the same time
  - Possibility to extract the real-time dynamics also from imaginary-time data
  - Euclidean correlators are accessible via lattice QCD
- For a mixed (time,momentum)-representation the connection between lattice correlators and spectral functions is given by:

\[
G_{ii}(\tau, \vec{p}, T) = \int_{0}^{\infty} \frac{d\omega}{2\pi} \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} \rho_{ii}(\omega, \vec{p}, T)
\]

- Unfortunately: Going from \( G_{ii}(\tau) \) to \( \rho_{ii}(\omega) \) is very difficult
  - Ill-posed problem
Approaching the problem

In the past there have been mainly two approaches to this problem:

- Extract the SPF via the Maximum Entropy Method (MEM)
  - Difficult estimation of systematics due to dependence on default model
- Fit the SPF using a physically motivated Ansatz and constrain using sensible additional information
  - Dependence on the Ansatz and additional observables

Note: All of the prior calculations were performed either using quenched Wilson-Clover or staggered fermions.

Here we show first dynamical results using two flavours of light Wilson-Clover fermions.

Dynamical lattice correlators

- Gauge configurations generated using the DD-HMC algorithm
- Two light flavours of $O(a)$-improved Wilson quarks
- Here: Concentrate on one point at $T \sim 250$ MeV and $m_q \sim 9$ MeV

- $\beta = 5.50$
- $M_\pi \sim 200$ MeV
- $N_\sigma = 64$
- $N_\tau = 16$
  - $T \sim 250$ MeV
- $N_\tau = 128$
  - $T \sim 0$ MeV
The reconstructed correlator

- The temperature dependence of the Kernel can be taken into account analytically:

\[
G_{\text{rec}}(\tau, T) = \sum_{\tau' = \tau, \Delta = N_{\tau}}^{N_{\tau'} - N_{\tau} - \tau} G(\tau', T = 0)
\]

- The reconstructed correlator gives a “thermal” correlator based on the zero temperature spectral function

- We can construct the reconstructed correlator using our \( N_\tau = 128 \) data!

- Unique possibility to compare zero and finite temperature spectral functions directly!

- Note: This was very fruitful in the case of charmonium
Results: The vector current correlator

\[ G_{ii}(\tau)/T^3 \]

\[ G_{ii}/G_{ii}^{\text{rec}} \]

- The ratio of reconstructed and thermal correlators shows they differ by \(\sim 10\%\) for the largest time separations.

\[ [G_{ii}(\tau)-G_{ii}^{\text{rec}}(\tau)]/T^3 \]

- The difference of the two is positive for large times and drops to negative values as \(\tau\) is decreased.
Results: The vector current correlator II

- The ratio $G_{ii}(\tau)/G_{ii}^{free,c/l}(\tau)$ is exactly connected via:

\[ G_{\mu\mu}(\tau) = -\chi_q T + G_{ii}(\tau) \]

- Dominant cut-off and physics regions become more clearly distinguished at $\tau \gtrsim 4$

- The ratio of thermal and free correlators decreases throughout

- Comparing continuum and lattice, ratios come together only at $\tau \geq 6$
Fitting the SPF

- Our philosophy is now to fit the SPF to our results using an appropriate Ansatz

  \[ G(\tau)/G_{\text{free}}(\tau) : \text{Transport } \rho_T(\omega) + \text{Free continuum } \rho_F(\omega) \]

  \[ \rho(\omega) = \rho_T(\omega) + \rho_F(\omega) \]

  \[ \rho_F(\omega) = \frac{3}{2\pi} (1 + \kappa) \omega^2 \tanh(\omega/4T) \]

  \[ [G(\tau) - G_{\text{rec}}(\tau)]: \text{Transport } \rho_T(\omega) + \text{Free continuum } \rho_F(\omega) + \text{Boundstate peak } \rho_B(\omega) \]

  \[ \Delta \rho(\omega) = \rho_T(\omega) + \rho_F(\omega) - \rho_B(\omega) \]

  \[ \rho_B(\omega) = \frac{\omega^2}{\pi} \cdot \frac{c_B g_B^2}{4(\omega^2 - m_B^2)^2 + g_B^2} + \frac{3}{2\pi} (1 + \kappa) \omega^2 \]
Fitting the SPF

- All the time the Ansatz for the transport contribution remains the same:

\[ \rho_T(\omega) = \frac{\omega}{\pi} \cdot \frac{c}{(\omega/g)^2 + 1} \]

- Note: It was pointed out that the asymptotics of the above Ansatz are incorrect and the transport contribution diverges in the correlator for very small times, as for large \( \omega \):

\[ \rho_T(\omega) \sim \frac{1}{\omega} \neq \frac{1}{\omega^2} \]

- For this reason we also fit to:

\[ \rho_T(\omega) = \frac{\tanh(\omega)}{\pi} \cdot \frac{c}{(\omega/g)^2 + 1} \]

\( \Rightarrow \) This Ansatz does have the correct behavior for large \( \omega \)

Fitting the SPF

- In the quenched case additional constraints were essential to obtain a good fit

  In \( G(\tau)/G_{\text{free}}(\tau) \) the ratio of the first two thermal Moments:

  \[
  R^{(2,0)}(\tau) = \frac{2 \cdot \int d\omega \rho(\omega) \text{csch}(\omega/2T)}{\int d\omega \omega^2 \rho(\omega) \text{csch}(\omega/2T)}
  \]

  In \([G(\tau) - G_{\text{rec}}(\tau)]\) we make use of a sum rule (derived from Ward Identities):

  \[
  \int d\omega \frac{\Delta \rho(\omega)}{\omega} = \int \frac{d\omega}{\omega} [\rho(\omega, T \neq 0) - \rho(\omega, T = 0)] = 0
  \]

  And also:

SPF: The reconstructed correlator case

- Fit-window: $\tau \geq 5$ ; Errors via covariance matrix

$\rightarrow$ We find very good agreement with the data
SPF: The free ratio case

- Fit-window: $\tau \geq 5$ ; Errors via covariance matrix

→ We find excellent agreement with the data
Comparing transport contributions

- In the next step we compare the transport contribution from both fits
  
  ➔ Note: All results have been rescaled by a factor 1/6
Comparing transport contributions

- Much larger errors in \([G(\tau) - G_{\text{rec}}(\tau)]\)
- Very small errors in \(G(\tau)/G_{\text{free}}(\tau)\), with both Ansaeetze
- Widths of \([G(\tau) - G_{\text{rec}}(\tau)]\) fits are larger than those of \(G(\tau)/G_{\text{free}}(\tau)\)
Comparing transport contributions

- Fits of $G(\tau)/G_{\text{free}}(\tau)$ lie very close to one another
- Larger spread of results for $[G(\tau) - G_{\text{rec}}(\tau)]$
- However: The intercept of all fits is compatible
Comparing transport contributions

- The intercept at \( \omega = 0 \) gives the electrical conductivity:

\[
0.38 [0.15] \lesssim \frac{1}{C_{em}} \frac{\sigma}{T} \lesssim 0.51 [0.88]
\]
Conclusion

- We analyzed the vector current correlation function using two-flavour lattice QCD with a pair of light Wilson-Clover quarks.

- Making use of zero temperature data we could study the thermal and reconstructed correlators in the light case for the first time.

- Using two different fit methods we were capable of estimating the electrical conductivity in fully dynamical QCD.

$$0.38[0.15] \lesssim \frac{1}{C_{em}} \frac{\sigma}{T} \lesssim 0.51[0.88]$$
Backup Slides
Current spectral functions

- The spectral functions (SPF) encode the physics of the system:

![Diagram showing spectral functions and particle states](image)
SPF: The reconstructed correlator case

- In $[G(\tau) - G_{\text{rec}}(\tau)]$ there are three possible scenarios:
  - Transport and complete cancelation of the boundstate
  - Boundstate and no emergence of transport
  - Combination of the two

→ The data supports the last scenario
Generic slide

- In the probability of finding a particle is given by the motion of a Brownian particle in the absence of an external field