Spectral densities in hot Yang-Mills theory

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Quark Confinement and the Hadron Spectrum X
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Background: Heavy ion collisions

Expansion of thermalizing plasma surprisingly well described in terms of a low energy effective theory — hydrodynamics

- UV physics encoded in transport coefficients: $\eta$, $\zeta$, ..
Background: Heavy ion collisions

RHIC observation: Hydrodynamics requires $\eta/s \lesssim 0.2$

- Lattice QCD: Small (with large error bars)
- Perturbative QCD: $\eta/s \sim 1/(g^4 \ln g) \gtrsim 1$
- AdS/CFT: $\eta/s = 1/(4\pi) \sim 0.08$ in two-derivative models
Background: Heavy ion collisions

Obvious questions:
- How to access the transport properties of the produced QGP from first principles but nonperturbatively?
- Is there a realistic chance to solve the problems lattice QCD faces?
- How ‘close’ is strongly coupled $\mathcal{N} = 4$ SYM to the system?
- Improvement from building gravity duals to nonconformal theories?
Introduction

Why spectral densities?

Motivation I: Transport coefficients

Kubo formulas: Transport coefficients obtainable from IR limit of retarded Minkowski correlators — viscosities from those of the energy momentum tensor $T_{\mu\nu}$:

$$\eta = \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} D_{12,12}^R(\omega, k = 0) \equiv \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, k = 0)}{\omega}$$

Problem: Lattice can only measure Euclidean correlators $\Rightarrow$ Nonperturbatively, spectral density available only through inversion of

$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh (\beta - 2\tau)\omega}{\sinh \frac{\beta \omega}{2}}$$

$\therefore$ To extract the IR limit of $\rho$, need to understand its behavior also at $\omega \gtrsim \pi T$ — very nontrivial challenge for lattice QCD, requiring perturbative input
Motivation I: Transport coefficients

For flavor current correlator, 5-loop vacuum result and accurate lattice data available ⇒ Model-independent analytic continuation of Euclidean correlator, resulting in predictions for flavor current spectral density and flavor diffusion coefficient; Burnier, Laine, 1201.1994

Results generalizable to other channels, in particular viscosities
Motivation II: Euclidean correlators

Euclidean correlators available from spectral density, but also directly computable on the lattice: Possibility for comparisons between lattice QCD, pQCD and AdS/CFT

Iqbal, Meyer, 0909.0582: Lattice data for correlators of $\text{Tr} F_{\mu\nu}^2$ in agreement with strongly coupled $\mathcal{N} = 4$ SYM, while leading order pQCD result completely off. How about NLO or other gravity duals?
Specifying the goals

Rest of this talk: Determine NLO perturbative expressions for spectral densities corresponding to different components of $T_{\mu\nu}$ in hot Yang-Mills theory

$$\theta \equiv g_B^2 F^a_{\mu\nu} F^a_{\mu\nu} \sim T^\mu_{\mu}, \quad \chi \equiv g_B^2 F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}, \quad \eta \equiv -2 F^a_{1\mu} F^a_{2\mu} = 2 T_{12}$$

Ultimately, try use the results to dig up transport coefficients from Euclidean lattice data

Related challenge: Study the effect of conformality and supersymmetry breaking in holographic setups by computing spectral densities in Improved Holographic QCD (IHQCD, ‘Kiritsis model’); compare results to pQCD and the lattice
Spectral densities from perturbation theory

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Setting up the calculation

The plan: Work within finite-$T$ SU(3) Yang-Mills theory

\[ S_E = \int_0^\beta d\tau \int d^{3-2\epsilon} x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \right\}, \]

write down diagrammatic expansions for the correlators

\[ G_\theta(x) \equiv \langle \theta(x) \theta(0) \rangle_c, \quad G_\chi(x) \equiv \langle \chi(x) \chi(0) \rangle, \quad G_\eta(x) \equiv \langle \eta(x) \eta(0) \rangle_c, \]

\[ \tilde{G}_\alpha(P) \equiv \int_x e^{-iP \cdot x} G_\alpha(x), \]

\[ \rho_\alpha(\omega) \equiv \text{Im} \tilde{G}_\alpha(p_0 = -i(\omega + i\epsilon), p = 0), \]

and evaluate the necessary integrals.
Setting up the calculation

End up computing two-loop two-point diagrams in dimensional regularization, the black dots representing the operators:
Computational methods I: Identifying the masters

Step 1: Perform Wick contractions and perform Lorentz algebra (typically with FORM)

Result: Expansion in terms of scalar ‘masters’

\[
\frac{\tilde{G}_\theta(P)}{4d_Ac_G^2g_B^4} = (D - 2) \left[ -J_a + \frac{1}{2} J_b \right] \\
+ g_B^2 N_c \left\{ 2(D - 2) \left[ -(D - 1)I_a + (D - 4)I_b \right] + (D - 2)^2 \left[ I_c - I_d \right] \right. \\
\left. + \frac{22 - 7D}{3} I_f - \frac{(D - 4)^2}{2} I_g + (D - 2) \left[ -3I_e + 3I_h + 2I_i - I_j \right] \right\},
\]

\[ J_a \equiv \oint \frac{P^2}{Q^2}, \quad J_b \equiv \oint \frac{P^4}{Q^2(Q - P)^2}, \quad I_a \equiv \oint \frac{1}{Q^2R^2}, \quad I_b \equiv \oint \frac{P^2}{Q^2R^2(R - P)^2}, \quad \ldots \]

\[ I_h \equiv \oint \frac{P^4}{Q^2R^2(Q - R)^2(R - P)^2}, \quad I_i \equiv \oint \frac{(Q - P)^4}{Q^2R^2(Q - R)^2(R - P)^2}, \quad \ldots \]

\[ I_i' \equiv \oint \frac{4(Q \cdot P)^2}{Q^2R^2(Q - R)^2(R - P)^2}, \quad I_j \equiv \oint \frac{P^6}{Q^2R^2(Q - R)^2(Q - P)^2(R - P)^2} \]
Computational methods II: Phase space integrals

Step 2: After performing Matsubara sum via cutting rules and taking imag. part, reduce master integrals to 3d phase space integrals ($E_{qr} \equiv |q - r|$):

\[
\rho_{\omega}(\omega) = \int_{q,r} \frac{\omega^6 \pi}{4 q r E_{qr}} \left\{ \right. \\
\left. \frac{1}{8q^2} \left[ \delta(\omega - 2q) - \delta(\omega + 2q) \right] \times \\
\times \left[ \left( \frac{1}{(q + r - E_{qr})(q + r)} - \frac{1}{(q - r + E_{qr})(q - r)} \right) (1 + 2n_q)(n_{qr} - n_r) \\
+ \left( \frac{1}{(q - r - E_{qr})(q + r) + E_{qr})(q - r)} \right) (1 + 2n_q)(1 + n_{qr} + n_r) \right] \\
\right. \\
+ \frac{1}{8r^2} \left[ \delta(\omega - 2r) - \delta(\omega + 2r) \right] \times \\
\times \left[ \left( \frac{1}{(q + r - E_{qr})(q + r)} - \frac{1}{(q - r - E_{qr})(q - r)} \right) (1 + 2n_r)(n_{qr} - n_q) \\
+ \left( \frac{1}{(q + r + E_{qr})(q + r)} - \frac{1}{(q - r - E_{qr})(q - r)} \right) (1 + 2n_r)(1 + n_{qr} + n_q) \right] \\
+ \left[ \delta(\omega - q - r - E_{qr}) - \delta(\omega + q + r + E_{qr}) \right] \\
\times \left[ \frac{n_r(1 + n_q + n_{qr}) - n_q n_{qr}}{(q + r + E_{qr})^2(q - r + E_{qr})(q - r - E_{qr})} \\
\frac{n_q(1 + n_q + n_{qr}) - n_{qr} n_r}{(q - r + E_{qr})^2(q - r + E_{qr})(q - r - E_{qr})} \\
\frac{n_r(1 + n_q + n_{qr}) - n_q n_{qr}}{(q - r - E_{qr})^2(q + r + E_{qr})(q + r - E_{qr})} \\
\frac{n_q(1 + n_q + n_{qr}) - n_{qr} n_r}{(q - r - E_{qr})^2(q + r + E_{qr})(q + r - E_{qr})} \right} \\
\left. \right\}
\]
Computational methods III: Form of the result

After quite some work: Final result in terms of $1d$ and $2d$ (numerically evaluated) integrals

\[
\frac{(4\pi)^3 \rho \int_j}{\omega^4(1 + 2n\frac{\omega}{2})} =
\int_0^{\frac{\omega}{4}} dq n_q \left[ \left( \frac{1}{q - \frac{\omega}{2}} - \frac{1}{q} \right) \ln \left( 1 - \frac{2q}{\omega} \right) - \frac{\frac{\omega}{2}}{q(q + \frac{\omega}{2})} \ln \left( 1 + \frac{2q}{\omega} \right) \right]
\]

\[
+ \int_{\frac{\omega}{4}}^{\frac{3\omega}{4}} dq n_q \left[ \left( \frac{2}{q - \frac{\omega}{2}} - \frac{1}{q} \right) \ln \left( 1 - \frac{2q}{\omega} \right) - \frac{\frac{\omega}{2}}{q(q + \frac{\omega}{2})} \ln \left( 1 + \frac{2q}{\omega} \right) - \frac{1}{q - \frac{\omega}{2}} \ln \left( \frac{2q}{\omega} \right) \right]
\]

\[
+ \int_{\frac{3\omega}{4}}^{\infty} dq n_q \left[ \left( \frac{2}{q - \frac{\omega}{2}} - \frac{2}{q} \right) \ln \left( \frac{2q}{\omega} - 1 \right) - \frac{\frac{\omega}{2}}{q(q + \frac{\omega}{2})} \ln \left( 1 + \frac{2q}{\omega} \right) + \left( \frac{1}{q} - \frac{1}{q - \frac{\omega}{2}} \right) \ln \left( \frac{2q}{\omega} \right) \right]
\]

\[
+ \int_{\frac{3\omega}{4}}^{\infty} dq \int_0^{\frac{\omega}{4} - |q - \frac{\omega}{4}|} dr \left( -\frac{1}{qr} \right) \frac{n_{\frac{\omega}{2} - q} n_q + r (1 + n_{\frac{\omega}{2} - r})}{n_r^2}
\]

\[
+ \int_{\frac{3\omega}{4}}^{\infty} dq \int_0^{\frac{\omega}{2}} dr \left( -\frac{1}{qr} \right) \frac{n_q - \frac{\omega}{2} (1 + n_{q-r})(n_q - n_{r+\frac{\omega}{2}})}{n_r n_{\frac{\omega}{2}}}
\]

\[
+ \int_{\frac{3\omega}{4}}^{\infty} dq \int_0^{\frac{\omega}{2}} dr \left( -\frac{1}{qr} \right) \frac{(1 + n_{q+\frac{\omega}{2}}) n_{q+r} n_{r+\frac{\omega}{2}}}{n_r^2}
\]

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Summary of current results (to NLO)

<table>
<thead>
<tr>
<th>OPEs</th>
<th>Spectral density</th>
<th>Coord. space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar</td>
<td>[1]</td>
<td>[2]</td>
</tr>
<tr>
<td>Pseudoscalar</td>
<td>[1]</td>
<td>[2]</td>
</tr>
</tbody>
</table>

[1] Mikko Laine, Mikko Vepsäläinen, AV, 1008.3263  
[5] Yan Zhu, AV, 121m.nnnn, m = 1(1)

Note: Inclusion of fermions possible in all cases.
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Bulk channel spectral density

\[
\frac{\rho_\theta(\omega)}{4d_A c_\theta^2} = \frac{\pi \omega^4}{(4\pi)^2} \left(1 + 2n_\omega \frac{\omega}{2}\right) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[ \frac{22}{3} \ln \frac{\bar{\mu}^2}{\omega^2} + \frac{73}{3} + 8 \phi_T(\omega) \right] \right\} + \mathcal{O}(g^8)
\]

\[
\frac{-\rho_\chi(\omega)}{16d_A c_\chi^2} = \frac{\pi \omega^4}{(4\pi)^2} \left(1 + 2n_\omega \frac{\omega}{2}\right) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[ \frac{22}{3} \ln \frac{\bar{\mu}^2}{\omega^2} + \frac{97}{3} + 8 \phi_T(\omega) \right] \right\} + \mathcal{O}(g^8)
\]

H. B. Meyer, 1002.3343
Bulk channel spectral density

\[ \rho_\theta(\omega) = \frac{\pi \omega^4}{(4\pi)^2} \left( 1 + 2n_\omega \right) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[ \frac{22}{3} \ln \frac{\mu^2}{\omega^2} + \frac{73}{3} + 8 \phi_T(\omega) \right] \right\} + O(g^8) \]

\[ -\rho_\chi(\omega) = \frac{\pi \omega^4}{(4\pi)^2} \left( 1 + 2n_\omega \right) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[ \frac{22}{3} \ln \frac{\mu^2}{\omega^2} + \frac{97}{3} + 8 \phi_T(\omega) \right] \right\} + O(g^8) \]
Bulk channel spectral density

Preliminary result: Comparison of $\text{Tr} \ F_{\mu\nu}^2$ correlator between perturbative Yang-Mills theory, $\mathcal{N} = 4$ SYM and IHQCD displays almost perfect agreement of the two holographic curves.
Bulk channel imaginary time correlator

In the imaginary time correlator, theoretical uncertainties (renormalization scale running) considerably suppressed

\[ G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\rho(\omega) \cosh \frac{(\beta - 2\tau)\omega}{2}}{\sinh \frac{\beta \omega}{2}} \]

![Graph showing the behavior of the correlator](image-url)
Bulk channel imaginary time correlator

For the imaginary time correlator, direct comparison with lattice results possible

\[ G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \cosh \left( \frac{\beta-2\tau}{2} \omega \right) \frac{\cosh \left( \frac{\beta}{2} \omega \right)}{\sinh \left( \frac{\beta}{2} \omega \right)} \]
Qualitatively, NLO results closer to lattice than LO ones
However: We computed time averaged correlator, not equal time
IHQCD computation underway
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Conclusions

- Perturbative input crucial for the eventual first principles determination of transport coefficients in the QGP
  - Analytic UV behavior of spectral densities available through tedious, but straightforward calculations
- NLO results derived for spectral density in the bulk channel, shear channel calculation just completed
  - Imaginary time correlators immediately available; in addition, time averaged spatial correlator determined in the bulk channel
- For Euclidean correlators, comparison of perturbative, lattice and holographic results possible
  - IHQCD vs. $\mathcal{N} = 4$ SYM comparison sheds light on the effects of conformal symmetry breaking