Center-symmetric effective theory for two-color QCD at high and moderate temperature

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Quark Confinement and the Hadron Spectrum
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High temperature: lattice QCD, resummed PT, EFT.
High density: phenomenological models, PT, EFT.
Aims and ingredients

• Dimensional reduction: separation of physical scales.
• Center symmetry: necessary for deconfinement physics.
• Two-color QCD: simplified treatment, allows direct check against lattice data at nonzero $\mu$.  

Construct a semi-analytic, model-independent approach valid at temperatures (almost) down to $T_c$. 
Dimensional reduction

- Matsubara formalism: bosonic (fermionic) fields are (anti)periodic in time with period $\beta = 1/T$.

- At high temperature the system is effectively 3d.
- $n \neq 0$ Matsubara modes: effective “mass” $\omega_n = 2\pi n T$.
- 4d Euclidean field theory at high temperature reduces to a 3d theory of the zero Matsubara mode.
Scales and degrees of freedom

- Dimensionally reduced theory of QCD: EQCD.
- Adjoint scalar $A^0_a$: “soft mass” $\propto gT$ by loop corrections.
- 3d gauge field $A_a$: massless in EQCD; nonperturbative physics at scale $\propto g^2T$.
- The EFT determines physics on length scales $\propto 1/gT$.

$$\mathcal{L}_{\text{EQCD}} = \frac{1}{4} (F_{ij}^a)^2 + \frac{1}{2} (\partial_i A_0^a)^2 + \frac{1}{2} m_E^2 (A_0^a)^2 + \frac{1}{8} \lambda_E (A_0^a A_0^a)^2$$


- Parameters of the effective Lagrangian can be obtained by directly integrating out heavy Matsubara modes.
Center Symmetry

- SU($N$) Yang–Mills theory has global $Z_N$ symmetry.
- Symmetry changes in the (de)confinement transition.
- Order parameter: expectation value of Polyakov loop.

$$\Omega(x) = \text{tr} \left\{ \mathcal{P} \exp \left[ ig \int_0^\beta d\tau A_0(\tau, x) \right] \right\}$$

- Dynamical fermions break center symmetry explicitly.
- EQCD breaks $Z_N$ explicitly by expanding around one of the $N$ degenerate minima.

Kurkela, Vienna (2009)
Two-color QCD I

- **No sign problem**, lattice simulations feasible.
- World where **baryons are bosons**.
- Invariance under the exchange
- Invariance of spectrum under
- Baryons and mesons belong to common multiplets of the **extended flavor SU(2N_f)** symmetry.
- No annoying three-body physics at low density.
- Gauge-invariant order parameter at high density.
Two-color QCD II

- BEC of scalar diquarks at moderate $\mu_B$.
- Crossover to BCS quark pairing at high $\mu_B$.
- Cold dense matter: useful hints about (de)confinement.
- Hot dilute matter: use lattice to test dim-nal reduction!

Zhang, TB, Rischke, JHEP 06 (2010)
Center-symmetric effective theory

- Construct an effective field theory that:
  - Preserves the $Z_N$ center symmetry.
  - Reduces to EQCD at high temperature.
  - Is superrenormalizable.
- Fix parameters by matching to (E)QCD.
- Worked out for SU(3) Yang–Mills:
- Worked out for SU(2) Yang–Mills:
- **Goal of this work**: add dynamical quarks.
Construction of the EFT I

- Degrees of freedom:
  - 3d spatial (magnetic) gluon field, $A_a(x)$.
  - Coarse-grained Polyakov loop field $Z(x)$.

- Gauge and center symmetry of the theory:

\[
\mathcal{L}(x) \rightarrow U(x) \mathcal{L}(x) U^\dagger(x)
\]
\[
A(x) \rightarrow U(x)[A(x) + i \nabla]U^\dagger(x)
\]
\[
\mathcal{L}(x) \rightarrow \pm \mathcal{L}(x)
\]
**Construction of the EFT II**

- **Advantages of two colors:**
  - Direct check against lattice simulation possible.
  - Coarse graining almost preserves unitarity:
    \[
    \mathcal{L}(x) = \frac{1}{2} \left[ \Sigma(x) + i\sigma_a \Pi_a(x) \right]
    \]

- Most general Lagrangian respecting the symmetries:
  \[
  \mathcal{L}_{\text{EFT}} = g_3^{-2} \left[ \frac{1}{2} \text{tr} F_{ij}^2 + \text{tr}(D_i \mathcal{L}^\dagger D_i \mathcal{L}) + V(\mathcal{L}) \right]
  \]
  \[
  V(\mathcal{L}) = b_1 \Sigma^2 + b_2 \Pi_a^2 + c_1 \Sigma^4 + c_2 (\Pi_a^2)^2 + c_3 \Sigma^2 \Pi_a^2 + d_1 \Sigma^3 + d_2 \Sigma \Pi_a^2
  \]

- Operators $d_{1,2}$ break center symmetry: **quark effect!**
Mass scales of the EFT

- 1 heavy mode (mass \( \propto T \)): \( \Sigma \), integrated out.
- 3 light modes (mass \( \propto gT \)): \( \Pi_a \), matched to \( A^0_a \) of EQCD.

\[
V(Z) = h_1 \tr(Z^\dagger Z) + h_2 (\tr Z^\dagger Z)^2 + g_3^2 \left[ \frac{1}{2} s_1 \Pi_a^2 + \frac{1}{4} s_2 (\Pi_a^2)^2 + s_3 \Sigma^4 + \frac{1}{2} s_4 \Sigma^3 + \frac{1}{2} s_5 \Sigma \Pi_a^2 \right]
\]

- Hard terms have extended \( SU(2)_L \times SU(2)_R \) symmetry.
- Spontaneously broken down to \( SU(2)_V \).
- \( \Pi_a \) are its pseudo-Nambu–Goldstone bosons.
Perturbative matching

- 5 of 7 parameters determined by matching to the perturbative one-loop Weiss effective potential.

\[ V_{\text{eff}}(\Pi_a) = \frac{4}{3} \pi^2 T^4 \left( \frac{g |\Pi|}{2\pi T} \right)^2 \left( 1 - \frac{g |\Pi|}{2\pi T} \right)^2 - \frac{4T^2}{\pi^2} \sum_{j=1}^{N_f} m_j^2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} K_2(n\beta m_j) \cosh(n\beta \mu_j) \cos \frac{ng |\Pi|}{2T} \]

Gross, Pisarski, Yaffe, Rev. Mod. Phys. 53 (1981)

- Remaining parameters are related to the heavy mode \( \Sigma \).

- They must be found by nonperturbative simulation.

- They have small effect on low-energy physics.
Predictions of the theory I

- Numerical lattice simulation to determine the critical temperature within the effective theory.

- Binder cumulant $B_4 = \frac{\langle \Sigma^4 \rangle}{\langle \Sigma^2 \rangle^2}$ to check universality of the 3-dimensional phase transition with $\nu = 0.63$.

Predictions of the theory II

- Domain wall solution (in absence of $Z_2$ breaking).
- Wall tension differs by 9% from the Yang–Mills value.

- In presence of $Z_2$ breaking: bubble solutions.

$$ R_c = \left( \frac{2}{3} \right)^{3/2} \frac{\pi^2}{\kappa}, \quad S_{\text{bubble}} = \frac{2^{13/2} \pi^7}{3^{11/2} \varphi^3 \kappa^2}, \quad \kappa = \frac{4}{\pi^2} \sum_{j=1}^{N_f} (\beta m_j)^2 K_2(\beta m_j) \cosh(\beta \mu_j) $$

- To do: thermodynamics at nonzero chemical potential.
Summary and outlook

• Constructed a low-energy EFT for two-color QCD.
• Quarks with arbitrary masses and chemical potentials.
• All but 2 parameters fixed by perturbative matching.
• Including center symmetry gets us closer to $T_c$.
• Domain wall structure of QCD correctly reproduced.
• Use the EFT to make predictions:
  ▸ Thermodynamics at moderate $T$ and low $\mu$.
  ▸ Phase structure at imaginary chemical potential.
  ▸ Combine with strong-coupling approaches.

Philipsen, Münster, Langelage, Lottini (2011)