Reevaluation of Neutron Electric Dipole Moment with QCD Sum Rules

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Contents of my talk

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I. Introduction

Electric dipole moment (EDM) for particle with spin $S$:

$$H = -d \mathbf{E} \cdot \frac{\mathbf{S}}{|\mathbf{S}|}$$

- EDM is $T$-odd and $P$-odd since
  $P: \mathbf{E} \rightarrow -\mathbf{E}, \; S \rightarrow S, \; T: \mathbf{E} \rightarrow \mathbf{E}, \; S \rightarrow -S$.
  EDM is sensitive to CP violation under CPT invariance.
- Current upper bounds on the electron and neutron EDMs:
  $$|d_e| < 1.7 \times 10^{-27} \text{ cm}, \quad |d_n| < 2.9 \times 10^{-26} \text{ cm}$$
- The EDMs have sensitivities to CP violation beyond the Standard Model, since the the CKM phase contributions are much smaller as
  $$|d_e|_{\text{KM}} < \sim 10^{-40} \text{ cm}, \quad |d_n|_{\text{KM}} \sim 10^{-32} \text{ cm}$$
CP-violating operators in QCD

(Flavor-conserving) CP-violating operators at parton level up to dimension 5 at hadronic scale (~1GeV):

\[ \mathcal{L}_{CP} = - \sum_{q=u,d,s} m_q \bar{q} i \gamma_5 q + \theta G \frac{\alpha_s}{8\pi} G_{\mu
u}^A \tilde{G}_{\mu\nu}^A \]

\[ - \frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} (F \cdot \sigma) \gamma_5 q - \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q \bar{q} g_5 (G \cdot \sigma) \gamma_5 q \]

\( \gamma_5 \) mass term \hspace{1cm} \( \theta \) term

\( F \cdot \sigma \equiv F_{\mu\nu} \sigma^{\mu\nu}, \ G \cdot \sigma \equiv G_{\mu\nu}^A \sigma^{\mu\nu} T^A \)

- (Physical) CP-violating parameters are \( \tilde{\theta} = \theta_G + \theta_Q, \ d_q, \ \text{and} \ \tilde{d}_q \).
  \( \theta_Q = \sum_{q=u,d,s} \theta_q \)

- We evaluate contribution to neutron EDM from terms up to dimension 5, with the QCD sum rules.
Sensitive to TeV-scale BSM

EDMs and CEDMs for quarks are sensitive to Beyond the Standard Model at TeV scale.

Supersymmetric Standard Model (SUSY SM)

Complex SUSY breaking parameters are CP violating and contribute to EDMs.

\[ d_d/e \sim \tilde{d}_d \sim \theta_{\text{CP}} \times 10^{-(24-25)} \text{cm} \times \left( \frac{m_{\text{SUSY}}}{1\text{TeV}} \right)^{-2} \]

Even flavor-violating parameters also contribute to the EDMs.
QCD sum rules

Correlation function of hadronic current $j(x)$:

$$\Pi(q^2) \equiv i \int d^4x \ e^{iq \cdot x} \langle 0 | T\{j(x)j^\dagger(0)\} | 0 \rangle$$

Dispersion relation of correlation function:

$$\Pi(q^2) = \frac{1}{\pi} \int_0^\infty ds \ \frac{\text{Im} \ \Pi(s)}{s - q^2 - i\epsilon}$$

For $-q^2 \gg 0$ the left-handed side of dispersion relation can be evaluated with perturbative QCD, while the right-handed side has information of hadronic states.
QCD sum rules

Dispersion relation of correlation function:

\[ \Pi(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \, \Pi(s)}{s - q^2 - i\epsilon} \]

Procedure in QCD sum rules

1. \( \Pi \) is evaluated with the Operator Product Expansion (OPE), where the long-distance quark-gluon interactions are parametrized in terms of universal vacuum condensates.

2. By taking the Borel transformation, which suppresses excited and continuum states, and constants contributions in the relation, information of the grand state is extracted.

3. Systematic errors are evaluated by modeling the excited and continuum states.
2. Evaluation of Neutron EDM with QCD sum rules

i. Basis of CP-violating operators
ii. Condensation under CP violation
iii. Phenomenological side of correlator
iv. Choice of neutron current
v. OPE
vi. Evaluation of neutron EDM
i. Basis of CP-violating operators

1. Rotated away the $\theta$ term to quark $\gamma_5$ mass terms with $U(1)_A$.
2. Take a basis of quark $\gamma_5$ mass terms with $SU(3)_A$ so that

$$\langle \Omega_{CP} | \mathcal{L}_{CP} | M_A \rangle = 0 \, , \quad (M^A = \pi, K, \eta) \, .$$

$$\mathcal{L}_{CP} = - \sum_{q=u,d,s} m_q \bar{q} i \tilde{\theta} \rho_q \gamma_5 q$$

$$- \frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} (F\sigma) \gamma_5 q - \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q \bar{q} g_s (G\sigma) \gamma_5 q \, .$$

where

$$\rho_u = \frac{m_*}{m_u} \left[ 1 + \frac{m_0^2}{2\theta} \left\{ \frac{\tilde{d}_u - \tilde{d}_d}{m_d} + \frac{\tilde{d}_u - \tilde{d}_s}{m_s} \right\} \right]$$

$$m_* = \frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_u m_s}$$

$$\rho_d = \frac{m_*}{m_d} \left[ 1 + \frac{m_0^2}{2\theta} \left\{ \frac{\tilde{d}_d - \tilde{d}_u}{m_u} + \frac{\tilde{d}_d - \tilde{d}_s}{m_s} \right\} \right]$$

$$\langle \bar{q} g_s (G\sigma) q \rangle = -m_0^2 \langle \bar{q} q \rangle$$

$$\rho_s = \cdots$$
ii. Condensation under CP violation

• Using axial anomaly, CP-violating contribution to the condensation of quark bilinear ($\Gamma$:4by4 matrix) is

\[
\langle 0|\bar{q}\Gamma q|0\rangle_{CP} = \frac{i\theta_G}{2} \rho_q \langle 0|\bar{q}\{\gamma_5, \Gamma\}q|0\rangle = 0 \quad (\theta_G \rightarrow 0)
\]

while the final result is independent of the operator basis.

• Equation of motion of quark is not automatic in condensation as

\[
\langle 0|\bar{q}\Gamma i\not{D}q|0\rangle_{\theta} = \begin{cases} 
\langle 0|\bar{q}\Gamma m_q q|0\rangle_{\theta} & \text{(for } \Gamma = 1, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}) \\
0 & \text{(for } \Gamma = \gamma_5) 
\end{cases}
\]
iii. Phenomenological side of correlator

- Neutron current $\eta_n(x)$ and one particle state under CP-violating BG:

$$\langle \Omega_{CP} | \eta_n(x) | N_{CP}(p, s) \rangle = \lambda_n e^{i \alpha_n \gamma_5} u_n(p, s) e^{-ip \cdot x}$$

- Correlator of neutron current under constant electromagnetic BG, $F$:

$$\Pi(q) \equiv i \int d^4x \ e^{iq \cdot x} \langle \Omega_{CP} | T\{\eta_n(x)\bar{\eta}_n(0)\} \rangle |\Omega_{CP}\rangle_F$$

Chiral invariant term proportional to neutron EDM $d_n$ is

$$\Pi^{(\text{phen})}(q) = \frac{1}{2} f(q^2) \{\tilde{F} \cdot \sigma, q\} + \ldots$$

where

$$f(q^2) = \left( \frac{\lambda_n^2 d_n m_n}{(q^2 - m_n^2)^2} + \frac{A(q^2)}{q^2 - m_n^2} + B(q^2) \right) \quad (q^2 \simeq m_n^2)$$
iv. Choice of neutron current

Neutron Current under CP-violating BG:

\[ \eta_n(x) = j_1(x) + \beta j_2(x) + i\epsilon[i_1(x) + \beta i_2(x)] \]

where

\[ j_1(x) = 2\epsilon_{abc} \left( d_a^T(x) C \gamma_5 u_b(x) \right) d_c(x) \]

and \( j_2(x) = 2\epsilon_{abc} \left( d_a^T(x) C u_b(x) \right) \gamma_5 d_c(x) \)

and P-odd currents, \( i_1(x) = \gamma_5 j_2(x) \) and \( i_2(x) = \gamma_5 j_1(x) \).

We take \( \beta = 1 \) since

1) Mixing to P-odd currents can be neglected.

\[
\langle \Omega_{CP} | T\{\eta_n(x)\bar{\eta}_n(0)\} | \Omega_{CP} \rangle_F |_{\gamma \text{ odd}} = \langle j_1, \bar{j}_1 \rangle + \beta[\langle j_1, \bar{j}_2 \rangle + \langle j_2, \bar{j}_1 \rangle] + \beta^2 \langle j_2, \bar{j}_2 \rangle + i\epsilon(1 - \beta^2)[\langle j_1, \bar{j}_2 \rangle - \langle j_2, \bar{j}_1 \rangle] \gamma_5 .
\]

2) Higher-order terms in OPE are suppressed.
While we evaluate LO and NLO terms, the NLO terms accidently vanish when $\beta = 1$. 
We get

\[ \Pi(q)^{(\text{OPE})} = \frac{1}{16\pi^2} \langle \bar{q}q \rangle \log \left( \frac{-q^2}{\Lambda^2} \right) \{ \bar{F} \sigma, \phi \} \Theta \]

where

\[ \Theta \equiv (4e_d m_d \rho_d - e_u m_u \rho_u) \chi \bar{\theta} + (4d_d - d_u) + (\kappa - \frac{1}{2} \xi)(4e_d \bar{d}_d - e_u \bar{d}_u) \]

We used condensations under electromagnetic BG.

\[ \langle \bar{q} \sigma_{\mu \nu} q \rangle_F = e_q \chi F_{\mu \nu} \langle \bar{q}q \rangle, \]

\[ g_s \langle \bar{q} G^A_{\mu \nu} T^A q \rangle_F = e_q \kappa F_{\mu \nu} \langle \bar{q}q \rangle, \]

\[ 2g_s \langle \bar{q} \gamma_5 \tilde{G}^A_{\mu \nu} T^A q \rangle_F = ie_q \xi F_{\mu \nu} \langle \bar{q}q \rangle \]
vi. Evaluation of neutron EDM

Sum rules after Borel transformation (M: Borel mass parameter)

\[ \lambda_n^2 d_n m_n - A M^2 = -\Theta \langle \bar{q} q \rangle \frac{M^4}{8\pi^2} e \frac{m_n^2}{M^2} \]

(single pole) (double pole)

\( M^4 e \frac{m_n^2}{M^2} \) in right-handed side

Double pole/Single pole in left-handed side
v. Evaluation of neutron EDM

Low-energy constant $\lambda_n(\Omega_{CP}|\eta_n(x)|N_{CP}(p, s)) = \lambda_n e^{i\alpha_n 5} u_n(p, s) e^{-ip\cdot x}$

1) Lattice evaluation (Y. Aoki et al, 08)

$$\lambda_n = -0.0436 \pm 0.0047_{(\text{stat})} \pm 0.0084_{(\text{syst})} \text{ GeV}^3$$

2) QCD sum rules (Leinweber, 97)

$$\lambda_n \simeq 0.022 \text{ GeV}^3$$

The lattice-predicted value gives more conservative prediction for neutron EDM.

$$d_n = 1.2 \pm 0.6^{+0.7}_{-0.4} \times 10^{-1} \Theta$$  \[\text{(pheno)(OPE)(lattice)}\]

For the center value

$$d_n = 4.2 \times 10^{-17} \tilde{\theta} [\text{e cm}] + 0.47d_d - 0.12d_u$$

$$+ e(-0.18\tilde{d}_u + 0.18\tilde{d}_d - 0.008\tilde{d}_s)$$
Conclusion and discussion

In this paper we evaluated the neutron EDM using the QCD sum rules. Here, we consider the contribution from QCD theta term, quark EDMs, and quark CEDMs.

We find that the neutron EDM becomes smaller by a factor 3~4 when using the lattice QCD value for the low energy constant, compared with the QCD sum rules’ one. Thus, this gives a more conservative bound on the new physics models.

When the Peccei-Quinn mechanism works, the theta parameter becomes a dynamical field (axion). It is non-vanishing when the quark CEDMs not vanishing. In the case, the neutron EDM becomes

\[ d_n = 1.2^{+0.6}_{-0.3} \pm 0.1^{+0.7}_{-0.4} \times 10^{-1} \Theta^{PQ} \]

The center value is

\[ d_n = 0.47d_d - 0.12d_u + e(0.17\tilde{d}_u + 0.35\tilde{d}_d) \]
CP-violating operators up to dim=6

$$\mathcal{L}_{\text{CPV}} = \sum_{i=1,2,4,5} \sum_q C_i^q(\mu) O_i^q(\mu) + C_3(\mu) O_3(\mu)$$

$$+ \sum_{i=1,2} \sum_{q' \neq q} \tilde{C}_i^{q'q}(\mu) \tilde{O}_i^{q'q}(\mu) + \frac{1}{2} \sum_{i=3,4} \sum_{q' \neq q} \tilde{C}_i^{q'q}(\mu) \tilde{O}_i^{q'q}(\mu).$$

$$\mathcal{O}_1^q = -\frac{i}{2} m_q \bar{q} e Q_q (F \cdot \sigma) \gamma_5 q,$$

$$\mathcal{O}_2^q = -\frac{i}{2} m_q \bar{q} g_s (G \cdot \sigma) \gamma_5 q,$$

$$\mathcal{O}_3 = -\frac{1}{6} g_s f_{ABC} \epsilon^{\mu \nu \rho \sigma} G^A_{\mu \lambda} G^B_{\lambda \nu} G^C_{\rho \sigma}$$

$$\mathcal{O}_4^q = \bar{q}_\alpha q_\alpha \bar{q}_\beta i \gamma_5 q_\beta,$$

$$\mathcal{O}_5^q = \bar{q}_\alpha \sigma^{\mu \nu} q_\alpha \bar{q}_\beta i \sigma_{\mu \nu} \gamma_5 q_\beta,$$

$$\mathcal{O}_1^{q'q} = \bar{q}_\alpha' q_\alpha' \bar{q}_\beta i \gamma_5 q_\beta,$$

$$\mathcal{O}_2^{q'q} = \bar{q}_\alpha' q_\beta' \bar{q}_\beta i \gamma_5 q_\alpha,$$

$$\mathcal{O}_3^{q'q} = \bar{q}_\alpha' \sigma^{\mu \nu} q_\alpha' \bar{q}_\beta i \sigma_{\mu \nu} \gamma_5 q_\beta,$$

$$\mathcal{O}_4^{q'q} = \bar{q}_\alpha' \sigma^{\mu \nu} q_\beta' \bar{q}_\beta i \sigma_{\mu \nu} \gamma_5 q_\alpha.$$

How to evaluate contribution from the Weinberg and four-quark operators?

(JH, Tsumura, Yang)
Backup
Condensates

\[ \langle \bar{q} g_s (G\sigma) q \rangle = -m_0^2 \langle \bar{q} q \rangle , \quad m_0^2 = 0.8 \text{ GeV}^2 \]

\[ \langle \bar{q} q \rangle = -(0.225 \text{ GeV})^3 \]

\[ \langle \bar{q} \sigma_{\mu \nu} q \rangle_F = \chi e_q F_{\mu \nu} \langle \bar{q} q \rangle , \quad \chi = -5.7 \pm 0.6 \text{ GeV}^{-2} \]

\[ g_s \langle 0 | \bar{q} G_{\mu \nu}^A T^A q | 0 \rangle_F = \kappa e_q F_{\mu \nu} \langle 0 | \bar{q} q | 0 \rangle , \quad \kappa = -0.34 \pm 0.1 \]

\[ 2g_s \langle 0 | \bar{q} \gamma_5 \tilde{G}_{\mu \nu}^A T^A q | 0 \rangle_F = i \xi e_q F_{\mu \nu} \langle 0 | \bar{q} q | 0 \rangle . \quad \xi = -0.74 \pm 0.2 \]