HADRON MASS EFFECTS IN POWER CORRECTIONS TO EVENT SHAPES

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In collaboration with Jesse Thaler and Iain Stewart     arXiv: 1209.3781

Builds on earlier work by Salam and Wicke  JHEP 0105 (2001) 061
OUTLINE

- Introduction
- Power Corrections
- Hadron mass effects on Power Corrections
- Anomalous dimension of Power Correction
- Comparison to Pythia 8 and Herwig++
- Conclusions
INTRODUCTION
Event Shapes $e^+ e^- \rightarrow \text{jets}$

Event shapes characterize in a geometrical way the distribution of hadrons in the final state.

Thrust is the most commonly studied event shape variable:

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

They are theoretically more friendly than a Jet algorithm.

DELPHI 2-jet event  \hspace{1cm} OPAL 3-jet event

Continuous transition from 2-jet to 3-jet, ... multi-jet events

**DELPHI 2-jet event**

**OPAL 3-jet event**

$Q = 91.2 \text{ GeV}$
We will concentrate on event shapes that are **not** recoil sensitive

and can be written in the dijet limit as

\[
e(N) = \frac{1}{Q} \sum_{i \in N} m_i^\perp f_e(r_i, y_i)
\]

\[
y = \frac{1}{2} \log \left( \frac{E + p_z}{E - p_z} \right)
\]

rapidity

\[
r = \frac{p^\perp}{m^\perp}
\]

transverse velocity

\[
m^\perp = \sqrt{p_T^2 + m^2}
\]

transverse mass

\[
\eta = \ln \left( \frac{\sqrt{r^2 + \sinh^2 y + \sinh y}}{r} \right)
\]
pseudo-rapidity

\[
v = \frac{\sqrt{r^2 + \sinh^2 y}}{\cosh y}
\]

velocity

**Event Shapes**  \( e^+ e^- \rightarrow \text{jets} \)

Excludes Jet Broadening (See talk by T. Becher)

All event shapes can be expressed in terms of these two variables.

**massless limit**

\[
v = r = 1
\]

\[
y = \eta
\]

\[
m^\perp = p^\perp
\]
Factorization theorem for event shape distributions

\[
\frac{1}{\sigma_0} \frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O}(e^0, \frac{\Lambda_{QCD}}{Q}) \quad [\text{Bauer, Lee, Fleming, Sterman}]
\]

\[
1 \frac{d\sigma}{\sigma_0 de} = H_Q \times J_e \otimes S_e + \mathcal{O}(e^0, \frac{\Lambda_{QCD}}{Q}) \quad [\text{Berger, Kuks, Sterman}]
\]

\[
e = e_c + e_s + e_\Lambda
\]

In the dijet limit the event shape decomposes in **collinear**, **soft** and **nonperturbative** modes. This translates into a factorization theorem for differential distributions.

\[
\gg \frac{\Lambda_{QCD}}{Q} \quad \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)
\]

Derived in the SCET framework

For more examples of SCET applications see talks by **A. Manohar**, **T. Becher**, **M. Beneke**, **I. Scimemi**, **M. Benzke**.
Factorization theorem for event shape distributions

\[
\frac{1}{\sigma_0} \frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O} \left( e^0, \frac{\Lambda_{\text{QCD}}}{Q} \right)
\]

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

\[
S_e(\ell) = \langle 0 | Y_n^\dagger Y_n^\dagger \delta(\ell - Q\hat{e}) Y_n \bar{Y}_n | 0 \rangle
\]

Soft Wilson lines

event shape operator

\[
S_e = \hat{S}_e \otimes F_e
\]

perturbative & nonperturbative

Leading power correction comes from soft function

\[
\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} \otimes F_e
\]

[VM, Thaler, Stewart]

[Bauer, Lee, Fleming, Sterman]

[Berger, Kuks, Sterman]

[Korchemsky, Tafat]

[Korchemsky, Sterman]
Naive OPE for nonperturbative corrections

\[ S_e(\ell) = \langle 0 \mid \bar{Y}_{\bar{n}}^\dagger Y_n^\dagger \delta(\ell - Q\hat{e}) Y_n \bar{Y}_{\bar{n}} \mid 0 \rangle \]

For \( e \gg \frac{\Lambda_{\text{QCD}}}{Q} \)

\[ \delta(\ell - Q\hat{e}) \simeq \delta(\ell) - \delta'(\ell)Q\hat{e} + \ldots \]

Correct up to \( \mathcal{O}(\alpha_s) \)

Shape function can be expanded on the tail

\[ F_e(\ell) \simeq \delta(\ell) - \Omega_1 \delta'(\ell) \]

\[ \Omega_1 = \langle 0 \mid \bar{Y}_{\bar{n}}^\dagger Y_n^\dagger Q \hat{e} Y_n \bar{Y}_{\bar{n}} \mid 0 \rangle \]

\[ \frac{d\sigma}{de} \simeq \frac{d\hat{\sigma}}{de} + \frac{\Omega_1}{Q} \frac{d}{de} \frac{d\sigma}{de} \simeq \frac{d\sigma}{de} \left( e - \frac{\Omega_1}{Q} \right) + \mathcal{O} \left( \frac{\Lambda_{\text{QCD}}}{Q\ell} \right)^2 \]

Leading nonperturbative correction in the tail is a shift of the distribution
POWER CORRECTIONS FOR EVENT SHAPES
Why are Power Corrections so important?

- Event shapes have been extensively used to determine $\alpha_s(m_Z)$
- Power Corrections play an essential role in that determination
- Also important effects in Jet Substructure [Feige, Schwartz, Stewart, Thaler 2012]
- Potentially, important effects at the LHC

$\alpha_s(m_Z)$ from global thrust fits

For other determinations see e.g. talks by A. Pich, P. Lepage, K. Petrov, D. Boito, M. Jamin, J. Schieck, X. Garcia Tormo

[Abbate, Fickinger, Hoang VM, Stewart] arXiv:1006.3080
arXiv:1204.5746
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\[ \alpha_s(m_Z) \text{ from global thrust fits} \]

\[ \alpha_s(m_Z) = 0.1135 \pm 0.0012 \]

\[ \pm \to \text{perturbative error} \]

\[ \text{All errors: } \alpha_s(m_Z) = 0.1135 \pm 0.0012 \]

[Abbate, Fickinger, Hoang VM, Stewart]

arXiv:1006.3080
arXiv:1204.5746
The main effect of the power correction is shifting the distribution to the right. The shift is proportional to \(1/Q\).
Hadron masses and Schemes

What can be measured when a particle hits the detector?

Ideally we would like **energy and momentum separately measured**, but that is **not always possible**.

If a **particle** is **not identified**, mass is not known, **no information on magnitude of momentum**.

One can assume all particles are pions [default scheme]

Alternatively one can use only energy and directions [E-scheme] \( |\vec{p}| \to E \)

---

This considerations are **irrelevant in perturbation theory**, but have **important consequences for power corrections**!
Approaches to Power Corrections

- **Monte Carlo Generators**
  - Pythia, Ariadne, Herwig, Powheg, ...
  - Use hadronization models
  - Hard to separate perturbative vs nonperturbative effects

- **Renormalon based**
  - Effective coupling model [Dokshitzer & Webber]
  - Dressed gluon [Gardi & Gruenberg]
  - Residual model dependence

- **Shape functions**
  - Factorization based [Korchemski, Sterman, Tafat]
  - SCET based [Hoang & Stewart; Lee & Sterman]
  - Derived directly from QCD
  - Operator definition
  - Systematically improvable
Studies of Universality

• **Dispersive approach** [Dokshitzer & Webber 1995]
  - Predicts universality for a bunch of event shapes, including recoil sensitive ones.
  - They are based on a model and on the one-gluon approximation. Modification of (effective coupling) below a cutoff scale.
  - Milan factor takes into account two-gluon effects. [Dokshitzer, Webber, Salam]

• **SCET-CSS approach** [Lee & Sterman 2006]
  - Predicts universality for non-recoil-sensitive event shapes.
  - They are model-independent, formulated in terms of QCD matrix elements.
  - Do not rely on one-gluon approximation.

\[ \Omega^e_1 = c_e \Omega^\rho_1 \]
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  - They are model-independent, formulated in terms of QCD matrix elements.
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Both approaches assume particles are massless!!
### Massless predictions for universality

<table>
<thead>
<tr>
<th></th>
<th>Formula</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thrust</strong></td>
<td>$\tau = 1 - \max_{\vec{n}} \frac{\sum_i</td>
<td>\vec{p}_i \cdot \vec{n}</td>
</tr>
<tr>
<td><strong>Two-Jetiness</strong></td>
<td>$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i</td>
<td>\vec{p}_i \cdot \vec{n}</td>
</tr>
<tr>
<td><strong>C-parameter</strong></td>
<td>$C = \frac{3}{2} \frac{\sum_{i,j}</td>
<td>\vec{p}_i</td>
</tr>
<tr>
<td><strong>Angularities</strong></td>
<td>$\tau_{(a)} = \frac{1}{Q} \sum_i E_i (\sin \theta_i)^a (1 -</td>
<td>\cos \theta_i</td>
</tr>
<tr>
<td><strong>Jet Masses</strong></td>
<td>$\rho_{\pm} = \frac{1}{Q^2} \left( \sum_{i \in \pm} p_i \right)^2$</td>
<td>$c_{\rho} = 1$</td>
</tr>
</tbody>
</table>
Massless Universality in SCET-CSS

In the massless limit one has

Transverse energy-flow operator

\[ \hat{\cal E}_T(\eta) \]

Measures all momenta flowing in a given rapidity

\[ \hat{e} = \int dy \, f_e(1, y) \cal E_T(y) \]

\[ \hat{e} | N \rangle = e(N) | N \rangle \]

Event shape operator

\[ \mathcal{E}_T(y) = \frac{1}{\cosh^3 y} \int_0^{2\pi} d\phi \lim_{R \to \infty} \int_0^{\infty} dt \, \hat{n}_i T_{0i}(t, R\hat{n}) \]

[In the massless limit one has]

\[ e(N) = \frac{1}{Q} \sum_{i \in N} p_i^\perp f_e(1, y_i) \]

\[ \mathcal{E}_T(y) | N \rangle = \sum_{i \in N} p_i^\perp \delta(y - y_i) | N \rangle \]

[Lee Sterman, Korchemsky Oderda Sterman, Sveshnikov and F.V. Tkachov Ore Sterman]
Massless Universality in SCET-CSS

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Massless Universality in SCET-CSS

In the massless limit one has

Transverse energy-flow operator

Event shape operator

Boost invariance requires this term is \( y \)-independent

Universal power correction

Calculable coefficient, depends on the event shape

Operator definition of power correction
HADRON MASS EFFECTS ON POWER CORRECTIONS
• Use the \textit{flux tube model} (later refined with QCD effects)
• Predict that hadron masses \textit{break universality}
• Find a \textit{privileged scheme} (E-scheme) which preserves universality
• Predict that hadron multiplicity translates into \textit{log}(Q) \textit{effects} on power corrections

\[ \Omega_1 \rightarrow \Omega_1 + K \left( \log \frac{Q}{\Lambda} \right)^{\frac{4CA}{\beta_0}} \]  

[Salam & Wicke 2001]
Salam & Wicke have studied mass effects on power corrections. Similar curves correspond to similar power corrections. HJM has very different power correction (PC) than the rest.

Mass effects treated as a correction to massless prediction.

All curves equal one for massless hadrons

**P-scheme**

\[ E_i \rightarrow |\vec{p}_i| \]

In the P-scheme all curves are very similar, approximate universality

**E-scheme**

\[ |\vec{p}_i| \rightarrow E_i \frac{\vec{p}_i}{|\vec{p}_i|} \]

In the E-scheme all curves are equal to each other, restores universality
Mass Effects in SCET

\[
e(N) = \frac{1}{Q} \sum_{i \in N} m_i \delta \eta f_e(r_i, y_i)
\]

One has to generalize the transverse energy flow operator

**Transverse flow operator**

\[
\hat{E}_T(r, y) \delta \nu \delta \eta
\]

\[
\hat{E}_T(r, y) = v(r, y)
\]

\[
\eta = \eta(r, y)
\]

measures momenta of particles with given velocity flowing at a given pseudo-rapidity

\[
\hat{e} = \frac{1}{Q} \int dy \, dr \, E_T(r, y)
\]

\[
E_T(v, \eta) = -\frac{v(1 - v^2 \tanh^2 y)^{3/2}}{\cosh \eta}
\]

\[
\lim_{R \to \infty} R^3 \int_0^{2\pi} d\phi \, \hat{n}_i \, T_{0i}(R, v \, R \, \hat{n})
\]
One has to generalize the transverse energy flow operator

\[ e(N) = \frac{1}{Q} \sum_{i \in N} m_i \ f_e(r_i, y_i) \]

\[ \Omega^e_1 = \int dr \ dy \ f_e(r, y) \langle 0 | \bar{Y}_{\bar{n}} Y_n^{\dagger} \mathcal{E}_T(r, y) Y_n \bar{Y}_{\bar{n}} | 0 \rangle \]
**Mass Effects in SCET**

\[ e(N) = \frac{1}{Q} \sum_{i \in N} m_i \mathcal{f}_e(r_i, y_i) \]

One has to generalize the transverse energy flow operator

\[ \Omega_1^e = \int dr \, dy \, f_e(r, y) \langle 0 | Y_n^\dagger Y_n^\dagger \mathcal{E}_T(r, y) Y_n Y_n^\dagger | 0 \rangle = c_e \int dr \, g_e(r) \Omega_1(r) \]

Operator definition of power correction

\[ \Omega_1(r) = \langle 0 | Y_n^\dagger Y_n^\dagger \mathcal{E}_T(r, 0) Y_n Y_n^\dagger | 0 \rangle \]

Same as for massless computation

\[ c_e = \int_{-\infty}^{\infty} dy \, f_e(1, y) \]

encodes all mass effects

Each \( g_e(r) \) defines a universality class of events with same power correction

\[ g_e(r) = \frac{1}{c_e} \int dy \, f_e(r, y) \]
Event shapes considered

mass scheme (default definition)

Thrust
Jet Masses
C-parameter
Angularities
2-Jettiness

Same color means same power correction
Event shapes considered

- Thrust
- Jet Masses
- C-parameter
- Angularities
- 2-Jettiness

P-scheme

\[ g_e(r) = \frac{p_{\perp}}{m_{\perp}} \]

C - parameter
Thrust
\( \tau_{-1} \)
\( \tau_{-\infty} \)
Jet Masses

Scheme changes event shape definition
Event shapes considered

Thrust
Jet Masses
C-parameter
Angularities
2-Jettiness

E-scheme

\[ g_e(r) \]

\[ r = \frac{p^\perp}{m^\perp} \]

C – parameter
Thrust
\( \tau_{-1} \)
\( \tau_{-\infty} \)
Jet Masses

Scheme changes
event shape definition
Effective parametrization

\[ h_n(r) = \sqrt{2n + 1} P_n(2x + 1) \]

\[ g_e(r) = \sum_{n=0}^{\infty} b_n^e h_n(r) \]

\( \Omega_1(r) \) can be expanded as well

\[ \Omega_1(r) = \Omega_1^p h_0(r) + \sqrt{3}(2\Omega_1^E - \Omega_1^p)h_1(r) + \Omega_1^\delta h_2(r) + \ldots \]

\[
\begin{align*}
\Omega_1^\tau &= 1.034 \Omega_1^E - 0.135 \Omega_1^p + 0.050 \Omega_1^\delta \\
\Omega_1^C &= 1.039 \Omega_1^E - 0.127 \Omega_1^p + 0.046 \Omega_1^\delta \\
\Omega_1^{\tau P} &= 1.022 \Omega_1^E - 0.156 \Omega_1^p + 0.064 \Omega_1^\delta
\end{align*}
\]

small correction
ANOMALOUS DIMENSION OF POWER CORRECTION
Anomalous dimension computation

\[ \Omega_1(r) = \langle 0 | Y_n^\dagger Y_n^\dagger \mathcal{E}_T(r, 0) Y_n Y_n^\dagger | 0 \rangle \]

One needs to compute diagrams that probe the operator

The measured gluon is off-shell
This probes values of \( r \neq 0 \)
Anomalous dimension computation

\[ \Omega_1(r) = \langle 0 \left| \overline{Y}_n^\dagger Y_n^\dagger \mathcal{E}_T(r, 0) Y_n \overline{Y}_n \right| 0 \rangle \]

One needs to compute diagrams that probe the operator

The measured gluon is off-shell
This probes values of \( r \neq 0 \)

Abelian contribution exactly vanish when adding real and virtual radiation

Self-energy diagrams are \( \text{IR and UV finite} \)
Anomalous dimension computation

\[ \Omega_1(r) = \langle 0 \left| Y_n^\dagger Y_n^\dagger \mathcal{E}_T(r, 0) Y_n \bar{Y}_n \right| 0 \rangle \]

One needs to compute diagrams that probe the operator.

The measured gluon is off-shell. This probes values of \( r \neq 0 \).

Only purely non-abelian diagrams contribute.

We obtain an IR finite anomalous dimension.
Results and consequences

\[ \gamma_{\Omega_1} = -\frac{\alpha_s C_A}{\pi} \log(1 - r^2) \]

r-dependent anomalous dimension
no mixing between various \( r \) values

RGE solution at NLL

\[
\Omega_1(r, \mu) = \Omega_1(r, \mu_0) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^\frac{2 C_A}{\beta_0} \log(1-r^2)
\]

\[
\sim \Omega_1(r, \mu_0) \left[ 1 - \frac{\alpha_s(\mu_0) C_A}{\pi} \log \left( \frac{\mu}{\mu_0} \right) \log(1 - r^2) \right]
\]

Expanded out result

Not a resummation formula for \( \Omega_1^e \)

\[
\Omega_1^e(\mu) = \int dr \ g_e(r) \Omega_1(r, \mu_0) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^\frac{2 C_A}{\beta_0} \log(1-r^2)
\]

Unknown function!

Using expanded out result

\[
\Omega_1^e(\mu) = \Omega_1^e(\mu_0) - \frac{\alpha_s(\mu_0) C_A}{\pi} \log \left( \frac{\mu}{\mu_0} \right) \Omega_{\log}^e(\mu_0)
\]

\[
\Omega_{\log}^e(\mu_0) = \int dr \ \log(1 - r^2) \ g_e(r) \Omega_1(r, \mu_0)
\]

New nonperturbative parameter
Matching computation

At one loop one has:

\[ \delta(\ell - Qe_{\text{pert}} - Qe_{\text{np}}) \]
\[ \simeq \delta(\ell - Qe_{\text{pert}}) - Qe_{\text{np}} \delta'(\ell - Qe_{\text{pert}}) \]
\[ \simeq \delta(\ell) - Q(e_{\text{np}} + e_{\text{pert}}) \delta'(\ell) \]

This corrects the naive OPE result

Full theory computation

Effective theory computation
(anomalous dimension)

Same diagrams as for anomalous dimension computation, but different measure

EFT diagrams have to be subtracted from full theory result

Matching coefficient compensates \( \mu \) dependence of \( \Omega_1 \)

\[
F_e(\ell) = \delta(\ell) + \int dr \, C^e_1(\ell, r, \mu) c_e g_e(r) \Omega_1(r, \mu) + O\left(\frac{\Lambda^2_{\text{QCD}}}{\ell^3}\right)
\]

\[
C^e_1(\ell, r, \mu) = -\delta'(\ell) + \frac{C_A \alpha_s(\mu)}{\pi} \ln(1-r^2) \frac{d}{d\ell} \left( \frac{1}{\mu} \left[ \frac{\mu}{\ell} \right] + \right)
\]

\[
+ \frac{\alpha_s(\mu)}{\pi} \delta'(\ell) \tilde{a}^e_1(r) \left[ O(\alpha_s^2) \right]
\]

Explicitly checked

Necessitates entire matching computation
COMPARISONS TO PYTHIA AND HERWIG
Comparisons to MC generators

Define generalized angularities, useful to compare to MC

\[
\tau(n,a) \equiv \sum_i m_i^n r^n e^{-|y_i|(1-a)} \begin{cases} 
   g(n,a) = r^n \\
   c(n,a) = \frac{2}{1-a}
\end{cases}
\]

We study the first moment of the distributions

Taking differences of classes we obtain:

\[
\Omega_1^0(\mu_Q) - \Omega_1^n(\mu_Q) = \frac{Q}{c_a} \left( \langle \tau(0,a) \rangle - \langle \tau(n,a) \rangle \right)
\]

<table>
<thead>
<tr>
<th>fit function basis</th>
<th>{ 1, r, (1 - r)^{-\frac{1}{4}} }</th>
</tr>
</thead>
</table>

resummed expression
expanded out expression
CONCLUSIONS
CONCLUSIONS

- Operator description of hadron mass effects on power corrections.

- These effects break universality. Not a simple a correction.

- Set of privileged classes in which there is universality. Approximate universality among classes (agreement with Salam & Wicke).

- Computation of anomalous dimension predicts log(Q) dependence. Complete matching computation is w.i.p.

- Comparisons to MC generators support our findings.