Low energy analysis of $\pi N$ scattering and the pion-nucleon sigma term with Covariant Baryon Chiral Perturbation Theory

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Part I

Introduction
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- $\pi N$ scattering is an important hadronic reaction that gives access to important questions related to strong interactions.

- At high energies:
  - Allows to study the baryonic spectrum of QCD together with their properties.

- At low energies:
  - Test the chiral dynamics of QCD.
  - Study the role of isospin violation.
  - Provides important information about the internal structure of the nucleon.

- At low energies, the spontaneously and explicitly broken chiral symmetry allow us to construct a perturbative theory for hadronic interactions $\Rightarrow$ ChPT.

- ChPT is an EFT that allows us to apply perturbation theory to processes involving the Goldstone bosons.

- Unfortunately, dealing with baryons in ChPT is not so easy...
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According to the power counting:

\[ \nu = \sum_i V_i (d_i + 2m_i - 2 + \frac{n_i}{2}) + 2L - \frac{E_N}{2} + 2 = 3 \]

However an explicit calculation (\( \mu = m_N \)) shows:

\[ \delta m_N^{(3)} = \frac{3g_A^2 m_N M^2}{32\pi^2 f^2} + \mathcal{O}(M^3) \]

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  - The IR description of the phase shifts are of the same quality as those of HBChPT. [JMA, JMC, JAO and LAR, PRC 83 (2011)]
  - Alters the analytical properties of the amplitude ⇒ Unphysical cuts.

It would be desirable to have a formulation consistent with the power counting of ChPT that preserves the good analytical properties of a covariant calculation.

⇒ **Extended-On-Mass-Shell (EOMS):**

- The terms that break the power counting (PCBT) are analytical in the quark masses and momenta ⇒ They can be canceled via a redefinition of the LECs. ⇒ We recover the power counting.
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The role of the $\Delta(1232)$ in $\pi N$ scattering

- The $\Delta(1232)$ is a resonance with quantum numbers $J = 3/2$ and $I = 3/2$ that dominates the $\pi N$ scattering at low energies.
- Most of the ChPT analyses of $\pi N$ scattering do not include it as an explicit degree of freedom arguing that its contribution can be absorbed in the LECs of the $\pi N$ Lagrangian (RS).
- However, the proximity of the $\Delta$ pole to the $\pi N$ threshold makes that the behavior of this resonance cannot be well reproduced by a finite polynomial $\Rightarrow$ Worsening of the convergence of the chiral series.
- This resonance can be included *consistently* in our EFT using the consistent formulation of chiral Lagrangians of Pascalutsa [Pascalutsa and Timmermans, PRC 60, (1999), Pascalutsa, PLB 503, (2001)].
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Part II

$\pi N$ scattering
We calculate the $\pi N$ scattering amplitude in covariant BChPT up to $O(p^3)$ exploring two possibilities:

- $\Delta$-ChPT: $\pi$ and $N$ are the only degrees of freedom $\Rightarrow$ Allows to compare with previous HBChPT and IR results.
- $\Delta$-ChPT: We include the $\Delta(1232)$ as an explicit degree of freedom using consistent Lagrangians $\Rightarrow$ We expect an improvement of the convergence of the chiral series.
  - $\Rightarrow$ Can solve various open problems of BChPT when studying the $\pi N$ scattering (convergence in the subthreshold region, $\sigma_{\pi N}$).

To fix the LECs of the chiral Lagrangians, we compare our theoretical amplitude to three different PWAs:

- The low energy PWA of Matsinos’ group (EM06) [Matsinos, Woolcock, Oades, Rasche, Gashi. NPA 778 (2006) 95-123].
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- The low energy PWA of Matsinos’ group (EM06) [Matsinos, Woolcock, Oades, Rasche, Gashi. NPA 778 (2006) 95-123].
Fits

\[ \sqrt{s} \text{ (GeV)} \]

**Red line:** $\Delta$-ChPT. **Green line:** $\Delta$-ChPT.
**Δ-ChPT Fits**

<table>
<thead>
<tr>
<th>LEC</th>
<th>KA85 Δ-ChPT</th>
<th>WI08 Δ-ChPT</th>
<th>EM06 Δ-ChPT</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>-0.80(6)</td>
<td>-1.004(30)</td>
<td>-1.000(8)</td>
<td>-1.26(14)</td>
<td>-1.50(7)</td>
<td>-1.47(2)</td>
</tr>
<tr>
<td>$c_2$</td>
<td>1.12(13)</td>
<td>1.010(40)</td>
<td>0.575(25)</td>
<td>4.08(19)</td>
<td>3.74(26)</td>
<td>3.63(2)</td>
</tr>
<tr>
<td>$c_3$</td>
<td>-2.96(15)</td>
<td>-3.040(20)</td>
<td>-2.515(35)</td>
<td>-6.74(38)</td>
<td>-6.63(31)</td>
<td>-6.42(1)</td>
</tr>
<tr>
<td>$c_4$</td>
<td>2.00(7)</td>
<td>2.029(10)</td>
<td>1.776(20)</td>
<td>3.74(16)</td>
<td>3.68(14)</td>
<td>3.56(1)</td>
</tr>
<tr>
<td>$d_1 + d_2$</td>
<td>-0.15(21)</td>
<td>0.15(20)</td>
<td>-0.34(5)</td>
<td>3.3(7)</td>
<td>3.7(6)</td>
<td>3.64(8)</td>
</tr>
<tr>
<td>$d_3$</td>
<td>-0.21(26)</td>
<td>-0.23(27)</td>
<td>0.276(43)</td>
<td>-2.7(6)</td>
<td>-2.6(6)</td>
<td>-2.21(8)</td>
</tr>
<tr>
<td>$d_5$</td>
<td>0.82(14)</td>
<td>0.47(7)</td>
<td>0.2028(33)</td>
<td>0.50(35)</td>
<td>-0.07(16)</td>
<td>-0.56(4)</td>
</tr>
<tr>
<td>$d_{14} - d_{15}$</td>
<td>-0.11(44)</td>
<td>-0.5(5)</td>
<td>0.35(9)</td>
<td>-6.1(1.2)</td>
<td>-6.8(1.1)</td>
<td>-6.49(2)</td>
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<tr>
<td>$d_{18}$</td>
<td>-1.53(27)</td>
<td>-0.2(8)</td>
<td>-0.53(12)</td>
<td>-3.0(1.6)</td>
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<td>-1.07(22)</td>
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| $h_A$ | 3.02(4) | 2.87(4) | 2.99(2) | – | – | – |
| $\chi^2_{d.o.f.}$ | 0.77 | 0.24 | 0.11 | 0.38 | 0.23 | 25.08 |

- $\Delta(1232)$ Breit-Wigner width $\Gamma_\Delta = 118(2)$ MeV (PDG) $\Rightarrow$ $h_A = 2.90(2)$
Part III

Subthreshold Region
Subthreshold Region

- The subthreshold contains points that are connected to important low energies theorems.
- For example, the value of $\bar{D}^+$ at the Cheng-Dashen point $(s = m_N^2, t = 2M_\pi^2)$ is directly related to the pion-nucleon sigma term.
- Up to now, ChPT analyses could not reproduce, from physical data, the subthreshold quantities extracted by the PWAs.
- To study the EOMS convergence, we calculate (among others) $d_{00}^+$, $d_{01}^+$ and $\Sigma$, which are defined by:

$$\bar{D}^+(\nu, t) = d_{00}^+ + d_{01}^+ t + d_{10}^+ \nu^2 + \ldots \quad (\nu \equiv \frac{s - u}{4m_N})$$

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- The amplitude fitted in the physical region can be extrapolated into the subthreshold one and compare with PWAs [1].

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- Good agreement between EOMS-BChPT+$\Delta(1232)$ and PWAs!

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<td>KA85 [2] $\Delta$-ChPT</td>
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Part IV

*The pion-nucleon $\sigma$-term*
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- $\sigma_{\pi N}$ is an observable of fundamental importance that embodies the internal scalar structure of the nucleon, related to:
  - Origin of the mass of ordinary matter.
  - Investigations of the QCD phase diagram and neutronic systems.
  - Used in estimations of DM-nucleon SI elastic scattering cross section.

- PWAs extrapolate $\Sigma = f_\pi^2 D^+$ to the Cheng-Dashen point and relate the $\Sigma$-term to $\sigma_{\pi N}$ through the relation:
  $$\Sigma = \sigma(2M_\pi^2) + \Delta_R = \sigma_{\pi N} + \Delta_\sigma + \Delta_R$$
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- Chiral symmetry allows to relate the $\sigma_{\pi N}$ to the LEC $c_1$.
- One can obtain this relation calculating $\sigma(t = 0)$ or by means of the Hellmann-Feynman Theorem:
  $$\sigma_{\pi N} = -4c_1 M_\pi^2 - \frac{3g_A^2 M_\pi^3}{16\pi^2 f_\pi^2 m_N} \left( \frac{3m_N^2 - M_\pi^2}{\sqrt{4m_N^2 - M_\pi^2}} \arccos \frac{M_\pi}{2m_N} + M_\pi \log \frac{M_\pi}{m_N} \right)$$
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  $$\Sigma = \sigma(2M^2_\pi) + \Delta_R = \sigma_{\pi N} + \Delta_\sigma + \Delta_R$$

- Chiral symmetry allows to relate the $\sigma_{\pi N}$ to the LEC $c_1$.
  - One can obtain this relation calculating $\sigma(t = 0)$ or by means of the Hellmann-Feynman Theorem:
    $$\sigma_{\pi N} = -4c_1 M^2_\pi - \frac{3g_A^2 M^3_\pi}{16\pi^2 f^2_\pi m_N} \left( \frac{3m^2_N - M^2_\pi}{\sqrt{4m^2_N - M^2_\pi}} \arccos \frac{M_\pi}{2m_N} + M_\pi \log \frac{M_\pi}{m_N} \right)$$

[Gasser, Leutwyler and Sainio, PLB 253, (1991)]

[Alarcon, Martin Camalich and Oller, PRD(R) 85 (2012)]
The pion-nucleon $\sigma$-term

- $\sigma_{\pi N}$ is an observable of fundamental importance that embodies the internal scalar structure of the nucleon, related to:
  - Origin of the mass of ordinary matter.
  - Investigations of the QCD phase diagram and neutronic systems.
  - Used in estimations of DM-nucleon SI elastic scattering cross section.

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- Good convergence of $\text{EOMS-BChPT} + \Delta(1232) \Rightarrow$ Reliable LECs $\Rightarrow$ Reliable $\sigma_{\pi N}$.

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<td>$c_1$ (GeV$^{-1}$)</td>
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The pion-nucleon $\sigma$-term

Higher order corrections:
- $\mathcal{O}(p^{7/2})$ (N$^2$LO):

\[ \Rightarrow -6 \text{ MeV} \text{ (to be compared with } -19 \text{ MeV at } \mathcal{O}(p^3)) \]

- $\mathcal{O}(p^4)$ (N$^3$LO):

\[ \Rightarrow -2 \cdots - 4 \text{ MeV} \]

(Extra contributions from $\mathcal{O}(p^4)$ LECs is estimated to be $\sim 1$ MeV)
The pion-nucleon $\sigma$-term

<table>
<thead>
<tr>
<th>$\sigma_{\pi N}$ (MeV)</th>
<th>LO</th>
<th>NLO</th>
<th>$N^2\text{LO}$</th>
<th>$N^3\text{LO}$</th>
</tr>
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<tr>
<td>78–62</td>
<td>−19</td>
<td>−6</td>
<td>−3(2)</td>
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⇒ Chiral expansion shows a clear convergent pattern!

• Comparison with independent phenomenology:
  • $h_A$ → Only WI08 $\Delta$-ChPT is compatible with the $\Delta(1232)$ BW width.
  • $\Delta_{\text{GT}}$ → WI08 $\Delta$-ChPT and EM06 $\Delta$-ChPT give a $\Delta_{\text{GT}}$ compatible with independent determinations (NN scattering and $\pi$-atoms).
  • $a_{0+}^+$ → Strongly constrains the value of $\sigma_{\pi N}$:

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[Baru, et. al., PLB 694 (2011)]
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<td>$\pi$-atom ($\pi^+ p$, $\pi^- p$)</td>
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$\sigma_{\pi N} = 59(7)$ MeV

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Gasser, et. al., PLB 253 (1991)
Part V

Summary and Conclusions
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- We performed a novel analysis in covariant BChPT within the EOMS scheme up to $\mathcal{O}(p^3)$ including the $\Delta$.
- We use different PWAs as an input to fix the LECs of the chiral Lagrangians.
- We show how EOMS-BChPT+$\Delta(1232)$ achieves the best convergence both in the physical and subthreshold regions.
  - Excellent description of the data up to 1.20 GeV.
  - Good description of the subthreshold region with amplitude fitted in the physical region.
  - Accurate and reliable value of $\sigma_{\pi N} \Rightarrow$
    \[
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Part VI

**Spares**
Section 1

Fits
Red line: $\Delta$-ChPT. Green line: $\not\Delta$-ChPT.
Fits

EM06

Red line: $\Delta$-ChPT. Green line: $\Delta$-ChPT.
Section 2

The Goldberger-Treiman Relation
The Goldberger-Treiman Relation

- The Goldberger-Treiman relation is a pre-PCAC relation that relies on the conservation of the spontaneously broken chiral symmetry.
- The non-exact conservation of this symmetry due to the quark masses leads to a deviation from this relation ($\Delta_{GT}$) that can be extracted from experimental information.

This deviation is usually defined as:

$$g_{\pi N} = \frac{g_A m_N}{f_\pi} (1 + \Delta_{GT})$$

Studies based on $\pi N$ and $NN$ PWA leads to $\Delta_{GT} = 1 - 3\%$

- In ChPT $\Rightarrow \Delta_{GT} = -\frac{2M^2_\pi d_{18}}{g_A} + \Delta_{loops}$
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- In ChPT $\Rightarrow \Delta_{GT} = -\frac{2M_\pi^2 d_{18}}{g_A} + \Delta_{loops}$
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The Goldberger-Treiman Relation

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<tbody>
<tr>
<td>$\Delta_{GT}$</td>
<td>$9(4)%$</td>
<td>$2(4)%$</td>
<td>$3.6(7)%$</td>
<td>$5.1(8)%$</td>
<td>$1.0(2.4)%$</td>
<td>$2.00(36)%$</td>
</tr>
<tr>
<td>$g_{\pi N}$</td>
<td>$14.03(52)$</td>
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<td>$13.53(10)$</td>
<td>$13.00(31)$</td>
<td>$13.13(5)$</td>
</tr>
<tr>
<td>$\Delta_{GT}$</td>
<td>$4.5(7)%$</td>
<td>$2.1(1)%$</td>
<td>$0.2(1.0)%$</td>
<td>1%</td>
<td></td>
<td>1.9(7)%</td>
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<td>$g_{\pi N}$</td>
<td>$13.46(9)$</td>
<td>$13.15(1)$</td>
<td>$12.90(12)$</td>
<td>$\simeq 13.0$</td>
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Taking the fits up to $\sqrt{s_{\text{max}}} = 1.20$ GeV in the $\Delta$-ChPT case.

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<td>14.03(52)</td>
<td>13.13(52)</td>
<td>13.34(10)</td>
<td>13.53(10)</td>
<td>13.00(31)</td>
<td>13.13(5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{GT}$</td>
<td>4.5(7)%</td>
<td>2.1(1)%</td>
<td>0.2(1.0)%</td>
<td>1%</td>
<td>1.9(7)%</td>
</tr>
<tr>
<td>$g_{\pi N}$</td>
<td>13.46(9)</td>
<td>13.15(1)</td>
<td>12.90(12)</td>
<td>$\simeq 13.0$</td>
<td>13.12(9)</td>
</tr>
</tbody>
</table>

Section 3

$\sigma_s$
σₚₚ

Using σₚₚ one can calculate:

\[ \sigma_s = \frac{1}{2m_N} \langle N | m_s \bar{s}s | N \rangle \]
\[ y = \frac{2 \langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle} \]

<table>
<thead>
<tr>
<th></th>
<th>σₓ (MeV)</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>σₓN = 59(7) MeV [1]</td>
<td>16(80)(60)</td>
<td>0.02(13)(10)</td>
</tr>
<tr>
<td>σₓN = 45(7) MeV [2]</td>
<td>-150(80)(60)</td>
<td>-0.28(13)(10)</td>
</tr>
</tbody>
</table>

Section 4

Nuclear Matter
\( \rho \cdot \sigma_{\pi N} \) controls the leading contribution in the density dependence of the quark condensate.

- \( \sigma_{\pi N} \approx 45 \text{ MeV} \Rightarrow \langle \bar{q}q \rangle \sim 0 \) for \( \rho \sim 3\rho_0 \)
- \( \sigma_{\pi N} \approx 60 \text{ MeV} \Rightarrow \langle \bar{q}q \rangle \sim 0 \) for \( \rho \sim 2\rho_0 \)

However to recover chiral symmetry in the medium is necessary \( f_t \rightarrow 0 \), which is controlled (LO) by \( c_2 + c_3 \).

- Taking into account our mean values for these LECs (for EOMS-ChPT+\( \Delta(1232) \)), that combination differs only a 10%.
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\begin{itemize}
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Section 5

$\Delta$-HBChPT
Summary of the results of [Fettes and Meißner, NPA 679 (2001)]:

<table>
<thead>
<tr>
<th>LEC</th>
<th>EM98</th>
<th>EM98( (g_{\pi N\Delta} = 1.05))</th>
<th>KA85( (g_{\pi N\Delta} = 1.05))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_1)</td>
<td>0.77(2)</td>
<td>0.39(2)</td>
<td>-0.44(1)</td>
</tr>
<tr>
<td>(g_{\pi N\Delta})</td>
<td>1.32(3)</td>
<td>1.05*</td>
<td>0.98(5)</td>
</tr>
<tr>
<td>(\Delta_{GT})</td>
<td>(input)</td>
<td>(input)</td>
<td>(input)</td>
</tr>
<tr>
<td>(\sigma_{\pi N}) (MeV)</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>51.1</td>
</tr>
</tbody>
</table>

"We remark that the sigma term can not be obtained directly from scattering data".

[Fettes and Meißner, NPA 679 (2001)]

Using sum rules that completely determine \(\sigma_{\pi N}\) from threshold parameters:

<table>
<thead>
<tr>
<th>LEC</th>
<th>EM98</th>
<th>KA85</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_1)</td>
<td>-0.18(2)</td>
<td>-0.35(9)</td>
</tr>
<tr>
<td>(g_{\pi N\Delta})</td>
<td>1.27(4)</td>
<td>1.00(8)</td>
</tr>
<tr>
<td>(\Delta_{GT})</td>
<td>(input)</td>
<td>(input)</td>
</tr>
<tr>
<td>(\sigma_{\pi N}) (MeV)</td>
<td>58.5(5.4)</td>
<td>45.5(2.7)</td>
</tr>
</tbody>
</table>
Section 6

Isospin breaking corrections
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The QCD Lagrangian contains an explicit chiral symmetry breaking term:

\[ H_{\text{mass}} = m_u \bar{u}u + m_d \bar{d}d \Rightarrow \]
\[ H_{\text{mass}} = \frac{m_u + m_d}{2}(\bar{u}u + \bar{d}d) + \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d) \]

- \( \frac{m_u + m_d}{2}(\bar{u}u + \bar{d}d) \) → Isospin symmetric.
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→ But \( m_u + m_d \) and \( m_u - m_d \) are of the same order!

- However, [Fettes and Meißner, NPA 693, 693 (2001)] shows that \( \pi N \) scattering receives corrections of the order of 2%.
- Concretely, for the \( S \)-waves (the most relevant for \( \sigma_{\pi N} \)) the corrections are of \( \approx 0.7\% \).
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J. A. Oller and U. G. Meißner,


J. Gegelia, G. Japaridze, K. Turashvili, Theoretical and Mathematical Physics, Vol. 101, No. 2, 1994 (Translated from Russian)


Baru, et. al., PLB 694 (2011).


