TRAPPED PHONONS

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In collaboration with C. Manuel and L.Tolos
Outline

• Introduction
• Shear viscosity
• Phonons
• Restricted geometries
• Conclusions
Introduction

Def.

**Phonon**: the Nambu-Goldstone boson (NGB) associated to the spontaneous breaking of a global U(1) symmetry

Standard phonon: the quantum of a compressional wave or of a lattice vibration

Our phonon can be something different
Introduction

Def.
**Phonon**: the Nambu-Goldstone boson (NGB) associated to the spontaneous breaking of a global U(1) symmetry

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Our phonon can be something different

We want to figure out the contribution of NGBs to the shear viscosity coefficient in restricted geometries
Shear viscosity

Fluid with a oscillating surface

Oss.
An external observer measures the damping of the oscillations of the plate (does not know whether below the surface there is a liquid or a gas)
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LIQUID
Variation of the energy momentum tensor

\[ \delta T_{ij} = -\eta \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{v} \right) \]
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\[
\begin{align*}
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\text{GAS} & \quad F_d = \frac{1}{2} \rho v^2 A \lambda
\end{align*}
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GAS
Drag force \( F_d = \frac{1}{2} \rho v^2 A \lambda \)

\[ \eta \propto \rho \tau_\perp \]

density

time for transport of momentum orthogonally to the plate
Shear viscosity

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Drag force
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F_d = \frac{1}{2} \rho v^2 A \lambda
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Oss.
If various independent processes exist, the one with the shortest timescale wins

\[\eta \propto \rho \tau_{\perp}\]
Restricted geometries

Compact star
Restricted geometries

Compact star

Optical trap
(ultracold fermionic atoms)
Restricted geometries

Phonons are trapped in a region of space
They interact among themselves and with the boundary
phonon-phonon interactions

3-ph (Belyaev) process

4-ph process
phonon-phonon interactions

3-ph (Belyaev) process

4-ph process

3-ph process is very sensitive to the equation of state

Ex.

\[ E_k = c_s k (1 + \gamma k^2) \]

if \( \gamma < 0 \) 3-ph process is not kinematically allowed
Emulation by ultracold fermionic atoms

Emulation
Use ultracold fermionic atoms in an optical trap to understand the general properties of NGBs

Phonons originate from the breaking of particle number
Considering \( E_k = c_s k (1 + \gamma k^2) \)

\[
\frac{\eta_{3\text{ph}}}{s_{\text{ph}}} \simeq 4.0 \times 10^{-9} \frac{T_F^8}{T^8} \quad \frac{\eta_{4\text{ph}}}{s_{\text{ph}}} \simeq 2.2 \cdot 10^{-7} \frac{T_F^8}{T^8}
\]

where \( \gamma \approx \frac{0.18}{k_F^2} \)

\( s_{\text{ph}} = \frac{2\pi^2 T^3}{45 c_s^3} \)
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data points
Duke group
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Missing effect of the boundary?
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Missing effect of the boundary?

Missing piece in the dispersion law?

diverge?

data points

Duke group

vanish?
Changing dispersion law

\[ E_k = c_s k (1 + \gamma k^2 + \delta k^4) \]
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With a negative \( \delta \) we can avoid the “universal” bound.
Changing dispersion law

$$E_k = c_s k (1 + \gamma k^2 + \delta k^4)$$

With a negative $\delta$ we can avoid the “universal” bound.

The analogous calculation for 4-ph processes is still missing!!
Mean free path

\[ E_k = c_s k (1 + \gamma k^2 + \delta k^4) \]

the added term does not help at all at low T
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At low temperature phonons are ballistic!!

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\[ \frac{l_{ph}}{R_x} \]

\[ T/T_F \]

At low temperature phonons are ballistic!!

MM, Manuel, Tolos
(submitted paper)
Mean free path

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At low temperature phonons are ballistic !!

Phonons can be described with hydrodynamics if \( \ell_{ph} \) is smaller than the typical size of the system

MM, Manuel, Tolos (submitted paper)
Boundary
Boundary

Knudsen layer

Boundary

$\ell_{ph}$

FLUID
Boundary

Knudsen layer

Boundary

\( \ell_{ph} \)

FLUID

Navier-Stokes
Boundary

Knudsen layer

Boundary

FLUID

Boltzmann

\( \ell_{ph} \)

Navier-Stokes
Knudsen number $K_n = \frac{\ell_{ph}}{R}$
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Hydrodynamic regime

\( K_n \ll 1 \)
Knudsen number \( K_n = \frac{\ell_{ph}}{R} \)

Hydrodynamic regime \( K_n \ll 1 \)

Ballistic regime \( K_n \gg 1 \)
Knudsen number \( K_n = \frac{\ell_{ph}}{R} \)

**Hydrodynamic regime**  \( K_n \ll 1 \)

**Intermediate regime**  \( K_n \sim 1 \)

**Ballistic regime**  \( K_n \gg 1 \)
Boundary

A boundary can absorb, diffuse or specular-reflect a particle

Simple boundary: no absorption
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Simple boundary: no absorption

specular-reflection
no shear stress
A boundary can absorb, diffuse or specular-reflect a particle.

Simple boundary: no absorption.

- **Specular-reflection**: no shear stress.
- **Diffusion shear stress**.
Boundary

A boundary can absorb, diffuse, or specular-reflect a particle.

Simple boundary: no absorption

Accommodation coefficient $\chi$

$\chi = 0$ all impinging particles are specular-reflected

$\chi = 1$ all impinging particles are diffused
Effective viscosity

Shear viscosity due to 3-ph and 4-ph processes

\[ \eta_{\text{bulk}} = \frac{1}{5} \rho_{\text{ph}} c_s l_{\text{ph}} \]

“Shear viscosity” due to phonon-boundary interaction

\[ \eta_{\text{ball}} = \frac{1}{5} \rho_{\text{ph}} c_s \chi(c_s \tau_b) \equiv \frac{1}{5} \rho_{\text{ph}} c_s a \]
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Effective shear viscosity

\[ \eta_{\text{eff}} = \left( \eta_{\text{bulk}}^{-1} + \eta_{\text{ball}}^{-1} \right)^{-1} \]
Effective viscosity

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Effective shear viscosity

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Two fitting parameters used:

- \( a \)
- \( \delta \)

MM, Manuel, Tolos
arXiv:1201.4006
CFL MATTER IN COMPACT STARS
Color-flavor locked (CFL) phase

Condensate
(Alford, Rajagopal, Wilczek hep-ph/9804403)

\[ \langle \psi_{\alpha_i} C \gamma_5 \psi_{\beta_j} \rangle \sim \Delta_{CFL} \epsilon_{I\alpha\beta} \epsilon_{Iij} \]

Using instantons or NJL models

\[ \Delta_{CFL} \approx (10 - 100) \text{ MeV} \]

\[ \mu \approx 400 \text{ MeV} \]
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Symmetry breaking

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2 \]

\[ \supset U(1)_Q \]

\[ \supset U(1)_{\bar{Q}} \]
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\[ \supset U(1)_{\tilde{Q}} \]

- Higgs mechanism: All gluons acquire “magnetic” mass
- \( \chi_{SB} \): 8 (pseudo) Nambu-Goldstone bosons (NGBs)
- \( U(1)_B \) breaking: 1 NGB
CFL Phonon

There is a massless NGB, $\phi$, associated to $U(1)_B/Z_2$

Quantum numbers $\phi \sim <\Lambda \Lambda >$ like the H-dibaryon of Jaffe, Phys. Rev. Lett. 38, 195 (1977)

Effective Lagrangian up to quartic terms

$$L_{\text{eff}}(\varphi) = \frac{3}{4\pi^2} \left[ (\mu - \partial_0 \varphi)^2 - (\partial_i \varphi)^2 \right]^2$$

Son, hep-ph/0204199
Application to compact stars

In principle the same reasoning used for phonons in ultracold fermionic atoms can be employed for compact stars. However in this case there is a large number of vortices.
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Dissipative processes due to vortex-phonon interaction damp r-mode oscillation for CFL stars rotating at frequencies < 1 Hz

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In principle the same reasoning used for phonons in ultracold fermionic atoms can be employed for compact stars. However in this case there is a large number of vortices.

Dissipative processes due to vortex-phonon interaction damp $r$-mode oscillation for CFL stars rotating at frequencies < 1 Hz.


Future project

Since CFL phonons are ballistic we need to understand their interaction with the CFL-nuclear matter boundary and whether this may lead to diffusive scattering.
Conclusion

• NGBs determine the low energy properties of many systems

• The presence of a boundary may significantly change the transport properties of a system

• The transition from the hydrodynamic to the ballistic regime does greatly reduce the shear viscosity

• An application to ultracold atoms has been shown

• An application to CFL matter requires the understanding of the interface between CFL and nuclear matter