Including Four-Gluon Interactions Into Landau / Maximally-Abelian Gauge Dyson-Schwinger Studies

Valentin Mader*, Reinhard Alkofer

Institut für Physik
Karl Franzens Universität Graz

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Dyson-Schwinger Equations

- equations of motion of Green functions of a quantum field theory
- one possible (infinite) set of defining equations for the Green functions
- by definition fully renormalized equations
Dyson-Schwinger Equations

- equations of motion of Green functions of a quantum field theory
- one possible (infinite) set of defining equations for the Green functions
- by definition fully renormalized equations
- practical calculations demand gauge-fixing and truncations
- truncations can interfere with "self-renormalization"
Maximally-Abelian Gauge

- discriminates between “Abelian” and “Non-Abelian” gauge fields

\[
A_\mu = \sum_r A^r_\mu T^r = \sum_i A^i_\mu T^i + \sum_a B^a_\mu T^a
\]

- Lorentz-covariant, Ghost-Antighost-symmetric
- separate gauge-fixing:
  - non-abelian: minimize \( \int d^4 x B^a_\mu B^a_\mu \)
  - abelian: maximize \( \int d^4 x A^i_\mu A^i_\mu \)
- Landau gauge: maximize \( \int d^4 x A^r_\mu A^r_\mu \)
IR-Analysis of the MAG

- use DSE’s and ERGE’s
- IR power-law ansätze
- no truncation involved

\[ Z_d \propto \left( p^2 \right)^{-\kappa}, \quad Z_{od} \propto \left( p^2 \right)^{\kappa}, \quad G_{od} \propto \left( p^2 \right)^{\kappa} \approx 0.74. \]
IR-Analysis of the MAG

- use DSE’s and ERGE’s
- IR power-law ansätze
- no truncation involved

Dressing functions behave like

\[ Z_d \propto (p^2)^{-\kappa}, \quad Z_{od} \propto (p^2)^{\kappa}, \quad G_{od} \propto (p^2)^{\kappa} \]

\[ \kappa \approx 0.74 \]

- Abelian Dominance
- Two loop terms are IR-leading
1 Introduction

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Yang-Mills propagator DSE’s in Landau Gauge

\[ -1 = \frac{1}{P^2 + m^2} + \frac{1}{P^2 + m^2} + \frac{1}{p^2 + m^2} + \frac{1}{p^2 + m^2} \]

\[ = \frac{1}{P^2 + m^2} + \frac{1}{p^2 + m^2} + \frac{1}{p^2 + m^2} + \frac{1}{p^2 + m^2} \]
Yang-Mills propagator DSE’s in Landau Gauge

\[
\begin{align*}
    \frac{1}{D} & = \frac{1}{D_1} + \frac{1}{D_2} + \frac{1}{D_3} \\
    & + \frac{1}{D_4} + \frac{1}{D_5} + \frac{1}{D_6}
\end{align*}
\]
Yang-Mills propagator DSE’s in Landau Gauge

\[ -1 = -1 + \]

\[ + + \]

\[ -1 \]

\[ -1 \]

\[ + + \]

\[ + + \]

\[ + + \]

\[ + + \]

\[ + + \]

\[ + + \]

\[ + + \]
Structure of Divergencies

\[ D(p^2) = \frac{Z(p^2)}{p^2 + Z_m m^2} \]

\[ D^{-1}(p^2) = c_1 \Lambda^2 + c_2 p^2 \log \left[ \frac{\Lambda^2}{p^2} \right] + \text{something finite} \]
Structure of Divergencies

\[ D(p^2) = \frac{Z(p^2)}{p^2 + Z_m m^2} \]

- quadratic divergences: modify gluon loop to cancel div's of ghost loop
  
  \[ Z_m(\Lambda^2, \mu^2)m^2(\mu^2) = 0 \Rightarrow \tilde{c}_1 = 0 \]

- logarithmic divergences: MOM-scheme
  
  \[ Z(p^2) \xrightarrow{p^2 \to \infty} \alpha_s(p^2)^{-\gamma} \]
Including Sunset

\[ I_{\text{sun}}(p^2) = \frac{Z_4}{6} g^4 N_c^2 \int_0^\Lambda d^4 q_1 \int_0^\Lambda d^4 q_2 \frac{Z(p_1^2)Z(p_2^2)Z(p_3^2)}{p_1^2 p_2^2 p_3^2} D_\Gamma(p, p_1, p_2, p_3) \mathcal{T} \]

\[ \mathcal{T} = \mathcal{T}(z_{01}, z_{02}, z_{03}, z_{12}, z_{13}, z_{23}) \]

- \[ I_{\text{sun}}(p^2) = c_1 \Lambda^2 + c_2 p^2 \log \left( \frac{\Lambda^2}{p^2} \right) + \text{something finite} \]

- different divergences appear in different combinations of \( z_{ij} \)'s (overlapping divergences)
Including Sunset

\[
I_{\text{sun}}(p^2) = \frac{Z_4}{6} g^4 N_c^2 \int_0^\Lambda d^4 q_1 \int_0^\Lambda d^4 q_2 \frac{Z(p_1^2)Z(p_2^2)Z(p_3^2)}{p_1^2 p_2^2 p_3^2} D_{\Gamma}(p, p_1, p_2, p_3) \mathcal{T}
\]

\[
\mathcal{T} = \mathcal{T}(z_{01}, z_{02}, z_{03}, z_{12}, z_{13}, z_{23})
\]

- solution: explicit counter term construction

\[
\tilde{\mathcal{T}} = \mathcal{T} - (\zeta - 4) \left( \lambda_1 + \lambda_2 \left( z_{12}^2 + 3z_{01}^2 \right) \right)
\]

- quadratic divergences entirely subtracted

\[
\tilde{I}_{\text{sun}}(p^2) = c_2 p^2 \log \left( \frac{\Lambda^2}{p^2} \right) + \text{something finite}
\]
Including Sunset

\[ D(p^2) = \frac{Z(p^2)}{p^2} \]
Summary

- Renormalization in non-perturbative functional methods demand novel appropriate techniques.
- In DSE’s one possibility is explicit counter-term construction.
- Explicit counter-term construction can also be applied for two-loop terms (sunset).
- In Landau gauge contributions of sunset-diagram not significant.
Outlook: Maximally Abelian Gauge

proposed self-consistent system to solve:

\[-1 \quad = \quad -1 +
\]

UV-behavior (given by one-loop terms)

IR-behavior (given by two-loop terms)

quadratic divergences subtracted

numerical implementation
Outlook: Maximally Abelian Gauge

proposed self-consistent system to solve:

\[ \begin{align*}
\cdots^{-1} &= \cdots^{-1} + \cdots^{-1} + \cdots^{-1} + \\
\cdots^{-1} &= \cdots^{-1} + \cdots^{-1} + \cdots^{-1} + \\
\end{align*} \]

✓ UV-behavior (given by one-loop terms)
✓ IR-behavior (given by two-loop terms)
✓ quadratic divergences subtracted
Outlook: Maximally Abelian Gauge

proposed self-consistent system to solve:

\[
\begin{align*}
\cdots^{-1} &= \cdots^{-1} + \cdots \\
\cdots^{-1} &= \cdots^{-1} + \cdots
\end{align*}
\]

- UV-behavior (given by one-loop terms)
- IR-behavior (given by two-loop terms)
- quadratic divergences subtracted
- numerical implementation
Thank You for Your attention!