Unquenching the gluon propagator with Schwinger-Dyson equations

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Quark Confinement and the Hadron Spectrum X
Munich 08|10|12-12|10|
Recent years have seen a fruitful synergy between lattice and SDE first principle calculations. Lattice studies focused on Green’s functions in the quenched approximation, while SDE studies mainly focused on Green’s functions in pure Yang-Mills. Time is ripe for the transition to real world QCD, with new lattice results incorporating flavor effects and a new SDE algorithm for estimating quark loop effects on the gluon propagator.
Apply the pinch technique to the Schwinger-Dyson equation of the gluon propagator

Graphs made out of **new vertices**, (inside **conventional props**) New vertices corresponds to **BFM vertices**

- **external gluons** dynamically converted into **background gluons**

**New Schwinger-Dyson equation** has a **special structure**

- **Subgroups** (one-/two-loop dressed gluon/ghost) are **individually transverse**

Express the **Schwinger-Dyson eq** in terms of a background-quantum identity

\[
\Delta^{-1}(q^2)[1 + G(q^2)]^2 P_{\mu\nu}(q) = q^2 P_{\mu\nu}(q) \sum_{i=0}^{10} (a_i)_{\mu\nu}
\]

\[
\hat{\Delta}(q^2) = [1 + G(q^2)]^{-2} \Delta(q^2)
\]

In 4d the function \( G \) is directly related to the inverse of the ghost dressing function

\[
F^{-1}(q^2) \approx 1 + G(q^2)
\]
Highly non-linear propagation of the effect

- The presence of quarks will also affect the original quenched diagrams
- Operating assumption: non-linear effects are suppressed wrt diagram \((a_{11})\)

Adding dynamical quarks gives

\[
\Delta_Q^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\hat{\Pi}^Q_{\mu\nu}(q) + i\hat{X}_{\mu\nu}(q)}{[1 + G_Q(q^2)]^2}
\]

Gauge invariant subset (also in the conventional formulation!) but

PT-BFM vertex satisfies a linear WI as opposed to the conventional STI

Many Abelian Ansätze available in the market (e.g., the time honored Ball-Chiu and/or Curtis-Pennington)

Suffix \(Q\) indicates the effects of quarks on quenched quantities
adding quarks to the SDE

Non-perturbative calculation of the term

\[ \hat{X}^{\mu\nu}(q) = -g^2 d_f \int_k Tr \left[ \gamma^\mu S(k) \hat{\Gamma}^\nu(k + q, -k, -q) S(k + q) \right] \]

\[ S^{-1}(k) = -i \left[ A(k^2) \not{k} - B(k^2) \right] = -i A(k^2) \left[ \not{k} - \mathcal{M}(k^2) \right] \]

The PT-BFM quark-gluon vertex satisfies

\[ p_3^\mu \hat{\Gamma}(p_1, p_2, p_3) = S^{-1}(-p_1) - S^{-1}(p_2) \]

valid for both BC and CP vertices

Make an Ansatz for the longitudinal part of the vertex

\[ \hat{\Gamma}(p_1, p_2, p_3) = L_1 \gamma^\mu + L_2 (\not{p}_1 - \not{p}_2) (p_1 - p_2)_\mu + L_3 (p_1 - p_2)_\mu + L_4 \sigma_{\mu\nu} (p_1 - p_2)_{\nu} \]

Solve the WI to get (Ball-Chiu vertex)

\[ L_1 = \frac{A(p_1) + A(p_2)}{2}; \quad L_2 = \frac{A(p_1) - A(p_2)}{2 \left( p_1^2 - p_2^2 \right)}; \quad L_3 = \frac{B(p_2) - B(p_1)}{p_1^2 - p_2^2}; \quad L_4 = 0 \]

A and B are obtained from solving the quark gap equation

adding quarks to the SDE

Use the BC vertex to get

\[
\hat{X}(q^2) = -\frac{2g^2}{d-1} \int_k \frac{1}{A_a A_b(k^2 - \mathcal{M}_a^2)[(k + q)^2 - \mathcal{M}_b^2]} \sum_{i=1}^{3} T_i(k, k + q)
\]

\[
T_1(k, k + q) = L_1 \{(2-d)(k^2 + k \cdot q) + d \mathcal{M}_a \mathcal{M}_b\}
\]

\[
T_2(k, k + q) = L_2 \{2[k \cdot (2k + q)] [(k + q) \cdot (2k + q)] - k \cdot (k + q)(2k + q)^2 + (2k + q)^2 \mathcal{M}_a \mathcal{M}_b\}
\]

\[
T_3(k, k + q) = L_3 \{\mathcal{M}_b [(2k + q) \cdot k] + \mathcal{M}_a [(2k + q) \cdot (k + q)]\}
\]

In the \( q \to 0 \) limit

\[
\hat{X}(0) = -\frac{2g^2}{d-1} \int_k \frac{1}{A^2(k^2 - \mathcal{M}^2)^2} \left\{ A \left[ (2-d)k^2 + d \mathcal{M}^2 \right] + 2A'k^2 \left( k^2 + \mathcal{M}^2 \right) - 4k^2 B' \mathcal{M} \right\}
\]

Use the seagull identity

\[
\int_k k^2 \frac{\partial f(k^2)}{\partial k^2} + \frac{d}{2} \int_k \Delta(k) = 0
\]

\[
f(k^2) = \left[ A(k^2) (k^2 - \mathcal{M}^2) \right]^{-1}
\]

The quark loop does not contribute to the gluon mass!

The seagull identity keeps the gluon massless in the absence of a DMG mechanism.

\[ f(k^2) = [A(k^2) (k^2 - \mathcal{M}^2)]^{-1} \]

\[ \int_k k^2 \frac{\partial f(k^2)}{\partial k^2} + \frac{d}{2} \int_k \Delta(k) = 0 \]

\[ f(k^2) = \left[ A(k^2) (k^2 - \mathcal{M}^2) \right]^{-1} \]
So what happens when quarks are present?

\[
\Delta^{-1}(q^2) = \frac{q^2 J(q^2) - m^2(q^2)}{\lambda^2(q^2)} 
\rightarrow \Delta^{-1}_Q(q^2) = q^2 J_Q(q^2) - m^2_Q(q^2)
\]

\(\hat{X}(q^2)\) does not affect the value of the mass and therefore contributes to \(J_Q(q^2)\)

\[q^2 J_Q(q^2) = q^2 J(q^2) + i \frac{\hat{X}(q^2)}{1 + G(q^2)}\]

However, it affects its value indirectly,

\[
\lambda^2(q^2) = m^2_Q(q^2) - m^2(q^2)
\]

First-principle determination requires the knowledge of the mass equation

Put everything together to write the master formula

\[
\Delta_Q(q^2) \approx \Delta(q^2) \frac{\Delta(q^2)}{1 + \left\{ i\hat{X}(q^2) [1 + G(q^2)]^{-2} - \lambda^2 \right\} \Delta(q^2)}
\]

Expresses the unquenched result as a deviation from the quenched case
adding quarks to the SDE

Two possibilities for treating $\lambda$

- **As a parameter** (get the qualitative features)
- **As a dynamical object** (get quantitative)

Couple to the master equation the recently obtained all-order gluon mass equation

$m^2(q^2) = \ell \int K(q^2, k^2, q \cdot k, \Delta, Y)$

**Gluons are heavier when quarks are included**

Suppression of the intermediate momentum region

Shows how fermions influence the gluon mass

As a **parameter** (get the qualitative features)

Gluons are heavier when quarks are included

Suppression of the intermediate momentum region

Couple to the master equation the recently obtained all-order gluon mass equation

$m^2(q^2) = \ell \int K(q^2, k^2, q \cdot k, \Delta, Y)$
Renormalize in the MOM scheme

\[ \Delta_{Q,R}(q^2) = \frac{\Delta_R(q^2)}{1 + \left\{ i \hat{X}_R(q^2) [1 + G_R(q^2)]^{-2} - \lambda^2 \right\} \Delta_R(q^2)} \]

\[ \hat{X}_R(q^2) = \hat{X}(q^2) - \frac{q^2}{\mu^2} \hat{X}(\mu^2) \]

\[ G_R(q^2) = G(q^2) - G(\mu^2) \]

\[ \hat{\Pi}_R(q^2) = \hat{\Pi}(q^2) - \frac{q^2}{\mu^2} \hat{\Pi}(\mu^2) \]

\[ \Delta_{Q,R}(\mu^2) = 1/\mu^2 \]

For large values of \( q^2 \)

\[ [\hat{\Delta}^{-1}(q^2)]^{(1)} = q^2 \left[ 1 + \frac{\alpha_s}{48\pi} (33 - 2N_f) \log(q^2/\mu^2) \right] \]

Correct one-loop result in the Landau gauge
adding quarks on the lattice

No systematic study of the IR sector with dynamical quarks

  - Symanzik improved action with 2+1 staggered fermions
  - Only gluon dressing; no propagator, no ghosts

  - Tadpole improved action with 2 dynamical overlap fermions

  - 2 Wilson clover fermions

Use ETwistedMassC configurations projected to the Landau gauge

- 2 light quarks and 2 light and two heavy quarks configurations

- Physical strange quark mass and somewhat large up/down masses: \( m_u/m_d \approx 2 - 5 \)

- Volumes up to \( 3^3 \times 6 \) \( [\text{fm}^4] \)

- Compensate for O(4) breaking artifacts (no cylindrical cut on (almost all) data but H(4) extrapolation)

- Study both the gluon and the ghost IR sector

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N_f=2 and N_f=2+1+1 lattice results

\[ \Delta_\alpha(q^2) \text{ [GeV}^2] \]

\[ q^2 \text{ [GeV}^2] \]

\[ \mu=4.3 \text{ [GeV]} \]

N_f=0, \( \beta=5.70 \), various Vs
N_f=2, \( \beta=3.90 \), V=24x48
N_f=2, \( \beta=4.20 \), V=48x96
N_f=2+1+1, \( \beta=1.99 \), V=32x64
N_f=2+1+1, \( \beta=1.95 \), V=48x96

Solution of the ghost SDE using a non constant ghost-gluon vertex

Fit
solving the quark gap equation

Up, down and strange quarks

Charm quark
Leading contribution comes from the quark-gluon vertex form factor $L_1$. 

\[ \hat{X}^{\mu\nu}(q) = \frac{q_{\mu}}{\mu} \]
Numerical results

Light quarks

\[ \Delta(q^2) \left[ \text{GeV}^{-2} \right] \]
numerical results
light quarks

\[ \Delta(q^2) [\text{GeV}^2] \]

- \( \mu = 4.3 \text{ [GeV]} \)
- \( N_f = 0, \beta = 5.70, \text{ various } V_s \)
- \( N_f = 0 \) fit
- \( N_f = 2 \) SDE solution

\[ q^2 [\text{GeV}^2] \]
numerical results
light quarks

\[ \Delta(q^2) \text{ [GeV}^2]\]

\[ q^2 \text{ [GeV}^2]\]

\[ \mu=4.3 \text{ [GeV]} \]

\[ N_f=0, \text{ fit} \]

\[ N_f=0, \beta=5.70, \text{ various } V \]

\[ N_f=0, \beta=3.90, V=24^3 \times 48 \]

\[ N_f=2, \beta=4.20, V=48^3 \times 96 \]

\[ N_f=2 \text{ SDE solution} \]
numerical results

heavier quarks
numerical results
heavier quarks

\[ \Delta(q^2) \text{ [GeV}^2] \]
New SDE algorithm
- Compute the unquenched gluon propagator as a deviation from the quenched one

PT-BFM framework
- Good control on basic field theoretical properties

Main results
- Quark loops suppress the IR region
- Very good agreement with lattice results

Many things to do
- Construct RGI combinations (gluon mass and effective charge) and study their flavor dependence
- Finite temperature, chemical potential,...