Meson Spectrum, Glueball Mass and QCD Effective Coupling within Infrared Confinement

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X Quark Confinement and the Hadron Spectrum
October 8-12, 2012, Garching, Muenchen
QCD behavior at large distances is an active field of research because many novel behaviors are expected at energies below 1\text{GeV} (IR region).

Understanding of a number of phenomena (quark confinement, QCD running coupling, glueball states, etc.) requires a correct description of hadron dynamics in the IR region.

QCD effective coupling, can provide a continuous interpolation between the asymptotical free state and the hadronization regime, where non-PT techniques must be employed (e.g. Yu.L.Dokshitzer et al., 1996).

A self-consistent and physically meaningful prediction of the QCD effective charge in the IR regime remains actual.

We investigate the QCD effective charge in the low-energy (below ~1 GeV) region by exploiting the hadron spectrum.
Contents:

- Relativistic QF model with infrared (IR) confinement
- Two-particle bound states.
- Conventional meson masses, Regge trajectories, …
- QCD effective coupling $\alpha_s$ in the low-energy domain
- IR-fixed point $\alpha_s(0)$
- The ground-state properties of the glueball (gg).
- Summary
Confinement

Confinement and dynamical symmetry breaking are crucial features of QCD.

**Color** confinement is the result of strong interaction, however, in the hadron scales (~ 200 MeV ~ 1fm) QCD becomes non-PT.

Moreover, there is no analytic proof that QCD should be color confining. The reason for confinement may be somewhat complicated.

Some different explanations of confinement:

- **Self-dual gluon background** [e.g., H.Leutwyler 1980; G.V.Efimov et al. 1995]
- **Confinement in lattice MC simulations** [e.g., C.D.Roberts 1994; F. Lenz 2004]
- **Confinement in string theory in higher-D** [e.g., R.Alkofer, J.Greensite, 2007]
- **IR Confinement** [e.g., C.S.Fischer, R.Alkofer, L.von Smekal 2002]
IR-finite Propagators

A.G. Williams et al. [2001]

Schwinger-Dyson eqs. + lattice QCD

- **Ansatz**: Quark and Gluon propagators are IR-confined functions in Euclidean space:

\[
\tilde{S}(\hat{p}) = \frac{1}{-i\hat{p} + m} = (i\hat{p} + m) \cdot \int_0^\infty dt \, e^{-t(\hat{p}^2 + m^2)} \\
\rightarrow (i\hat{p} + m) \cdot \int_0^\infty dt \, e^{-t(\hat{p}^2 + m^2)} = \frac{i\hat{p} + m}{\hat{p}^2 + m^2} \left(1 - e^{-\frac{(\hat{p}^2 + m^2)}{\Lambda^2}}\right)
\]

\[
D(x) = \frac{1}{4\pi^2 x^2} = \int_0^\infty ds \, e^{-s x^2} \rightarrow \int_{\Lambda^2}^\infty ds \, e^{-s x^2} = \frac{e^{-\frac{x^2 \Lambda^2}{4\pi^2}}}{4\pi^2 x^2} \\
\tilde{D}(p) = \frac{1}{p^2} \rightarrow \frac{1 - e^{-\frac{p^2}{\Lambda^2}}}{p^2}
\]

\(\Lambda \rightarrow 0\): deconfinement
QCD Effective Coupling

**THEORY:** QCD predicts a dependence of $\alpha(Q)$ on energy scale $Q$. This *dependence* is described theoretically by the RG equations.

**EXPERIMENT:** but its *actual value* must be obtained from experiment. It is well determined experimentally at relatively high energies $Q > 2 \text{ GeV}$.

<table>
<thead>
<tr>
<th>Process</th>
<th>$Q$ (GeV)</th>
<th>$\alpha_s$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$ decays</td>
<td>$1.78$</td>
<td>$0.330 \pm 0.014$</td>
<td>S. Bethke (2009)</td>
</tr>
<tr>
<td>$\bar{Q}Q$ states</td>
<td>$4.1$</td>
<td>$0.239 \pm 0.012$</td>
<td>S. Davies (2003)</td>
</tr>
<tr>
<td>$\gamma$ decays</td>
<td>$4.75$</td>
<td>$0.217 \pm 0.021$</td>
<td>A. Penin (1998)</td>
</tr>
<tr>
<td>$\bar{Q}Q$ states</td>
<td>$7.5$</td>
<td>$0.1923 \pm 0.0024$</td>
<td>S. Bethke (2009)</td>
</tr>
<tr>
<td>$\gamma$ decays</td>
<td>$9.46$</td>
<td>$0.184 \pm 0.015$</td>
<td>S. Bethke (2009)</td>
</tr>
<tr>
<td>$e^+e^-$ jets</td>
<td>$14.0$</td>
<td>$0.170 \pm 0.021$</td>
<td>P. A. M. Fernandez (2002)</td>
</tr>
</tbody>
</table>

$\alpha_s(2 < Q < 180) \rightarrow \text{measured}$

$\alpha_s(Q \rightarrow \infty) \rightarrow 0$

$\alpha_s(Q << 1) \rightarrow \alpha_s^0$

D. Zwanziger 1992
A. Williams 2001

$\alpha_s(Q < 1) \rightarrow ?$
The Model

- Consider a relativistic quantum-field model of quark-gluon interaction.

\[ L = -\frac{1}{4} \left( F_{\mu\nu}^A - g f^{ABC} A_\mu^B A_\nu^C \right)^2 + \sum_f \left( \bar{q}_f \left[ \gamma_\alpha \partial_\alpha - m_f + g \Gamma_\alpha^C A_{\alpha}^C \right] q_f \right) \]

\[ F_{\mu\nu}^B \equiv \partial_\mu A_\nu^B - \partial_\nu A_\mu^B \quad \Gamma_\alpha^C \dot\equiv i\gamma_\alpha t^C \]

- Partition Functional written in terms of quark and gluon variables

\[ Z = \int \int \delta \bar{q} \delta q \int \delta A \exp \left\{ -\int dx \ L[\bar{q}, q, A] \right\} \]

Assumptions (in hadronization region):
- Quark and gluon propagators are infrared confined.
- The coupling remains weak (\( \alpha_s < 1 \)). Ladder BSE is sufficient to estimate the meson masses with reasonable accuracy.
Two-particle Bound States

“Nonexotic” states:

Quark is a fermion and has spin=1/2. In the quark model \((q\bar{q})\) bound states are classified in \(J^{PC}\) multiplets. For a pair with spin \(s=\{0,1\}\) and angular momentum \(l\) the parity is \(P = (-1)^{l+s}\) and the total spin is \(|l - s| < J < |l + s|\). The most established sectors of hadron spectroscopy is the ground states, namely, the pseudoscalar (0−+) and vector (1−−) mesons.

“Exotic” states:

QCD predicts the existence of pure \((gg, ggg, \ldots)\) bound states, and they are called the glueballs. The gluon spin is 1, the ground state glueball is predicted by lattice gauge theories to be the scalar 0+++ , the first excited state is the tensor 2+++.
Quark-Antiquark States

• LO contributions to quark-antiquark and two-gluon bound states:

\[
Z_{(\bar{q}q)} = \int \int \delta \bar{q} \delta q \exp \left\{ -\left( \bar{q} S^{-1} q \right) + \frac{g^2}{2} \left\langle \left( \bar{q} \Gamma A q \right) \left( \bar{q} \Gamma A q \right) \right\rangle_D \right\}
\]

\[
Z_{(AA)} = \left\langle \exp \left\{ -\frac{g}{2} \left( f AAF \right) \right\} \right\rangle_D \quad \left\langle (\bullet) \right\rangle_D = \int D A \ e^{-\frac{1}{2} (AD^{-1}A)} (\bullet)
\]

• Allocate one-gluon exchange between quark-colored currents
• Isolate color-singlet combination
• Perform Fierz transformation \((J = S, P, A, V, T)\)
• Go to centre-of-masses frame (due to different quark masses)
• Orthonormalized system: \(\{U_Q\}\) with quantum numbers \(Q = \{n, l, \ldots\}\):
  • Local quark currents and vertices with given quantum numbers
  • Diagonalization on colorless quark currents
  • Gaussian representation: a new path integration over auxiliary fields \(B_N\)
• Explicit path-integration over quark variables and write the effective action
• Hadronization Ansatz: $B_N$ fields are identified as meson fields

• Generating functional in terms of meson field variables.  
  Isolate all quadratic (kinetic part) field configurations:

$$Z_N = \int \prod_{N} \delta B_N \exp \left\{ -\frac{1}{2} \left( B_N \left[ 1 + g^2 \text{Tr}(V_N S V_N S) \right] B_N \right) + W_{\text{resid}} \left[ B_N \right] \right\}$$

• Diagonalization of the quadratic part is equivalent to the solution of the ladder Bethe-Salpeter equation on the orthonormalized system $\{U_N\}$

$$g^2 \text{Tr}(V_N S V_{N',S}) = \left( U_N \lambda U_{N'} \right) = \lambda_N (-p^2) \delta^{JJ'} \delta^{QQ'}$$

• Symmetric Bethe-Salpeter kernel is defined:  [ G.G., PRD81 (2010) ]

$$\lambda_N (-p^2) = \frac{4 g^2 C_J}{9} \int \frac{d^4 k}{(2\pi)^4} \left\{ V(k) \right\}^2 \cdot \text{Tr} \left\{ \Gamma_J \hat{S} \left( k + \xi_1 \hat{p} \right) \Gamma_J \hat{S} \left( k - \xi_2 \hat{p} \right) \right\}$$

• Renormalization:

$$B_{\text{REN}}(x) = \sqrt{-\lambda_N (M_N^2)} \cdot B_N(x)$$

$$\left\langle B_N \left| 1 + \lambda_N (-p^2) \right| B_N \right\rangle = \left\langle B_N \left| 1 + \lambda_N (M_N^2) - \lambda_N (M_N^2)(p^2 + M_N^2) \right| B_N \right\rangle$$

$$= \left\langle B_{\text{REN}} \left| (p^2 + M_N^2) \right| B_{\text{REN}} \right\rangle$$
Meson Mass Equation

- The meson mass may be derived from the equation:

\[ 1 = \frac{8 \alpha_s C_f}{3\pi^3} \int d^4k \ V_j(k) \cdot \Pi_j(p, k) \cdot V_j(-k), \quad (p^2 = -M_j^2) \]

\[ \Pi_j(k) = -\frac{1}{4!} Tr \left\{ \Gamma_j \tilde{S} \left( \hat{k} + \xi_1 \hat{p} \right) \Gamma_j \tilde{S} \left( \hat{k} - \xi_2 \hat{p} \right) \right\} \]

\[ V_j(k) = \int dx \sqrt{D(x)} \ U_j(x) \ e^{ikx} \]

\[ V_j(-k) = \int dx \sqrt{D(x)} \ U_j(x) \ e^{-ikx} \]
Asymptotical Regge-type behaviour:

\[ M_l^2 \approx M_0^2 + (l + 1) \cdot \text{const} \quad \text{for} \quad l \geq 3 \quad \text{and} \quad J = V \]

Vector mesons are heavier than pseudoscalars at same quark contents:

\[ 1 \approx C_J \cdot \exp(M_J^2) \cdot (M_J^2 + \text{const}) \]
\[ 1 = C_P > C_V = 1 / 2 \]

The coupling is bounded from above:

\[ \alpha_s(M) = 1 / \lambda_J(M^2) \leq \alpha_s^{\text{max}} \]

Finite behaviour of the running coupling at origin:

\[ \alpha_s(0) \leq \alpha_s^{\text{max}} < \infty \]
b) Numerical results for conventional meson masses (in MeV)

\[ J^{PC} = 0^{-+} \]

<table>
<thead>
<tr>
<th>P-mesons</th>
<th>PDG-2010</th>
<th>Our estim.</th>
<th>V-mesons</th>
<th>PDG-2010</th>
<th>Our estim.</th>
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<tbody>
<tr>
<td>D (~uc)</td>
<td>1870</td>
<td>1904</td>
<td>( \rho ) (uu)</td>
<td>770</td>
<td>770</td>
</tr>
<tr>
<td>( D_s ) (~sc)</td>
<td>1970</td>
<td>1970</td>
<td>( K^* ) (us)</td>
<td>892</td>
<td>893</td>
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<tr>
<td>( \eta_c ) (~cc)</td>
<td>2980</td>
<td>3049</td>
<td>( \overline{D}^* ) (uc)</td>
<td>2010</td>
<td>2025</td>
</tr>
<tr>
<td>B (~ub)</td>
<td>5279</td>
<td>5135</td>
<td>( \overline{D}_s^* ) (~sc)</td>
<td>2112</td>
<td>2097</td>
</tr>
<tr>
<td>( B_s ) (~sb)</td>
<td>5370</td>
<td>5255</td>
<td>( J/\psi ) (~cc)</td>
<td>3097</td>
<td>3097</td>
</tr>
<tr>
<td>( B_c ) (~cb)</td>
<td>6286</td>
<td>6246</td>
<td>( B^* ) (~ub)</td>
<td>5325</td>
<td>5185</td>
</tr>
<tr>
<td>( \eta_b ) (~bb)</td>
<td>9389</td>
<td>9391</td>
<td>( Y ) (~bb)</td>
<td>9460</td>
<td>9460</td>
</tr>
</tbody>
</table>

Model parameters (in MeV):

\[ \Lambda \approx 226 \]

\[ m_{ud} \approx 115.3 \quad m_s \approx 392.3 \]

\[ m_c \approx 1529.8 \quad m_b \approx 4733.7 \]
Conventional Meson Mass Estimates

- Our estimates
- PDG-2011 data

|relative errors| < 2.4%

Graph showing the masses of vector and pseudoscalar mesons, with markers for different mesons and the PDG-2011 data.
Predicted Behaviors of QCD Running Coupling

\[ \alpha_s(M) \] vs. \( M \) (in MeV)

\[ \alpha_s(Q) \] vs. \( Q \) [GeV]

QCD \( \alpha_s(M_Z) = 0.1184 \pm 0.0007 \)

July 2009
● We revealed a new analytic IR-fixed point:

\[
\alpha_s(0) = \alpha_s^0 = \frac{3\pi}{16 \cdot \ln(2)} \approx 0.85 \quad \rightarrow \quad \alpha_s^0 / \pi = \frac{3}{16 \cdot \ln(2)} \approx 0.27
\]

● Our result is in reasonable agreement with often-quoted estimates:

\[
\begin{align*}
\alpha_s^0 / \pi &= 0.19 - 0.25 \quad [S.\text{Godfrey} \ 1985], \\
\alpha_s^0 / \pi &= 0.265 \quad [T.\text{Zhang} \ 1991], \\
\alpha_s^0 / \pi &= 0.26 \quad [F.\text{Halzen} \ 1993], \\
\left< \alpha_s^0 / \pi \right>_{1\text{GeV}} &= 0.2 \quad [M.\text{Baldicchi} \ 2008]
\end{align*}
\]
Our estimate for $\alpha_s$ compared with other predictions:

- PT (dotted), [M. Baldicci et al. 2008]
- 3-loop analytic coupling (solid)
- Massive 1-loop analytic (dashed)
Glueballs

**Theoretical status:** The existence of glueballs is predicted by QCD because of the self-interaction of gluons. The lightest glueball should be a scalar particle.

- Lattice calculations,
- QCD sum rules,
- Tube model,
- Constituent glue model
- Holographic QCD model

**Experimental status:** Signatures naively expected for glueballs:
- no place in (q-qbar) nonets,
- enhanced production in gluon-rich channels of radiative decays,
- decay branching fractions incompatible with (q-qbar) states,

**Predictions:** expecting glueballs and multiquark states in the mass range:

\[ M_G \approx 1.2 - 1.8 \text{ GeV} \]

\[ J^{PC} = 0^{++} \]
**Scalar (Lowest State) Glueball**

\[ J^{PC} = 0^{++} \]

- Isolate color-singlet term in the di-gluon current:

\[
t_{ik}t_{jl} = \frac{N_c^2 - 1}{2N_c^2} \delta^{il}\delta^{jk} - \frac{1}{N_c} t_{il} t_{jk}
\]

\[
f^{ABE} f^{A'B'E} = \frac{2}{3} (\delta^{AA'} \delta^{BB'} - \delta^{AB'} \delta^{BA'}) + d^{AA'E} d^{BB'E} - d^{AB'E} d^{BA'E}
\]

- The lowest-state glueball mass is defined from:

\[
1 - \frac{\pi^2 g^2}{8} \int dz e^{izp} \Pi(z) = 0, \quad -p^2 = M_G^2
\]

\[
\Pi(z) \equiv \int\int dx dy \, U(x) \sqrt{W(x)} \, D \left( z + \frac{x + y}{2} \right) D \left( z - \frac{x + y}{2} \right) \sqrt{W(y)} U(y)
\]
the estimated value of scalar glueball mass:

\[ M_G \approx 1792 \pm 35 \text{ MeV} \]

Compared with often quoted predictions:

<table>
<thead>
<tr>
<th>Mass (MeV)</th>
<th>Source, Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1665</td>
<td>Quenched lattice QCD (D.Weingarten 1999)</td>
</tr>
<tr>
<td>1750±130</td>
<td>Quenched lattice QCD (C.Morningstar, M.Peardon 1999)</td>
</tr>
<tr>
<td>1490</td>
<td>Anisotropic QLQCD (N.Ishii et al. 2006)</td>
</tr>
<tr>
<td>1710±50±58</td>
<td>QLQCD Infinite volume, continuum limit (Y.Chen 2006)</td>
</tr>
<tr>
<td>1670±90</td>
<td>Analytic confinement (G.Ganbold PRD79, 2009)</td>
</tr>
<tr>
<td>1790±60</td>
<td>UKQCD Collaboration (2011)</td>
</tr>
</tbody>
</table>

the scalar glueball size - ‘radius’:

\[
r_G \approx \frac{1}{\Lambda} \left( \frac{\int d^4 x \cdot x^2 \cdot W(x)}{\int d^4 x \cdot W(x)} \right)^{1/2} = \frac{\sqrt{2}}{\Lambda} \approx \frac{1}{160 \text{ MeV}} \sim 1.25 \text{ fm}
\]
Summary:

- The conventional meson spectrum and the lowest-state glueball may be reasonably described in the framework of a simple relativistic quantum-field model of quark-gluon interaction based on infrared confinement.

- The behavior of the QCD running coupling in the low-energy region may be explained reasonably by fitting the known meson masses.

- Our approach exhibits a new, independent, and specific IR-finite behavior of QCD coupling.

- The model is able to address simultaneously different sections of the low-energy particle physics. Consideration can be extended to other problems (exotic mesons, other glueballs, baryons, multiquark and mixed states, hadron decay processes, …etc.).
The speaker’s attendance at this conference was sponsored by the Alexander von Humboldt Foundation.

- http://www.humboldt-foundation.de