Going beyond the propagators of Landau gauge Yang-Mills theory

Markus Q. Huber, Lorenz von Smekal

Institute of Nuclear Physics, Technical University Darmstadt

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$X_{th}$ Confinement and the Hadron Spectrum
Non-perturbative Landau gauge Green functions

Non-perturbative propagators of Landau gauge Yang-Mills theory:

- information about confinement
- input for phenomenological calculations (QCD phase diagram, bound states, ...) → sessions B, D
- QCD phase diagram:
  phase transitions (e.g. via condensates, Polyakov loop),
  no sign problem for functional methods, but equations get more complicated;
  also vertices change, e.g., ghost-gluon vertex at non-zero temperature [Fister, Pawlowski, 1112.5440]

Calculated with

- Monte-Carlo simulations,
- functional renormalization group,
- Dyson-Schwinger equations,
- ...
Dyson-Schwinger equations (DSEs) of gluon and ghost propagators:

\[
D_{gl}(p) = \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}, \quad D_{gh} = -\frac{G(p^2)}{p^2}
\]

- Equations of motion of correlation functions.
- Infinite tower of coupled integral equations.
- Contain three-point and four-point functions:
  - ghost-gluon vertex, three-gluon vertex, four-gluon vertex
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Truncated propagator Dyson-Schwinger equations

Standard truncation:

\[
\begin{align*}
\text{using bare ghost-gluon vertex and three-gluon vertex model}
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Ghost-gluon vertex

Various lattice results [Cucchieri, Maas, Mendes, PRD77; Ilgenfritz, BJP37]
OPE analysis [Boucaud et al., JHEP 1112]
Modeling via ghost DSE [Dudal, Oliveira, Rodriguez-Quintero, 1207.5118]
Semi-perturbative DSE analysis [Schleifenbaum et al., PRD72]
Ghost-gluon vertex

\[ \Gamma_{\mu}^{A\bar{c}c,abc}(k; p, q) := ig f^{abc} (p_{\mu} A(k; p, q) + k_{\mu} B(k; p, q)) \]

Note:

\[ B(k; p, q) \] is irrelevant in Landau gauge, but it is not the pure longitudinal part.

IR and UV consistent truncation:
Solutions of DSEs: Decoupling and scaling

- Two types of solutions with functional methods that differ only in deep IR [Boucaud et al., JHEP 0806, 012; Fischer, Maas, Pawlowski, AP 324]:
  - scaling [von Smekal, Alkofer, Hauck PRL97],
  - decoupling [Aguilar, Binosi, Papavassiliou PRD78]

- Lattice calculations find only decoupling type solution for $d = 3, 4$ and scaling for $d = 2$

$$d = 2$: Analytic and numerical arguments from DSEs for scaling only [Cucchieri, Dudal, Vandersickel, PRD85; MQH, Maas, von Smekal, 1207.0222] as well as from analysis of Gribov region [Zwanziger, 1209.1974].
Ghost-loop-only truncation

**Scaling solution:** Ghost loop is IR dominant by power counting [von Smekal, Hauck, Alkofer, 79; Alkofer, Fischer, Llanes-Estrada, PLB611]

**Decoupling solution:** No ghosts $\rightarrow$ no gluon "mass" [Aguilar, Binosi, Papavassiliou, JHEP1201];

ghost loop alone, i.e., $\Gamma^{A^3} = 0$?

\[ i_1 i_2 - 1 = + i_1 i_2 - 1 + i_1 i_2 \]

\[ \begin{array}{c|c|c}
-4 & 0.01 & 1 \\
100 & 10 & 10^4
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
0.0 & 0.5 & 1.0 & 1.5 & 2.0
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
Z H p^2 & L L
\end{array} \]

$\Rightarrow$ Ghost loop alone sufficient to obtain decoupling behavior!

Note: Gluon loop only $\rightarrow$ Mandelstam solution, IR divergent gluon propagator [Mandelstam PRD20]
**Ghost-loop-only truncation**

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Three-gluon vertex: Ultraviolet

Ansatz that reproduces the correct UV behavior of the gluon propagator [Fischer, Alkofer, Reinhardt, PRD65]:

\[
D^A^3(x, y, z) = \frac{1}{Z_1} \frac{[G(y)G(z)]^{1-a/\delta-2a}}{[Z(y)Z(z)]^{1+a}}
\]

\(y\) and \(z\) are the momenta in the gluon loop, i.e., not Bose symmetric in \(y\), \(z\) and \(x\).
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Bose symmetrized version:

$$D^{A^3, UV}(x, y, z) = \frac{1}{Z_1} G(x + y + z)^\alpha Z(x + y + z)^\beta$$

Fix $\alpha$ and $\beta$:

1. UV behavior of gluon propagator
2. IR behavior of three-gluon vertex?
Three-gluon vertex: Ultraviolet

Ansatz that reproduces the correct UV behavior of the gluon propagator [Fischer, Alkofer, Reinhardt, PRD65]:

\[ D^A_3(x, y, z) = \frac{1}{Z_1} \frac{[G(y)G(z)]^{1-a/\delta-2a}}{[Z(y)Z(z)]^{1+a}} \]

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Fix \(\alpha\) and \(\beta\):

1. UV behavior of gluon propagator
2. IR behavior of three-gluon vertex → yes, but . . .
Three-gluon vertex: Infrared

Hints from lattice data [Cucchieri, Maas, Mendes, PRD77]:

Three-gluon vertex might have a zero crossing.

\((d = 2, 3: \text{zero crossing seen} \ [\text{Cucchieri, Maas, Mendes, PRD77; Maas, PRD75}])\)
Three-gluon vertex: Infrared

Hints from lattice data [Cucchieri, Maas, Mendes, PRD77]:
Three-gluon vertex might have a zero crossing.
($d = 2, 3$: zero crossing seen [Cucchieri, Maas, Mendes, PRD77; Maas, PRD75])

⇒ add IR part:

$$D^{A^3,IR}(x, y, z) = h_{IR} G(x + y + z)^3 (f^{3g}(x)f^{3g}(y)f^{3g}(z))^4$$

IR damping function $f^{3g}(x) := \frac{\Lambda_{3g}^2}{\Lambda_{3g}^2 + x}$

Parameter for zero crossing: $h_{IR} < 0$

New three-gluon vertex:

$$D^{A^3}(x, y, z) = D^{A^3,IR}(x, y, z) + D^{A^3,UV}(x, y, z)$$
Propagators: Improving the three-gluon vertex

ghost-gluon vertex: bare

original three-gluon vertex
Propagators: Improving the three-gluon vertex

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Bose symmetric three-gluon vertex
Propagators: Improving the three-gluon vertex

ghost-gluon vertex: bare

original three-gluon vertex
Bose symmetric three-gluon vertex
Bose symmetric three-gluon vertex with IR part

⇒ Improved three-gluon vertex adds additional strength in the mid-momentum regime.
Ghost-gluon vertex

\[ \Gamma^{A\bar{c}c,abc}_\mu (k; p, q) := ig f^{abc} (p_\mu A(k; p, q) + k_\mu B(k; p, q)) \]

basis choice: \( p^2, k^2, \varphi \)
Ghost-gluon vertex

$$\Gamma_{\mu}^{\bar{c}c,abc}(k;p,q) := igf^{abc}(p_{\mu}A(k;p,q) + k_{\mu}B(k;p,q))$$

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basis choice: \( p^2, k^2, \phi \)
**Ghost-gluon vertex: Selected configurations**

**Fixed angle:**

\[ \cos(H_jL) = 0.111 \]

**Fixed momentum:**

\[ p^2 = 0.00681292 \ g^2 \]

Results for \( SU(2) \)
Ghost-gluon vertex: Comparison with lattice data

Symmetric configuration $k^2 = q^2 = p^2$:

\[ \Gamma^{A \bar{c} c}(p^2, p^2, p^2) \]

Orthogonal configuration $k^2 = 0, q^2 = p^2$:

\[ \Gamma^{A \bar{c} c}(0, p^2, p^2) \]

lattice data: [Cucchieri, Maas, Mendes, PRD77]

Results for $SU(2)$
Propagators and dynamic ghost-gluon vertex

Results for $SU(3)$

bare ghost-gluon vertex, Bose non-symmetric three-gluon vertex

lattice: $32^4$
lattice: $48^4$ [Sternbeck, hep-lat/0609016]
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bare ghost-gluon vertex,
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Results for $SU(3)$
Propagators and dynamic ghost-gluon vertex

- More realistic three-gluon vertex improves mid-momentum behavior.
- Dynamic ghost-gluon vertex influences ghost in IR
  Note: Solution with dynamic ghost-gluon vertex requires improved three-gluon vertex for stability (gluon loop otherwise too strong).
- Missing part around 2 GeV probably due to two-loop terms
  → talk of Valentin Mader
Summary & Conclusions

- **Ghost dominance** for decoupling and scaling solution (ghost-loop-only analysis).

- More realistic three-gluon vertex:
  - Bose symmetric and IR features from lattice
    - Yields more support in mid-momentum regime.
    - Reduces gap to lattice results.

- Dynamic inclusion of ghost-gluon vertex:
  - rather small changes for gluon
  - influences ghost in the IR
  - improved three-gluon vertex required

- Ghost-gluon vertex results agree well with available lattice results.
Inclusion of three-point functions:
- test of truncations,
- required for quantitative results and
- likely also for some aspects of non-zero temperature and density calculations

Automatization tools:
- **DoFun** [Alkofer, MQH, Schwenzer, CPC180; MQH, Braun, CPC183]
- **CrasyDSE** [MQH, Mitter, CPC183]
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Thank you very much for your attention.