Topological and magnetic properties of the QCD vacuum probed by overlap fermions

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QCD vacuum

\[ G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \]

\[ \rho_R \neq \rho_L \]

Positive topological charge density

Negative topological charge density
Huge electromagnetic fields, never observed before!

Black curves are from W.-T. Deng and X.-G. PRC 85, 044907
Visible effects

Left-handed

Right-handed
Visible effects

- Spins parallel to $\vec{B}$

Left-handed

Right-handed
Visible effects

Left-handed

Right-handed

- Spins parallel to B
- Momenta antiparallel
Visible effects

- Spins parallel to $B$
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to $B$

Left-handed

Right-handed

Kharzeev, McLerran, Warringa (2007)
Our task

- Chiral condensate in magnetic fields
- Magnetization of the QCD vacuum
- Imbalance between left-/right-handed quarks
- Fluctuations of the electric current
- Conductivity of the vacuum
- Dimensionality of the topological structures
- Resolution-dependent observables
Step 1: Lattice action

\[ S = -\beta \sum_{x, \mu > \nu} \left\{ \frac{5}{3} \frac{P_{\mu \nu}}{u_0^4} - r_g \frac{R_{\mu \nu} + R_{\nu \mu}}{12 u_0^6} \right\} + c_g \beta \sum_{x, \mu > \nu > \sigma} \frac{C_{\mu \nu \sigma}}{u_0^6} \]

\[ P_{\mu \nu} = \frac{1}{3} \text{Re } \text{Tr} \]

\[ R_{\mu \nu} = \frac{1}{3} \text{Re } \text{Tr} \]

\[ C_{\mu \nu \sigma} = \frac{1}{3} \text{Re } \text{Tr} \]

\[ r_g = 1 + 0.48 \alpha_s(\pi/a) \]

\[ c_g = 0.055 \alpha_s(\pi/a) \]

Lüscher and Weisz (1985), see also Lepage hep-lat/9607076
Step 2: Monte Carlo

• Heat bath for SU(2)

• Use the standard algorithm for each subgroup of SU(3). Cabibbo & Marinari (1982)

\[ a_1 = \begin{pmatrix} \alpha_1 \\ 1 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 \\ \alpha_2 \end{pmatrix}, \quad a_3 = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \]

• Overrelaxation. Adler (1981)

• Cooling (smearing) for some particular cases. DeGrand, Hasenfratz, Kovács (1997)
Step 3: Fermions & B-field

\[
D_{ov} = \frac{1}{a} \left( 1 - \frac{A}{\sqrt{A^\dagger A}} \right)
\]

\[
A = 1 - a \, D_W(0)
\]

Neuberger overlap operator (1998)

\[
\langle \bar{\Psi} \hat{\Gamma} \Psi \rangle \sim \text{Tr} \left[ \hat{\Gamma} \, D_{ov}^{-1} \right]
\]

\[
\hat{\Gamma} \in \{1, \gamma^5, \gamma^\mu, \sigma_{\mu\nu}, \ldots\}
\]

Buividovich, Chernodub, Luschevskaya, Polikarpov (2009)
Chiral condensate

\[ \Sigma_{\text{fit}} = \Sigma_0 \left[ 1 + \left( \frac{eB}{\Lambda_B^2} \right)^{\nu} \right] \]

$14^4$, $T < T_c$, $a = 0.105$ fm

Power-function fit
Chiral condensate

\[ \Sigma_{\text{fit}} = \Sigma_0 \left[ 1 + \left( \frac{eB}{\Lambda_B^2} \right)^\nu \right] \]

\[ \Sigma_0 = [230 \pm 5 \text{ MeV}]^3 \]

\[ \nu = 1.57 \pm 0.23 \]
\[ \rho_5 = \bar{\Psi} \gamma^5 \Psi \]

\[ j^\mu = \bar{\Psi} \gamma^\mu \Psi \]

\[ \langle \bar{\Psi} \sigma_{12} \Psi \rangle = \mu_z (qB) \langle \bar{\Psi} \Psi \rangle \]

\[ \langle \bar{\Psi} \sigma_{03} \Psi \rangle = \epsilon_z (qB) \langle \bar{\Psi} \Psi \rangle \]
CONFINEMENT

\[ G_{i,j}(\tau) = \int d^3 \vec{x} \langle J_i(\vec{0}, 0) J_j(\vec{x}, \tau) \rangle \]

DECONFINEMENT (T=350 MeV)
... and its spectral function

Conductivity

Meson masses

\[
G_{ij} = \int_0^\infty \frac{d\omega}{2\pi} \left[ \frac{\omega}{2T} \cosh(\omega(\tau - \frac{1}{2T})) \right] \rho_{ij}(\omega)
\]
Electrical conductivity

\[ \sigma_{ij} = \lim_{\omega \to 0} \frac{\rho_{ij}(\omega)}{4T} \]

What does it mean?

- There are similar effects for $T > T_c$ and thus the local CP-violation is present in the both confinement and deconfinement phases.
- Above $T_c$ vacuum is a conductor.
- Below $T_c$ vacuum is either an insulator (for $B = 0$) or an anisotropic conductor (for strong $B$).
- $\langle j^2_\mu \rangle \neq 0$ might be an evidence of a macroscopic current.
- More in-plane dileptons (i.e. $\perp \vec{B}$).
Where is it localized?

Negative topological charge density

Positive topological charge density
Inverse Participation Ratio

Observables:

\[ \rho_\lambda(x) = \psi^*_\lambda^\alpha(x) \psi_{\lambda \alpha}(x) \quad \text{"Chiral condensate" for eigenvalue } \lambda \]

\[ \rho^5_\lambda(x) = \left(1 - \frac{\lambda}{2}\right) \psi^*_\lambda^\alpha(x) \gamma^5_{\alpha \beta} \psi_{\lambda \beta}(x) \quad \text{"Chirality" = Topological charge density} \]

Inverse Participation Ratio (inverse volume of the distribution):

\[ \text{IPR} = N \sum_x \rho_i^2(x) \]

\[ \sum_x \rho_i(x) = 1 \]

Unlocalized: \( \rho(x) = \text{const}, \quad \text{IPR} = 1 \)
Localized on a site: \( \text{IPR} = N \)
Localized on fraction \( f \) of sites: \( \text{IPR} = 1/ f \)

Fractal dimension (performing a number of measurements with various lattice spacings):

\[ \text{IPR}(a) = \frac{\text{const}}{a^d} \]
Localization of zero-modes

Definition:

$$\text{IPR}_0 = N \left[ \frac{\sum_x (\rho_0(x))^2}{\left( \sum_x \rho_0(x) \right)^2} \right]_{\lambda=0}$$

$$\rho_\lambda(x) = \psi_\lambda^\ast(x) \psi_{\lambda\alpha}(x)$$

$$\rho_5(x) = \left( 1 - \frac{\lambda}{2} \right) \psi_\lambda^\ast(x) \gamma_5 \psi_\lambda^\beta(x)$$
Topological charge density

Definition 1:

\[
IPR_0^5 = N \left[ \frac{\sum_x |\rho_0^5(x)|^2}{\left(\sum_x |\rho_0^5(x)|\right)^2} \right]^{\lambda},
\]

Definition 2:

\[
IPR_0^5 = N \left[ \frac{\sum_x (\rho_0^5(x))^2}{\left(\sum_x \rho_0(x)\right)^2} \right]^{\lambda}
\]

\[\lambda = 0\]

\[\lambda \neq 0\]
Fractal dimension

Our result: $d = 2 \div 3$
and after cooling $d \sim 4$

$d = 1$: monopoles
$d = 2$: vortices
$d = 3$: domain walls
$d = 4$: instantons

On the low-dimensional defects in QCD see also
[hep-ph/0410034]
Thank you for the attention!

and

Have a good time!