Chiral Magnetic Effect and Holography

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Based on work with
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Kalaydzhyan, I.K., Phys. Rev. Lett. 106 (2011) 211601,
Gahramanov, Kalaydzhyan, I.K., PRD 85 (2012) 126013

Xth Quark Confinement and the Hadron Spectrum,
TUM Department of Physics, 7-12 October 2012
Motivation: Observed charge asymmetry in HICs

Heavy-ion collision:

Excess of positive charge (above $\Psi_R$)

Excess of negative charge (below $\Psi_R$)
Chiral magnetic effect

1. In presence of a magnetic field $B$, momenta of the quarks align along $B$

2. Topological charge induces chirality

3. Positively/negatively charged quarks move up/down (charge separation!)

4. An electric current is induced along the magnetic field $B$

$$\vec{J} = C \mu_5 \vec{B}$$

CME is a candidate for explaining an observed charge asymmetry in HIC

Does the CME hold at strong coupling?

$$\Rightarrow \text{AdS/CFT}$$

CME in HICs:


$$\Delta N_5 \equiv (N_L - N_R)|_\infty - (N_L - N_R)|_{-\infty} = 2N_fQ$$

$$Q = \frac{g^2}{32\pi^2} \int d^4xF^{\alpha\nu}_\mu \tilde{F}^{\mu\nu}_\alpha$$

[in fig.: $[0-4] - [2-2] = 2 \cdot 2 \cdot (-1)$]
Overview

Outline:

I. CME in hydrodynamics
II. Fluid/gravity model of the CME
III. CME in anisotropic fluids
   \((v_2\text{-dependence})\)

Conclusions
Part I: CME in hydrodynamics
Hydrodynamics vs. fluid/gravity model

Hydrodynamics

- Multiple-charge model
  - $U(1)^n$ plasma with triangle anomalies
  - Son & Surowka (2009), Neiman & Oz (2010)

  $n=2$

- Two-charge model
  - $U(1)_V \times U(1)_A$ plasma

  recover CME (and other effects)

Fluid/gravity model

- Holographic $n$-charge model
  - 5D AdS black hole geometry with $n$ U(1) charges

- Holographic two-charge model
  - $n$-charge model reduced to two charges

  recover holographic CME, etc.
Hydrodynamical model with $n$ anomalous U(1) charges

$U(1)^n$ plasma with triangle anomalies:

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda \quad (a = 1, 2, \ldots, n)$$

$$\partial_\mu j^a_\mu = C^{abc} E^b \cdot B^c$$

stress-energy tensor $T^{\mu\nu}$ and U(1) currents $j^{a\mu}$:

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + Pg^{\mu\nu} + \ldots$$

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \ldots$$

“New” transport coefficients (not listed in Landau-Lifshitz)

- vortical conductivities $\xi^a_\omega$  
  Erdmenger, Haack, Kaminski, Yarom (2008)

- magnetic conductivities $\xi^{ab}_B$  
  Son & Surowka (2009)

first found in a holographic context (AdS/CFT)

Son & Surowka (2009)

E- & B-fields, vorticity:

$$E^{a\mu} = F^{a\mu\nu} u_\nu$$

$$B^{a\mu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F^{a}_{\alpha\beta}$$

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$
First-order transport coefficients

\[ j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^a_{B} B^{b\mu} + \ldots \]

vortical and magn. conductivities:

\[ \xi^a_\omega = C^{abc} \mu^b \mu^c - \frac{2}{3} \rho^a C^{bcd} \frac{\mu^b \mu^c \mu^d}{\epsilon + P} + \mathcal{O}(T^2) \]

\[ \xi^{ab}_B = C^{abc} \mu^c - \frac{1}{2} \rho^a C^{bcd} \frac{\mu^c \mu^d}{\epsilon + P} + \mathcal{O}(T^2) \]

chemical potentials

Conductivities are non-zero only in fluids with triangle anomalies!

Son & Surowka (2009), Neiman & Oz (2010)
Two charge case (n=2): $U(1)_A \times U(1)_V$

$U(1)_A$: provides chemical potential $\mu_5$ (chirality)
$U(1)_V$: measures the electric current

Hydrodynamical equations:

\[
\partial_\mu T^{\mu\nu} = F^{V\nu\lambda} j_\lambda \\
\partial_\mu j_5^{\mu} = -\frac{C}{4} F^{\nu\mu} \tilde{F}^{\nu\mu\nu} \\
\partial_\mu j^{\mu} = 0
\]

axial gauge field switched off! ($A^A_\mu = 0$)

Constitutive equations:

\[
j^{\mu} = \rho u^{\mu} + \kappa_\omega \omega^{\mu} + \kappa_B B^{\mu} \\
j_5^{\mu} = \rho_5 u^{\mu} + \xi_\omega \omega^{\mu} + \xi_B B^{\mu}
\]

CME coefficient

CME coefficient

Identifications:

\[
A^A_\mu = A^1_\mu, \quad A^V_\mu = A^2_\mu, \\
j_5^{\mu} = j^{1\mu}, \quad j^{\mu} = j^{2\mu} \\
\mu_5 = \mu^1, \quad \mu = \mu^2, \\
\rho_5 = \rho^1, \quad \rho = \rho^2, \\
\kappa_\omega = \xi_\omega^2, \quad \kappa_B = \xi_B^{22}, \\
\xi_\omega = \xi_\omega^1, \quad \xi_B = \xi_B^{12}
\]

C-parity allows for:

\[
C^{111} = C/3 \quad (AAA) \\
C^{122} = C^{221} = C^{212} = C/3 \quad (AVV) \\
C^{121} = C^{211} = C^{112} = 0 \quad (VAA) \\
C^{222} = 0 \quad (VVV)
\]
(Chiral) magnetic and vortical effects

constitutive equations:

\[ j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu \]
\[ j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu \]

transport coefficients (conductivities):

\begin{align*}
\text{CVE} & : \quad \kappa_\omega / \mu = 2 C \mu_5 \left( 1 - \frac{\mu \rho}{\epsilon + P} \left[ 1 + \frac{\mu_5^2}{3 \mu^2} \right] \right) , \quad \kappa_B = C \mu_5 \left( 1 - \frac{\mu \rho}{\epsilon + P} \right) \\
\text{QVE} & : \quad \xi_\omega / \mu = C \mu \left( 1 - 2 \frac{\mu_5 \rho_5}{\epsilon + P} \left[ 1 + \frac{\mu_5^2}{3 \mu^2} \right] \right) , \quad \xi_B = C \mu \left( 1 - \frac{\mu_5 \rho_5}{\epsilon + P} \right)
\end{align*}

C=chiral since CVE, CME prop. to chiral chemical potential \( \mu_5 \)
Q=quark since QVE, QME prop. to quark chemical potential \( \mu \)
(QME also called chiral separation effect (CSE))

\( \mu \omega \) creates an effective magnetic field *Kharzeev and Son (2010)*
Part II: Fluid/gravity model of the CME
Kalaydzhyan & I.K., PRL 106 (2011) 211601
Hydrodynamics vs. fluid/gravity model

Hydrodynamics

Multiple-charge model
$U(1)^n$ plasma with triangle anomalies
Son & Surowka (2009)
Neiman & Oz (2010)

$n=2$

Two-charge model
$U(1)_V \times U(1)_A$ plasma
recover CME (and other effects)

Fluid/gravity model

Holographic $n$-charge model
5D AdS black hole geometry with
$n$ U(1) charges

$n=2$

Holographic two-charge model
$n$-charge model reduced to two charges
recover holographic CME, etc.
Gravity: Holographic computation

Strategy: quark-gluon plasma is strongly-coupled $\Rightarrow$ use AdS/CFT to compute the transport coefficients relevant for the anomalous effects (CME, etc.)

- find a 5d charged AdS black hole solution with several $U(1)$ charges

\[ U(1)_V \times U(1)_A \]

- duality:

\[
(m, q_a) \longleftrightarrow (T, \mu_a)
\]

mass $m$
U(1) charges $q_a$

\[ T \sim r_+ \quad \text{Hawking temperature} \]
\[ \mu^a \equiv A^a_0(r_+) - A^a_0(\infty) \]

- use fluid-gravity methods to holographically compute the transport coefficients $\kappa_\omega, \kappa_B, \xi_\omega, \xi_B$ (i.e. CME and other effects)
AdS black hole solution with multiple U(1) charges

Five-dimensional $U(1)^n$ Einstein-Maxwell theory with cosmological term:

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R + 12 - F_{MN}^a F^{aMN} + \frac{S_{abc}}{6\sqrt{-g}} \varepsilon^{PKLMN} A_P^a F_{KL}^b F_{MN}^c \right]$$

**Fields:**
- metric $g_{MN}$ ($M, N = 0, ..., 4$)
- $n$ U(1) gauge fields $A_M^a$
  ($a = 1, ..., n$)
- cosmological constant $\Lambda = -6$

The information of the anomalies is encoded in the Chern-Simons coefficients:

$$S_{abc} = 4\pi G_5 C_{abc}$$

Son & Surowka (2009)
Boosted AdS black hole solution

5d AdS black hole solution (0\textsuperscript{th} order solution):

\[ ds^2 = -f(r)u_\mu u_\nu dx^\mu dx^\nu - 2u_\mu dx^\mu dr + r^2 (\eta_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu \]

\[ A^a = (A^a_0(r) u_\mu + A^a_\mu) dx^\mu \]

with

\[ f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q_a)^2}{r^4} \]

\[ A^a_0(r) = -\frac{\sqrt{3}q^a}{2r^2} \]

\( u_\mu \) = four-velocity of the fluid
First-order transport coefficients

We use the standard procedure of Bhattacharyya et al. (2008) to holographically compute the transport coefficients $\xi^a_\omega$ (Torabian & Yee (2009) for $n=3$) and $\xi^{ab}_B$:

1. Vary 4-velocity and background fields (up to first order):

   \[
   u_\mu = (-1, x^\nu \partial_\nu u_i), \quad A^a_\mu = (0, x^\nu \partial_\nu A^a_i)
   \]

   The boosted black-brane solution ($0^{th}$ order sol.) is no longer an exact solution, but receives higher-order corrections.

   Ansatz for first-order corrections:

   \[
   ds^2 = (-f(r) + \tilde{g}_{tt}) dt^2 + 2 (1 + \tilde{g}_{tr}) dt dr + r^2 (dx^i)^2 + \tilde{g}_{ij} dx^i dx^j - 2 x^\nu \partial_\nu u_i dr dx^i \\
   + 2 \left( (f(r) - r^2) x^\nu \partial_\nu u_i + \tilde{g}_{ti} \right) dt dx^i,
   \]

   \[
   A^a = \left(-A^a_0(r) + \tilde{A}_t^a\right) dt + \left(A^a_0(r)x^\nu \partial_\nu u_i + x^\nu \partial_\nu A^a_i + \tilde{A}_i^a\right) dx^i
   \]

   Need to determine first order corrections $\tilde{g}_{tt}, \tilde{g}_{tr}, etc.$
First-order transport coefficients (cont.)

2. Solve equations of motion (system of Einstein-Maxwell equations) and find the first-order corrections to the metric and gauge fields:

\[ \bar{g}_{\tau r} = g_{\tau t} = \bar{A}_i^a = 0 \]
\[ \bar{g}_{\tau i}(r) = f(r) \int_{r_+}^{r} dr' \frac{1}{r'(f(r'))^2} \left( \int_{r_+}^{r'} dr'' I(r'') - r'_+ f'(r_+) C_i \right) \]
\[ \bar{A}_i^a(r) = \int_{r_+}^{r} dr' \frac{1}{r' f(r')} [Q_i^a(r') - Q_i^a(r_H) - C_i r + A_0^a(r_+) + r'_+ \bar{g}_{\tau i}(r') A_0^a(r')] \]

(lengthy calculation, \( I(r) \), \( Q_i^a(r) \), \( C_i \) functions of \( A_0^a(r) \), \( f(r) \), \( u_i \))

3. Read off energy-momentum tensor and \( \text{U}(1) \) currents from the near-boundary expansion of the first-order corrected background (e.g. Fefferman-Graham coordinates):

\[ T_{\mu \nu} = \frac{g^{(4)}_{\mu \nu}(x)}{4 \pi G_5} + c.t \]
\[ j_\mu^a = \frac{\eta^{\mu \nu} A^{(2)}_{a \nu}(x)}{8 \pi G_5} + \hat{j}_\mu^a \]
\[ ds^2 = \frac{1}{z^2} \left( g_{\mu \nu}(z, x) \, dx^\mu dx^\nu + dz^2 \right) , \]
\[ g_{\mu \nu}(z, x) = \eta_{\mu \nu} + g^{(2)}_{\mu \nu}(x) \, z^2 + g^{(4)}_{\mu \nu}(x) \, z^4 + ... \]
\[ A_{\mu}^a(z, x) = A_{\mu}^{a(0)}(x) + A_{\mu}^{a(2)}(x) \, z^2 + ... \]
First-order transport coefficients (cont.)

4. Determine the vortical and magnetic conductivities $\xi^a_\omega$ and $\xi^{ab}_B$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} + \ldots$$

$$j^{a\mu} = \rho^a u^\mu + \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \ldots$$

use identities (from zeroth order solution):

$$P \equiv m/16\pi G_5$$

$$\rho_a \equiv \sqrt{3} q_a/16\pi G_5$$

$$\Rightarrow \frac{\sqrt{3} q^a}{4m} = \frac{\rho^a}{\epsilon + P} \quad (\epsilon = 3P)$$

transport coefficients:

$$\xi^a_\omega = \frac{4}{16\pi G_5} \left( S^{abc} \mu^b \mu^c - \frac{2}{3} \frac{\rho^a}{\epsilon + P} S^{bcd} \mu^b \mu^c \mu^d \right)$$

$$\xi^{ab}_B = \frac{4}{16\pi G_5} \left( S^{abc} \mu^c - \frac{1}{2} \frac{\rho^a}{\epsilon + P} S^{bcd} \mu^c \mu^d \right)$$

$$\mu^a \equiv A_0^a(r_H) - A_0^a(\infty)$$

$$S_{abc} = 4\pi G_5 C_{abc} \quad \Rightarrow \quad \text{We recover the hydrodynamic result!}$$
Holographic magnetic and vortical effects

Using the same identifications as in hydrodynamics, but now for the holographically computed transport coefficients, we get

\[
\begin{align*}
\kappa_\omega &= 2C \mu \mu_5 \left( 1 - \mu \frac{\sqrt{3}q}{4m} \left[ 1 + \frac{\mu_5^2}{3\mu^2} \right] \right), \\
\kappa_B &= C \mu_5 \left( 1 - \mu \frac{\sqrt{3}q}{4m} \right), \\
\xi_\omega &= C \mu^2 \left( 1 - 2\mu_5 \frac{\sqrt{3}q_5}{4m} \left[ 1 + \frac{\mu_5^2}{3\mu^2} \right] \right), \\
\xi_B &= C \mu \left( 1 - \mu_5 \frac{\sqrt{3}q_5}{4m} \right)
\end{align*}
\]

Result: CME, CVE, etc. are realized in an n-charged AdS black hole model (plus background gauge field), when appropriately reduced to a two-charge model (n=2).

Other AdS/QCD models:
Lifschytz and Lippert (2009), Yee (2009),
Part III: CME in anisotropic fluids

Gahramanov, Kalaydzhyan & I.K.,
Possible dependence of the charge asymmetry on $v_2$

Event-by-event anisotropy ($v_2^{\text{obs}}$) dependence (low $p_T$)

Investigate the charge asymmetry as a function of the anisotropy $v_2^{\text{obs}}$ of the measured particles in mid-central 20–40% centrality collisions ($B \approx \text{const.}$). Consider (rare) events with different $v_2^{\text{obs}}$.

Observations:
- same-sign particles are emitted more likely in UD direction the larger $v_2^{\text{obs}}$
- same-sign particles are emitted less likely in LR direction the larger $v_2^{\text{obs}}$ (the dependence is significantly weaker for opposite-sign particles)
- $\Rightarrow$ strong $v_2^{\text{obs}}$ dependence of the difference between UD and LR

$\Rightarrow$ charge separation depends approx. linearly on $v_2^{\text{obs}}$ (apparently in contradiction with the CME)
Build-up of the elliptic flow and momentum anisotropy

Central question: In anisotropic fluids, does the chiral conductivity depend on $\nu_2$?

Sketch of the time-evolution of the momentum anisotropy $\varepsilon_P$:

\[
\varepsilon_P = \frac{P_T - P_L}{P_T + P_L}
\]

at freeze-out:

$\nu_2 \approx \varepsilon_P / 2$

Our model describes a state after thermalization with unequal pressures $P_T \neq P_L$. We do not model the full evolution of $\varepsilon_P$. 

sketch based on Huovinen, Petreczky (2010)
Hydrodynamics of an anisotropic fluid

Anisotropic fluid with *n* anomalous U(1) charges

stress-energy tensor $T^\mu\nu$ and U(1) currents $j^{a\mu}$:

$$T^\mu\nu = (\epsilon + P_T)u^\mu u^\nu + P_T g^{\mu\nu} - (P_T - P_L)v^\mu v^\nu + \tau^{\mu\nu}$$

$$j^{a\mu} = \rho^a u^\mu + \nu^{a\mu}$$

orthogonality and normalization:

$$u_\mu u^\mu = -1, \quad v_\mu v^\mu = 1, \quad u_\mu v^\mu = 0$$

local rest frame:

$$u^\mu = (1, 0, 0, 0) \quad v^\mu = (0, 0, 0, 1)$$

$$T^\mu\nu = \begin{pmatrix}
\epsilon & 0 & 0 & 0 \\
0 & P_T & 0 & 0 \\
0 & 0 & P_T & 0 \\
0 & 0 & 0 & P_L
\end{pmatrix}$$

thermodynamic identity:

$$\epsilon + P_T = Ts + \mu \rho$$
Chiral magnetic and vortical effects

Anisotropic fluid with one axial and one vector U(1)

Repeating the hydrodynamic computation of Son & Surowka (for \( n=2 \)), we find the following result for the chiral magnetic effect:

\[
\Delta j^\mu = \kappa_B B^\mu, \quad \kappa_B = C' \mu_5 \left( 1 - \frac{\mu \rho}{\epsilon + P_T} \right)
\]

or, for small \( \varepsilon_p \),

\[
\kappa_B \approx C' \mu_5 \left( 1 - \frac{\mu \rho}{\epsilon + \bar{P}} \left[ 1 - \frac{\varepsilon_p}{6} \right] \right)
\]

as before \quad \text{anisotropy-dependent}

\( \bar{P} = \frac{2P_T + P_L}{3} \)

multiple charge case (\( n \text{ arb.} \)):

\[
\xi^{ab}_B = C'^{abc} \mu^c - \frac{1}{2} \frac{\rho^a}{\epsilon + P_T} C^{bcd} \mu^c \mu^d
\]
Boosted anisotropic AdS black hole solution (w/ \( n \) U(1)’s)

5d AdS black hole solution (ansatz):

\[
ds^2 = (r^2 w_T(r) P_{\mu\nu} - f(r) u_\mu u_\nu) \, dx^\mu dx^\nu - 2u_\mu dx^\mu dr
- r^2 (w_T(r) - w_L(r)) v_\mu v_\nu \, dx^\mu dx^\nu,
\]

\[
A^a = (A_0^a(r) u_\mu + A_\mu^a) \, dx^\mu
\]

asymptotic solution (close to the boundary):

\[
f(r) = r^2 - \frac{m}{r^2} + \sum_a \frac{(q_a)^2}{r^4} + \mathcal{O}(r^{-6})
\]

\[
w_T(r) = 1 + \frac{m\zeta}{4r^4} + \mathcal{O}(r^{-8})
\]

\[
w_L(r) = 1 - \frac{m\zeta}{2r^4} + \mathcal{O}(r^{-8})
\]

\[
A_0^a(r) = -\frac{\sqrt{3}q_a}{2r^2} + \mathcal{O}(r^{-10})
\]

no analytic solution  \( \Rightarrow \) numerical solution
Numerical solution for the AdS black hole background

Shooting techniques provide numerical plots for the functions $f(r)$, $A_0(r)$, $w_T(r)$, $w_L(r)$ for $\zeta=10$:

outer horizon: $r_+ = 1$
First-order transport coefficients

The holographic computation of the transport coefficients is very similar to that in the isotropic case.

Magnetic conductivities (result):

\[ \xi_{B}^{ab} = \frac{4}{16\pi G_{5}} \left( S^{abc} \mu^{c} - \frac{1}{2} C(r_{+}) S^{bcd} \mu^{c} \mu^{d} \right) \]

\[ \mu^{a} \equiv A_{0}^{a}(r_{H}) - A_{0}^{a}(\infty), \quad S_{abc} = 4\pi G_{5} C_{abc} \]

\[ C(r_{+}) = \frac{r_{+} \partial_{r_{+}} A_{0}^{a}(r_{+})}{r_{+}(f'(r_{+}) - 4 \sum_{a} A_{0}^{a}(r_{+}) \partial_{r_{+}} A_{0}^{a}(r_{+}))} \frac{1}{w_{L}(r_{+})^{1/2}} \]

\[ = \frac{\sqrt{3}}{4m} \frac{1}{1 + \frac{1}{4} \zeta} q_{a} = \frac{\rho^{a}}{\varepsilon + P_{T}} \]

⇒ find agreement with hydrodynamics if the orange factors agree
(needs to be shown numerically)
Numerical agreement with hydrodynamics

Numerical plot $w_L(r_+)$ as a function of the anisotropy:

$$w_L(r_+) = (1 + \frac{1}{4}\zeta)^2$$
Conclusions

I presented two descriptions of the CME (and related effects) in

a) isotropic plasmas \( P = P_T = P_L \):

i) \textbf{hydrodynamic model:} \( \text{U}(1)_A \times \text{U}(1)_V \) fluid with triangle anomaly

ii) \textbf{holographic fluid-gravity model:} 5d AdS-Reissner-Nordstrom-like solution with two \( \text{U}(1) \) charges

Agreement was found between both models.

b) anisotropic plasmas \( P_T \neq P_L \):

- experimental data suggests possible \( v_2 \)– dependence of the charge separation
- Does the chiral magn. conductivity \( \kappa_B \) depend on \( v_2 \)?
- we constructed anisotropic versions of the above \( \text{U}(1)_A \times \text{U}(1)_V \) models and found

\[
\kappa_B \approx C \mu_5 \left( 1 - \frac{\mu \rho}{\epsilon + \tilde{P}} \left[ 1 - \frac{\varepsilon_p}{6} \right] \right) \quad (\tilde{P} = \frac{2P_T + P_L}{3})
\]

Is the observed charge asymmetry a combined effect of the CME (1\(^{\text{st}}\) term in \( \kappa_B \)) and the dynamics of the system (2\(^{\text{nd}}\) term)?