Nuclear Physics from Lattice QCD and EFT

William Detmold
From quarks to nuclei
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
From quarks to nuclei

- Nuclear physics: an emergent phenomenon of the Standard Model
- How exactly do nuclei emerge from the SM? (QCD and EFT for LQCD)
  - Issues
  - Recent progress
  - Future intersections
Quantum chromodynamics

- NP from the SM, but focus on QCD
- Lattice QCD: quarks and gluons
  - Formulate problem as functional integral over gluonic degrees of freedom on $\mathbb{R}^4$
  - Discretise and compactify system
  - Integrate via importance sampling (average over important gluon configs)
  - Undo the harm done in previous steps
- Major computational challenge ...
QCD: meson/baryon spectrum

Ground state $B=0,1$ spectrum of QCD

points correspond to different sets of calculations
QCD: meson/baryon spectrum

Ground state B=0,1 spectrum of QCD

Time to move up the periodic table

[A Kronfeld, 1209.3468]

points correspond to different sets of calculations
QCD Spectroscopy

- Measure correlator \( (\chi = \text{object with q\# of hadron}) \)

\[
C_2(t) = \sum_x \langle 0 | \chi(x, t) \overline{\chi}(0, 0) | 0 \rangle
\]

- Unitarity: \( \sum_n |n\rangle \langle n| = 1 \)

\[
= \sum_x \sum_n \langle 0 | \chi(x, t) | n \rangle \langle n | \overline{\chi}(0, 0) | 0 \rangle
\]

- Hamiltonian evolution

\[
= \sum_x \sum_n e^{-E_n t} e^{ip_n \cdot x} \langle 0 | \chi(0, 0) | n \rangle \langle n | \overline{\chi}(0, 0) | 0 \rangle
\]

- Long times only ground state survives

\[
t \to \infty \quad e^{-E_0(0)t} | \langle 0; 0 | \overline{\chi}(x_0, t) | 0 \rangle |^2 = Z e^{-E_0(0)t}
\]

\( \chi(x) = \overline{u}(x) \gamma_5 d(x) \)
Effective mass

- Construct \( M(t) = \ln \left[ C_2(t)/C_2(t + 1) \right] \xrightarrow{t \to \infty} M \)
- Plateau corresponds to energy of ground state
- Fancier techniques able to resolve multiple eigenstates
Nuclear physics from LQCD
Nuclear physics from LQCD

- Can we compute the mass of $^{208}$Pb in QCD?
Nuclear physics from LQCD

• Can we compute the mass of $^{208}\text{Pb}$ in QCD?
• Yes
Nuclear physics from LQCD

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$$\langle 0 | T q_1(t) \ldots q_{624}(t) \bar{q}_1(0) \ldots \bar{q}_{624}(0) | 0 \rangle$$
Nuclear physics from LQCD

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- Long time behaviour gives ground state energy up to EW effects

$$t \to \infty \quad \# \exp(-M_{Pb}t)$$
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Long time behaviour gives ground state energy up to EW effects

$$t \to \infty \implies \# \exp(-M_{Pb}t)$$

But...
An \((\text{exponentially hard})^2\) problem?
An (exponentially hard)$^2$ problem?

- Complexity: number of Wick contractions $= (A+Z)!(2A-Z)!$

```
  \[ a_i(t_1)a_j(t_1)a_i(t_1)a_i(t_2)a_i(t_2)a_j(t_2)a_j(t_2) \]
```
An (exponentially hard)$^2$ problem?

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- Dynamical range of scales (numerical precision)
An (exponentially hard)² problem?

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An (exponentially hard)² problem?

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- Importance sampling: statistical noise exponentially increases with \(A\)
The trouble with baryons

- Importance sampling of QCD functional integrals
  - correlators determined stochastically
- Variance in single nucleon correlator \((C)\) determined by
  \[
  \sigma^2(C) = \langle CC^\dagger \rangle - |\langle C \rangle|^2
  \]
- For nucleon:
  \[
  \frac{\text{signal}}{\text{noise}} \sim \exp \left[-\left(M_N - \frac{3}{2}m_\pi \right)t\right]
  \]
- For nucleus A:
  \[
  \frac{\text{signal}}{\text{noise}} \sim \exp \left[-A(M_N - \frac{3}{2}m_\pi \right)t\right]
  \]

[Lepage '89]
The trouble with baryons

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• For nucleon:

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• For nucleus A:

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[LePage '89]
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[Lepage '89]
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[Lepage '89]
The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)

\[ \Xi^0 \Xi^0 n \]

@ \( m_\pi = 390 \text{ MeV} \)
The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)

Golden window of time-slices where signal/noise const

@ $m_t = 390$ MeV
No? trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)

The trouble with baryons

No?

\[ \pi \]

@ \( m_\pi = 390 \text{ MeV} \)

Golden window of time-slices where signal/noise const
No trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)

Golden window of time-slices where signal/noise const

Interpolator choice can be used to suppress noise
Multi-baryon systems

- Scattering*/bound systems

- Focus on (strong interaction) bound states

- Dibaryons: H, deuteron, \( \Xi \Xi \)

- \( ^3 \)H, \( ^4 \)He and hypernuclei: \( ^4 \)He\( \Lambda \), \( ^4 \)He\( \Lambda \Lambda \), ...

- Correlators for significantly larger A

- Caveat: at unphysical quark masses and no electroweak interactions

* See talk of S Beane (Tues F3)
Bound states at finite volume

• Two particle scattering amplitude in infinite volume

\[ A(p) = \frac{8\pi}{M} \frac{1}{p \cot \delta(p) - ip} \]

bound state at \( p^2 = -\gamma^2 \) when \( \cot \delta(i\gamma) = i \)

• Scattering amplitude in finite volume (Lüscher method)

\[ \cot \delta(i\kappa) = i - i \sum_{\vec{m} \neq 0} e^{-|\vec{m}|\kappa L} \frac{1}{|\vec{m}|\kappa L} \]

\[ \kappa \xrightarrow{L \to \infty} \gamma \]

• Need multiple volumes

• More complicated for n>2 body bound states
H-dibaryon

- R Jaffe [1977]: chromo-magnetic interaction
  \[ \langle H_m \rangle \sim \frac{1}{4} N(N - 10) + \frac{1}{3} S(S + 1) + \frac{1}{2} C_c^2 + C_f^2 \]
  most attractive for spin, colour, flavour singlet
- H-dibaryon (uuddss) J=I=0, s=-2 most stable
  \[ \Psi_H = \frac{1}{\sqrt{8}} \left( \Lambda\Lambda + \sqrt{3}\Sigma\Sigma + 2\Xi N \right) \]
- Bound in a many hadronic models
- Experimental searches
  - Emulsion expts, heavy-ion, stopped kaons
  - No conclusive evidence for or against

KEK-ps (2007)
\[ K^+ \, ^{12}_C \rightarrow K^+ \, \Lambda\Lambda \, X \]
H dibaryon in QCD

• Early quenched studies on small lattices: mixed results
  [Mackenzie et al. 85, Iwasaki et al. 89, Pochinsky et al. 99, Wetzorke & Karsch 03, Luo et al. 07, Loan 11]

• Semi-realistic calculations

  • “Evidence for a bound H dibaryon from lattice QCD”
    PRL 106, 162001 (2011)
    $N_f=2+1$, $a_s=0.12$ fm, $m_\pi=390$ MeV, $L=2.0, 2.5, 3.0, 3.9$ fm

  • “Bound H dibaryon in flavor SU(3) limit of lattice QCD” *
    PRL 106, 162002 (2011)
    $N_f=3$, $a_s=0.12$ fm, $m_\pi=670, 830, 1015$ MeV, $L=2.0, 3.0, 3.9$ fm

• NB: Quark masses unphysical, single lattice spacing

  * use a somewhat different method
H dibaryon in QCD

- Extract energy eigenstates from large Euclidean time behaviour of two-point correlators

\[
C_\Lambda(t) = \sum_x \langle 0 | \chi(x, t) \bar{\chi}(0) | 0 \rangle \xrightarrow{t \to \infty} Z_\Lambda e^{-M_\Lambda t}
\]

\[
C_{\Lambda\Lambda}(t) = \sum_x \langle 0 | \phi(x, t) \bar{\phi}(0) | 0 \rangle \xrightarrow{t \to \infty} Z_{\Lambda\Lambda} e^{-E_{\Lambda\Lambda} t}
\]

- Correlator ratio allows direct access to energy shift

\[
R(t) = \frac{C_{\Lambda\Lambda}(t)}{C_\Lambda(t)} \xrightarrow{t \to \infty} \tilde{Z} e^{-\Delta E_{\Lambda\Lambda} t}
\]
After volume extrapolation, H bound at unphysical quark masses.

Quark mass extrapolation is uncertain and unconstrained.

\[ B_{H}^{\text{lin}} = +4.9 \pm 4.0 \pm 8.3 \text{ MeV} \]

Other extrapolations possible.

[Solan, Thomas & Young PRL. 107 (2011) 092004, Haidenbauer & Meissner 1109.3590]

Suggests H is weakly bound or just unbound.

* 230 MeV point preliminary (one volume)
Deuteron

- Deuteron also investigated
- NPLQCD
- PACS-CS
- More work needed at lighter masses

![Graph showing ΔE(3S₁) vs. m_π² for various models and experimental results. The graph includes data points for different collaborations and calculations, with error bars for some points. The x-axis represents m_π² in GeV², ranging from 0 to 2.5. The y-axis represents ΔE(3S₁) in GeV, ranging from -0.040 to 0.040.]

[experiment]
Fukugita et al. [7]
NPLQCD mixed [8]
Aoki et al. [11]
PACS-CS V_oo [2]
NPLQCD 2+1f V_oo [3]
NPLQCD 3f V_max [4]
This work 2+1f V_oo

[Unlabeled graph showing ΔE(L) vs. 1/L for different masses. The graph includes data points for Nf=2+1 and Nf=0, with corresponding masses and energy differences. The x-axis represents 1/L, ranging from 0 to 3. The y-axis represents ΔE(L) in GeV, ranging from -0.024 to -0.004.]

N_f = 2+1 m_π = 0.51 GeV
N_f = 0 m_π = 0.80 GeV

[Graph showing m_π dependence of ΔE(∞) for 3S₁ channel. The closed, open, and cross symbols denote the 2+1/3 flavor (quenched) result. The results of Refs. [2, 3] are extrapolated values in the infinite volume limit. The experimental result (star) is also presented for comparison.]

[Yamazaki et al. 1207.4277]
Deuteron

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![Diagram showing the dependence of $\Delta E(3S_1)$ on $m^2_\pi$](Yamazaki et al. 1207.4277)
Many baryon systems

• Many baryon correlator construction is messy and expensive

• Techniques learnt in many-pion studies
  [WD & M. Savage; WD, K Orginos, Z. Shi]

• New tricks
  [T. Doi & M. Endres.; WD, K Orginos]

• Enables study of few (and many) baryon systems

• NPLQCD collaboration
  • Unphysical SU(3) symmetric world @ $m_{s}^{\text{phys}}$
  • Multiple big volumes, single lattice spacing

NB: SU(3) symmetry leads to unphysical degeneracies
Nuclei (A=2)

\[ \Lambda \Lambda \]

\[ n \Xi(3S_1) \]

\[ n p(3S_1) \]

\[ n \Sigma(3S_1) \]
Nuclei (A=2)

\[
\begin{align*}
\text{deuteron} & \quad \text{nn} & \quad \text{H–dib} & \quad \text{n}\Lambda (1s0) & \quad \text{n}\Lambda (3s1) & \quad \text{n}\Sigma (1s0) & \quad \text{n}\Sigma (3s1) & \quad \text{n}\Xi (3s1) & \quad \text{p}\Xi (3s1)
\end{align*}
\]

\[
\begin{align*}
\Delta E \text{ (MeV)}
\end{align*}
\]
Nuclei (A=3,4)

\[ \text{\(b \Delta E\)} \]

\[ t/b \]

\[ ^3\text{He} \]

\[ ^4\text{He} \]

\[ ^3\Lambda\text{He} \]

\[ ^4\Lambda\Lambda\text{He} \]
Nuclei (A=3,4)

- Empirically investigate volume dependence
- Need to ask if this is a 2+1 or 3+1 or 2+2 etc scattering state

\[
\begin{align*}
\Delta E \text{ (MeV)} & \\
3\text{He} \left( \frac{3^+}{2} \right) & -50 & \Sigma + n + p & -50 & d + \Sigma & -50 & \Sigma N(3s1) + N & -50
\end{align*}
\]

- L=24 , |p|=0
- L=32 , |p|=0
- L=48 , |p|=0
Nuclei (A=3,4)

- Empirically investigate volume dependence
- Need to ask if this is a 2+1 or 3+1 or 2+2 etc scattering state
Nuclei (A=2,3,4)
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d, nn, $^3$He, $^4$He

- **PACS-CS**: bound d, nn, $^3$He, $^4$He
- Previous quenched work
- Recent unquenched study at $m_\pi=500$ MeV
- **HALQCD**
  - Extract an NN potential
  - Strong enough to bind H, $^4$He at $m_{PS}=490$ MeV SU(3) pt
  - d, nn not bound
Nuclei ($A=4,...$)

Quark-quark determinant based contraction method

(low statistics, single volume)
Nuclei (A=4,...)

Quark-quark determinant based contraction method

$^4$He (SP)

(low statistics, single volume)
Nuclei (A=4,...)

Quark-quark determinant based contraction method

$^{8}\text{Be (SP)}$

(low statistics, single volume)
Nuclei \( (A=4,\ldots) \)

Quark-quark determinant based contraction method

\( ^{12}\text{C} \) (SP)

(low statistics, single volume)
Nuclei \((A=4,...)\)

Quark-quark determinant based contraction method

\[ ^{16}\text{O (SP)} \]

(low statistics, single volume)
Nuclei (A=4,...)

Quark-quark determinant based contraction method

$^{28}$Si (SP)

(low statistics, single volume)
Effective field theory for LQCD
Effective field theory for LQCD

- What is the role of EFT in LQCD calculations?
Effective field theory for LQCD

- What is the role of EFT in LQCD calculations?
- Quark mass extrapolation? ... not for long ...
Effective field theory for LQCD

• What is the role of EFT in LQCD calculations?
  • Quark mass extrapolation? ... not for long ...
  • Volume dependence
    • $R_4$ formulation of LQCD calculations restricts what can be studied
Effective field theory for LQCD

• **What is the role of EFT in LQCD calculations?**
  - Quark mass extrapolation? ... not for long ...
  - Volume dependence
    • $R_4$ formulation of LQCD calculations restricts what can be studied
  - Central for quantities where we do not know how to do QCD calculation – ex reactions
    • LECs determined from LQCD used to make predictions
Scattering from volume effects
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- *Lüscher method*: use two particle energy levels at finite volume to extract two-body interaction [Lüscher 1985, 1990]

- Philosophically: matching QCD onto a hadronic description
Scattering from volume effects

- *Lüscher method*: use two particle energy levels at finite volume to extract two-body interaction [Lüscher 1985, 1990]
- Philosophically: matching QCD onto a hadronic description
- Much work on extensions/modifications for studying
  - Resonances [Feng et al; Döring et al; ]
  - Boosted systems [Rummikainen & Gottlieb; Kim et al; Feng et al., Davoudi & Savage; Göckeler et al,...]
  - Inelastic/multi-channel systems [Bernard et al.; Döring et al.; Briceno&Davoudi; Hansen&Sharpe; Hall et al; Liu]
Three nucleon systems

- Kreuzer & Hammer [PLB 694 (2011) 424] studied three-nucleon systems in pionless EFT (valid at low energies, p<\(m_\pi\))
  [see also Luu Lattice2008; Polejaeva & Rusetsky [12]; Kreuzer & Grießhammer [12]]

- Lagrangian involves two and three nucleon interactions

\[ \mathcal{L} = \begin{array}{c} \text{two nucleon interaction} \\ + \\ \text{three nucleon interaction} \end{array} \]

- Infinite volume: solve Faddeev equations to find bound states

- Finite volume: replace loops by momentum sums and propagators by periodic versions and solve

- Boundary conditions impose cubic symmetry: decompose into irreps of double cover of octahedral group \(2O\)
Triton at finite volume

- Binding energy of triton tuned to physical value at infinite volume

- Lattice calculations at different volumes would constrain the three body EFT interaction $H(\Lambda)$

The Triton in Finite Volume

Results

$E_3^\infty = -8.4818$ MeV

Size of the triton $\sim 2f_{m}$

Results are renormalized

Shift at volumes typical in Lattice QCD already more than 100%

![Graph showing the binding energy of the triton as a function of volume.](Image)

Comparison to data from Chiral EFT on the lattice [Epelbaum et al, 10]: study higher partial waves, higher orders

Simon Kreuzer (GWU)

Three-Body Physics in a Finite Volume
More baryons

• Philosophical approach is similar to three baryon case
• Lattice calculations and extract energies of N baryon system
• Match EFT calculation in FV for a varying LECs to extract them
• \( V \rightarrow \infty \) EFT calculation to extract binding/scattering information
• Problems (has not really been attempted)
  • Four and higher body EFT calculations are computationally demanding
  • Convergence of EFT must be carefully investigated
• Lattice EFT calculations (E Epelbaum’s talk) could be used
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• Nuclear physics is an emergent phenomenon of the Standard Model
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• Nuclear physics is an emergent phenomenon of the Standard Model

• What does it take to make this a quantitative statement?
From quarks to nuclei

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- QCD and EFT
From quarks to nuclei

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• What does it take to make this a quantitative statement?

  • QCD and EFT

  • Bright future
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  • Strong connections to experimental programs
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    - Spectroscopy and beyond
From quarks to nuclei

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• *What does it take to make this a quantitative statement?*

• QCD and EFT

• *Bright future*

• Strong connections to experimental programs
  • Spectroscopy and beyond

• Answer questions that experiments have not and cannot
From quarks to nuclei

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• What does it take to make this a quantitative statement?
  • QCD and EFT

• Bright future
  • Strong connections to experimental programs
    • Spectroscopy and beyond
  • Answer questions that experiments have not and cannot
    • Quantitatively address changes in NP with parameters of the SM
From quarks to nuclei

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