The effect of an infrared divergent quark-antiquark interaction kernel on other Green functions

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Motivation

**Quark Confinement:**
*Linearly rising potential between static colour sources!*

**Experiment:**
Hadrons fall *approximately* on Regge trajectories

**Lattice:**
Wilson loop follows area law

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Motivation

Quark Confinement \( \Rightarrow \)
gauge-invariant static potential \( V(r) = \sigma r + \ldots, \sigma \approx 1 \text{ GeV/fm} \)

This potential is contained in gauge-fixed correlation functions, i.e., QCD Green’s functions

Then:

Infrared divergence of the quark–antiquark scattering kernel?

Functional eqs. (DSEs, ERG, nPI, \ldots) & Slavnov-Taylor ids.: Consequences for all other Green’s functions \ldots!?!
Landau gauge QCD Green’s functions

▶ Green’s functions of colour-charged fields: **gauge fixing required!**
▶ Least number of Green’s functions and best investigated gauge: **Landau gauge**
▶ Historically: **Confining or confined gluons?**

▶ Lattice Landau gauge gluon propagator & decoupling and scaling solutions of functional eqs.: **Confined, i.e., IR suppressed, gluon propagator**

▶ Functional eqs.: **Quark-gluon vertex IR enhanced**
M. Hopfer (Talk on Thursday)
▶ **Hypothesis: 4-point function IR divergent**
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**A note on other gauges with confining and (?) confined gluons:**

- Maximally abelian gauge:
  IR enhanced colour-diagonal & IR suppressed off-diagonal gluons
  *cf. V. Mader (Talk today)*
- Coulomb gauge:
  IR enhanced $D_{00}$ & IR suppressed spatial gluon propagator
  *cf. talks by H. Reinhardt and P. Watson*

- Lattice Landau gauge gluon propagator &
  decoupling and scaling solutions of functional eqs.:
  **Confined, i.e., IR suppressed, gluon propagator**

- Functional eqs.:
  **Quark-gluon vertex IR enhanced**
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There are several solutions on the lattice!

A. Sternbeck (Talk on Thursday)

- **Functional eqs.:**
  - Quark-gluon vertex IR enhanced

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Hypothesis: 4-point function IR divergent
Assuming an IR divergent 4-point function

How is Confinement described by Green’s functions?

► Assume quark 4-point function to be maximally IR singular, \( i.e., \propto 1/k^4 \):

\[
p_1 \rightarrow p_3 \propto \frac{1}{(p_1 - p_3)^4}\bigg|_{\text{reg.}}
\]

► Put \textit{e.g.} DSE for 4-quark function:

\[
\begin{align*}
\begin{array}{ccc}
\begin{tikzpicture}
\fill (0,0) circle (0.1cm);
\draw[thick, ->] (-1,1) -- (1,1);
\draw[thick, ->] (-1,-1) -- (1,-1);
\end{tikzpicture}
& =
\begin{tikzpicture}
\fill (0,0) circle (0.1cm);
\draw[thick, ->] (-1,1) -- (1,1);
\draw[thick, ->] (-1,-1) -- (1,-1);
\end{tikzpicture}
& +
\begin{tikzpicture}
\fill (0,0) circle (0.1cm);
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\end{tikzpicture}
\end{array}
\end{align*}
\]
Consequences an IR divergent 4-point function

For simplicity: Analysis first for fundamentally charged scalar!

Consistency requirements:

- Boundedness of higher $n$-point functions to $1/k^4$ $\Rightarrow$ matter-gluon vertex less singular $\Rightarrow$ **colour structure**

  

- One-gluon exchange fails to reproduce this colour structure!

- All 4-point functions (4-gluon, ghost-gluon, matter-gluon, matter-ghost) inherit the $1/k^4$ singularity in specific colour channels.

- Higher $n$-point functions contain contributions $\propto 1/k^4$ with $k$ being the momentum transfer between two coloured clusters.

- Propagators and 3-point functions protected by cancellations.

Decoupling theorem circumvented by IR singularities:

One heavy fundamental charge induce changes in the IR behaviour of YM Green’s functions!?!?
Assumption of confining IR singularity in matter-matter scattering kernel leads to several wanted features.

No decoupling of infinitely heavy charges?

Further to be clarified:

• Absence of van-der-Waals forces?
• Casimir scaling? $N$-ality?
• Relation to dynamical chiral symmetry breaking / restoration?
• …
Landau gauge Green’s functions:

- Linear confinement ("1/k^4") consistently possible in \( n \geq 4 \)-point functions.
- Decoupling theorem circumvented by IR singularities. (?)

Outlook:

- Tensor structures of quark(-gluon/ghost) \( n \)-point function?
- Absence of confining interaction between colour singlets?
- Explicit calculation for a consistently truncated system: Nature of confining force? Hadrons as bound states?
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51st Schladming Winter School: Extreme QCD in and out of Equilibrium
Feb. 23 – March 2, 2013

Invited speakers: