Physical unitarity of a massive Yang-Mills theory without the Higgs field from a viewpoint of confinement

Kei-Ichi Kondo
(Chiba University, Japan)

(Chiba University, Japan)

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Introduction

We consider a “massive” Yang-Mills theory without the Higgs field. A motivation comes from the non-pertubative issues of the Yang-Mills theory or QCD.

- Confinement and Green functions: The deep infrared behaviors of gluon and ghost Green functions are believed to be intimately connected to color confinement [Kugo and Ojima, 1978] [Gribov, 1978]. The decoupling solution [Boucaud et al., 2007] for gluon and ghost propagators supported by recent simulations is well fitted by a massive Yang-Mills theory, i.e., Curci-Ferrari (CF) model [Tissier and Wschebor, 2010, 2011] [Serreau and Tissier, 2012]. The CF model is much simpler than the Gribov-Zwanziger model. How color confinement is understood from the CF model?

- Glueball mass spectrum: In the potential model of [Cornwall and Soni, 1982], glueballs are identified with bound states of massive gluons described by a naive mass term. How the mass and spin of the resulting glueball are related to those of the constituent gluons? Can we introduce a BRST-invariant mass term for gluons?

- Vacuum condensates: On-shell BRST invariant version of vacuum condensate of mass dimension two $\langle A_\mu A_\mu \rangle$ has been constructed [Kondo, 2001]. Can we construct the full (off-shell) BRST invariant version?
Curci-Ferrari model

As a “massive” Yang-Mills theory without the Higgs field, we consider the Curci-Ferrari (CF) model with a (mass) parameter $M$ which reduces in the $M \to 0$ limit to the Yang-Mills theory written in terms of the Yang-Mills field $A_\mu$, the Faddeev-Popov (FP) ghost field $C$, antighost field $\bar{C}$ and the Nakanishi-Lautrup (NL) field $N$.

$$L^{\text{tot}}_{\text{YM}} = L_{\text{YM}} + L_{\text{GF+FP}} + L_m,$$

$$L_{\text{YM}} = -\frac{1}{4} F_{\mu
u} \cdot F^{\mu\nu},$$

$$L_{\text{GF+FP}} = \frac{\beta}{2} N \cdot N + N \cdot \partial^\mu A_\mu - \frac{\beta}{2} g N \cdot (i\bar{C} \times C)$$

$$+ i\bar{C} \cdot \partial^\mu D_\mu[A]C + \frac{\beta}{4} g^2 (i\bar{C} \times C) \cdot (i\bar{C} \times C)$$

$$= N \cdot \partial^\mu A_\mu + i\bar{C} \cdot \partial^\mu D_\mu[A]C + \frac{\beta}{4} (\bar{N} \cdot \bar{N} + N \cdot N),$$

$$L_m = \frac{1}{2} M^2 A_\mu \cdot A^\mu + \beta M^2 i\bar{C} \cdot C,$$  \hspace{1cm} (1)

where $\beta$ corresponds to a gauge-fixing parameter in the $M \to 0$ limit and $\bar{N} = -N + gi\bar{C} \times C$. 
Figure 1: The coupled Schwinger–Dyson equations for the gluon and ghost propagators in Yang-Mills theory with the conventional Lorentz gauge fixing. (a) ghost equation, (b) gluon equation. Here $\Delta$ denotes the full ghost propagator and $D$ the full gluon propagator, while $\Gamma$ denote the four types of vertices. Two-loop diagrams are enclosed by the broken line.
The decoupling solution [Boucaud et al., 2007] for gluon and ghost propagators currently supported by recent simulations rather than the scaling solution is well fitted by a massive Yang-Mills theory, i.e., Curci-Ferrari (CF) model [Tissier and Wschebor, 2010, 2011] [Serreau and Tissier, 2012] The CF model is much simpler than the Gribov-Zwanziger model.

Figure 2: The $SU(2)$ results of [Tissier and Wschebor, 2010] (red curve) are compared with lattice data of [Cucchieri and Mendes, 2008] (blue points). (left panel) gluon propagator. (right panel) ghost dressing function. The best fit is obtained for $g = 7.5$ and $M = 0.68 \text{GeV}$ for $SU(2)$.

How color confinement is understood from the CF model?
The aim of this talk is to reconsider physical unitarity of a massive Yang-Mills theory without introducing the Higgs field.

For this purpose, we start from the Curci-Ferrari (CF) model[1976], which is a massive extension of the massless Yang-Mills theory in the most general Lorenz gauge having both the BRST and anti-BRST symmetry.

• The CF model is renormalizable. [Curci and Ferrari, 1976] [de Boer, Skenderis, van Nieuwenhuizen and Waldron, 1996]

Nevertheless,

• The CF model does not seem to satisfy the physical unitarity. [Curci and Ferrari, 1976] [Ojima, 1982] [de Boer, Skenderis, van Nieuwenhuizen and Waldron, 1996]

• The CF model is not invariant under the usual BRST transformation. However, it can be made invariant under a modified BRST-like transformation. But, the modified BRST transformation is not nilpotent.

The nilpotency leads to the physical unitarity in the usual massless Yang-Mills theory due to Kugo and Ojima.

Is the nilpotency necessary to recover the physical unitarity in the massive case?
What is the physical unitarity?

The $S$-matrix or the scattering operator $S$ is unitary:

$$1 = S^\dagger S = SS^\dagger. \quad (2)$$

This means that for any (initial) state $\Psi_b \in \mathcal{V}$ and any (final) state $\Psi_a \in \mathcal{V}$,

$$\langle \Psi_b | \Psi_a \rangle = \langle \Psi_b | S^\dagger S | \Psi_a \rangle = \sum_{\Phi_n \in \mathcal{V}} \langle \Psi_b | S^\dagger | \Phi_n \rangle \langle \Phi_n | S | \Psi_a \rangle, \quad (3)$$

by inserting the complete set of states $\{ \Phi_n \}$ in the total state space $\mathcal{V}$:

$$1 = \sum_{\Phi_n \in \mathcal{V}} \langle \Phi_n \rangle \langle \Phi_n \rangle.$$

On the other hand, the physical unitarity of the $S$-matrix means that the $S$ matrix is unitary on the physical subspace

$$\mathcal{V}_{\text{phys}} := \{ | \text{phys} \rangle \in \mathcal{V}; \langle \text{phys} | \text{phys} \rangle \geq 0 \} \subset \mathcal{V} \quad (4)$$

for any state $\Psi_a, \Psi_b \in \mathcal{V}_{\text{phys}}$,

$$\langle \Psi_b | \Psi_a \rangle = \sum_{\Phi_n \in \mathcal{V}_{\text{phys}}} \langle \Psi_b | S^\dagger | \Phi_n \rangle \langle \Phi_n | S | \Psi_a \rangle, \quad (5)$$
The unitarity of the $S$-matrix is rewritten in terms of the scattering amplitude defined by

$$S = 1 + iT,$$  \hspace{1cm} (6)

into the relation:

$$-i(T - T^\dagger) = TT^\dagger.$$ \hspace{1cm} (7)

Then the unitarity relation reads for any state $\Psi_a, \Psi_b \in \mathcal{V}$:

$$\text{Im} \langle \Psi_b | T | \Psi_a \rangle := \frac{1}{2i} \left( \langle \Psi_b | T | \Psi_a \rangle - \langle \Psi_b | T^\dagger | \Psi_a \rangle \right) = \frac{1}{2} \sum_{\Phi_n \in \mathcal{V}} \langle \Psi_b | T | \Phi_n \rangle \langle \Phi_n | T^\dagger | \Psi_a \rangle.$$ \hspace{1cm} (8)

On the other hand, the physical unitarity requires that for any physical state $\Psi_a, \Psi_b \in \mathcal{V}_{\text{phys}}$, only the physical states contribute to the intermediate states:

$$\text{Im} \langle \Psi_b | T | \Psi_a \rangle = \frac{1}{2} \sum_{\Phi_n \in \mathcal{V}_{\text{phys}}} \langle \Psi_b | T | \Phi_n \rangle \langle \Phi_n | T^\dagger | \Psi_a \rangle.$$ \hspace{1cm} (9)

In other words, the physical unitarity in gauge theories states that all the unphysical modes cancel in the intermediate states.

The imaginary part is calculated by the Cutkosky cutting rule.
§ 3  Physical unitarity in the Yang-Mills theory

\[ M = 0 \]

Figure 3: In the \( M = 0 \) case, diagrams contributing to the amplitude \( T(A \rightarrow A) \) to the order \( g^2 \) are given by (a) vector boson loop, (b) ghost–antighost loop, (c) boson tadpole.

Figure 4: In the \( M = 0 \) case, the physical unitarity of the amplitude \( T(T^\pm \rightarrow T^\pm) \) to the order \( g^2 \) is checked according to the Cutkosky rule using the diagrams: (a) vector boson loop, (b) ghost–antighost loop.
Figure 5: In the $M = 0$ case, mode cancellations occur. In the amplitude, two diagrams (a) from the longitudinal mode $L$ and the scalar mode $S$ are cancelled by a ghost-antighost $C, \bar{C}$ diagram (b).

In the massless case $M = 0$, for the amplitude $T \rightarrow T$ for the transverse mode $T$, two diagrams (a) from the longitudinal model $L$ and the scalar mode $S$ are cancelled by a ghost-antighost $C, \bar{C}$ diagram (b).

\[ \begin{array}{l}
T^+, T^- \ , L, S \ (\cong N : \text{non- \textbf{propagating}}) , C, \bar{C}; \\
\end{array} \]  \hfill (10)

In the massive case $M \neq 0$, for the amplitude $U \rightarrow U$ for a physical particle $U$, a diagram (a) from the scalar mode is not sufficient to cancel the ghost-antighost $C, \bar{C}$ diagram (b).

\[ \begin{array}{l}
T^+, T^- , L \ , S \cong N ; C, \bar{C} \\
\end{array} \]  \hfill (11)
Define the physical spin-1 vector mode $U_\mu$ by

$$U_\mu := A_\mu - \frac{1}{M^2} \partial_\mu N, \quad \text{or} \quad A_\mu = U_\mu + \frac{1}{M^2} \partial_\mu N$$

Figure 6: In the $M \neq 0$ case, the physical unitarity of the amplitude $T(U \to U)$ for the physical spin-1 vector mode $U$ to the order $g^2$ is checked according to the Cutkosky rule using the diagrams: (a) vector boson loop, (b) ghost–antighost loop.

Figure 7: In the $M \neq 0$ case, the incomplete mode cancellation among unphysical modes to the order $g^2$ prevents us from ensuring the physical unitarity. In the amplitude, a diagram (a) from the scalar mode $S$ is overcancelled by a ghost-antighost diagram (b), leaving one half of (b) nonvanishing.
The original gluon propagator has the manifestly renormalizable form:

\[
\langle \tilde{A}^A_\mu (k) \tilde{A}^B_\nu (-k) \rangle = i \delta^{AB} \left[ -\frac{g_{\mu\nu}}{k^2} - \frac{k_\mu k_\nu}{k^2 + i\epsilon} - \frac{\beta k_\mu k_\nu}{k^2 - \beta M^2 + i\epsilon} \right], 
\]

(13)

The original gluon propagator is decomposed into the spin-1 and spin-0 parts:

\[
\langle \tilde{A}^A_\mu (k) \tilde{A}^B_\nu (-k) \rangle = \langle \tilde{K}^A_\mu (k) \tilde{K}^B_\nu (-k) \rangle + M^{-4} k_\mu k_\nu \langle \mathcal{N}^A (k) \mathcal{N}^B (-k) \rangle \\
= i \delta^{AB} \left[ -\frac{g_{\mu\nu}}{k^2 - M^2 + i\epsilon} - \frac{k_\mu k_\nu}{M^2} - \frac{\beta k_\mu k_\nu}{k^2 - \beta M^2 + i\epsilon} \right], 
\]

(14)

since there are no mixing propagators: \( \langle \tilde{K}^A (k) \mathcal{N}^B (k) \rangle = 0 = \langle \mathcal{N}^A (k) \tilde{K}^B (-k) \rangle \).

The spin-1 part is reproduced from

\[
\langle \tilde{U}^A_\mu (k) \tilde{U}^B_\nu (-k) \rangle = i \delta^{AB} \left[ -\frac{g_{\mu\nu}}{k^2 - M^2 + i\epsilon} \right], 
\]

(15)
The modified BRST transf.

The ordinary BRST transformation

\[
\begin{align*}
\delta A_\mu(x) &= \mathcal{D}_\mu [A] C(x), \\
\delta C(x) &= -\frac{g}{2} C(x) \times C(x), \\
\delta \bar{C}(x) &= i N(x), \\
\delta N(x) &= 0,
\end{align*}
\] (16)

satisfies

\[
\delta \mathcal{L}_{YM} = 0, \quad \delta \mathcal{L}_{GF+FP} = 0
\] (17)

However,

\[
\delta \mathcal{L}_m = M^2 \partial^\mu C \cdot A_\mu \neq 0 \implies \delta \mathcal{L}_{CF} \neq 0,
\] (18)

However, a modified BRST transformation \( \delta' \) exits: Then we require (up to the total derivative term)

\[
0 = \delta' (\mathcal{L}_{GF+FP} + \mathcal{L}_m),
\] (19)
while $\delta' \mathcal{L}_m \neq 0$ and $\delta' \mathcal{L}_{GF+FP} \neq 0$.

Thus a modified BRST transformation is found to be

\[
\begin{cases}
\delta' A_\mu(x) = D_\mu [A] C(x), \\
\delta' C(x) = -\frac{g}{2} C(x) \times C(x), \\
\delta' \bar{C}(x) = i \bar{N}(x), \\
\delta' \bar{N}(x) = M^2 C,
\end{cases}
\] (20)

which modifies the BRST transformation of the NL field and reduces to the usual BRST transformation in the limit $M \rightarrow 0$.

Similarly, a modified anti-BRST transformation $\bar{\delta}'$ exists:

\[
\begin{cases}
\bar{\delta}' A_\mu(x) = D_\mu [A] \bar{C}(x), \\
\bar{\delta}' \bar{C}(x) = -\frac{g}{2} \bar{C}(x) \times C(x), \\
\bar{\delta}' C(x) = i \bar{N}(x), \\
\bar{\delta}' \bar{N}(x) = -M^2 \bar{C}(x), \text{ or } \bar{\delta}' \bar{N}(x) = g \bar{N}(x) \times \bar{C}(x) + M^2 \bar{C}(x),
\end{cases}
\] (21)

which reduces to the usual anti-BRST transformation in the limit $M \rightarrow 0$. 

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However, the modified BRST transformation violates the nilpotency:

\[
\delta' \delta' \mathcal{A}_\mu(x) = 0, \\
\delta' \delta' \mathcal{C}(x) = 0, \\
\delta' \delta' \bar{\mathcal{C}}(x) = i\delta' \mathcal{N}(x) = iM^2 \mathcal{C}(x) \neq 0, \\
\delta' \delta' \mathcal{N}(x) = M^2 \delta' \mathcal{C}(x) = -M^2 \frac{g}{2} \mathcal{C}(x) \times \mathcal{C}(x) \neq 0. \tag{22}
\]

The nilpotency is violated also for the modified anti-BRST transformation:

\[
\bar{\delta}' \bar{\delta}' \mathcal{A}_\mu(x) = 0, \\
\bar{\delta}' \bar{\delta}' \bar{\mathcal{C}}(x) = 0, \\
\bar{\delta}' \bar{\delta}' \mathcal{C}(x) = i\bar{\delta}' \bar{\mathcal{N}}(x) = -iM^2 \bar{\mathcal{C}}(x) \neq 0, \\
\bar{\delta}' \bar{\delta}' \bar{\mathcal{N}}(x) = -M^2 \bar{\delta}' \mathcal{C}(x) = M^2 \frac{g}{2} \bar{\mathcal{C}}(x) \times \bar{\mathcal{C}}(x). \tag{23}
\]

In the limit \( M \to 0 \), the modified BRST and anti-BRST transformations reduce to the usual BRST and anti-BRST transformations and become nilpotent.
§ Requirements for a massive Yang-Mills field

We want to construct a massive spin-1 vector boson field $\mathcal{K}_\mu(x)$ such that

(i) $\mathcal{K}_\mu$ has a modified BRST-invariance (off mass shell): physical field

$$\delta' \mathcal{K}_\mu = 0.$$  \hfill (24)

(ii) $\mathcal{K}_\mu$ satisfies the divergenceless (on mass shell): 3 d.o.f.

$$\partial_\mu \mathcal{K}_\mu = 0.$$  \hfill (25)

(iii) $\mathcal{K}_\mu$ obeys the adjoint transformation under the color rotation:

$$\mathcal{K}_\mu(x) \rightarrow U \mathcal{K}_\mu(x) U^{-1}, \quad U = \exp[i\epsilon^A Q^A],$$  \hfill (26)

The total Lagrangian is invariant under the (infinitesimal) global gauge transformation or color rotation defined by

$$\delta \Phi(x) \equiv [\epsilon^C i Q^C, \Phi(x)] = \epsilon \times \Phi(x), \quad \text{for} \quad \Phi = A_\mu, N, C, \bar{C},$$

$$\delta \varphi(x) \equiv [\epsilon^C i Q^C, \varphi(x)] = -i \epsilon \varphi(x),$$  \hfill (27)
§ Identification of a massive spin-one boson

Such a field $K_\mu$ is obtained by a nonlinear but local transformation from the original fields of the CF model:

$$K_\mu = A_\mu - \frac{1}{M^2} \partial_\mu N - \frac{g}{M^2} A_\mu \times N + \frac{g}{M^2} \partial_\mu C \times \bar{C} + \frac{g^2}{M^2} (A_\mu \times C) \times \bar{C}, \quad (28)$$

In the Abelian limit or the lowest order, $K_\mu$ reduces to the Proca field for massive vector:

$$K_\mu \rightarrow A_\mu - \frac{1}{M^2} \partial_\mu N \equiv U_\mu. \quad (29)$$

$U_\mu$ is invariant under the Abelian version of the modified BRST, but is not invariant in the non-Abelian BRST transformation.

The new field $K_\mu$ is converted to a simple form:

$$K_\mu(x) = A_\mu(x) + \frac{1}{M^2} i \delta' \bar{\delta}' A_\mu(x). \quad (30)$$
(i) BRST invariance:

\[ \delta' \mathcal{K}_\mu = \delta' A_\mu + \frac{i}{M^2} \delta' (\partial_\mu \bar{C} + g A_\mu \times \bar{C}) \]

\[ = \delta' A_\mu + \frac{i}{M^2} \delta' (\partial_\mu \bar{C}' + g \delta' A_\mu \times \bar{C}' + g A_\mu \times \delta' \bar{C}') \]

\[ = \delta' A_\mu + \frac{i}{M^2} (\partial_\mu \delta'^2 \bar{C}' + g \delta'^2 A_\mu \times \bar{C}' - g \delta' A_\mu \times \delta' \bar{C}' \]

\[ + g \delta' A_\mu \times \delta' \bar{C}' + g A_\mu \times \delta'^2 \bar{C}') \]

\[ = \delta' A_\mu + \frac{i}{M^2} D_\mu [A] \delta'^2 \bar{C}' = 0, \quad (31) \]

where we have used \( \bar{\delta}' A_\mu = D_\mu [A] \bar{C} \), \( \delta'^2 A_\mu = 0 \), \( \delta'^2 \bar{C}' = iM^2 C \), and \( \delta' A_\mu = D_\mu [A] C \).

(iii) is trivial in the Lie-algebra form;

\[ \mathcal{K}_\mu = A_\mu - \frac{1}{M^2} \partial_\mu N + i \frac{g}{M^2} [A_\mu, N] - i \frac{g}{M^2} [\partial_\mu C, i \bar{C}] - \frac{g^2}{M^2} [[A_\mu, C], i \bar{C}]. \quad (32) \]
(ii) The equations of motion, i.e., field equations are obtained

\[
\frac{\delta \mathcal{L}_{\text{tot}}^{\text{CF}}}{\delta A_\mu} = \mathcal{D}^\nu [\mathcal{A}] \mathcal{F}_{\nu\mu} - \partial_\mu \mathcal{N} + g i \partial_\mu \bar{C} \times C + M^2 A_\mu + g J_\mu = 0,
\]

where we have used the left derivative and defined the matter current \( J_\mu \) by

\[
J_\mu^A = -i (T^A \varphi)_a \frac{\partial \mathcal{L}_{\text{tot}}}{\partial (\partial_\mu \varphi_a)}.
\]

The conserved Noether current associated with color symmetry \( \partial_\mu J_\mu^{\text{color}} = 0 \) is given by

\[
J_\mu = A_\nu \times \mathcal{F}^{\nu\mu} + i \partial_\mu \bar{C} \times C - \mathcal{D}^\mu [\mathcal{A}] C \times i \bar{C} + A_\mu \times \mathcal{N} + J_\mu.
\]

Using this result, the equation of motion for \( A_\mu \) is cast into the Maxwell-like form:

\[
\partial_\nu \mathcal{F}^{\nu\mu} + g J_\mu^{\text{color}} + i \delta'^i \bar{\delta}^l A^\mu + M^2 A_\mu = 0.
\]

Then we use \( \partial_\mu \partial_\nu \mathcal{F}^{\nu\mu} = 0 \) and \( \partial_\mu J_\mu^{\text{color}} = 0 \).
§ Physical unitarity in the CF model!? 

In the massive case $M \neq 0$, the physical unitarity of the amplitude $T(\mathcal{H} \rightarrow \mathcal{H}')$ to the order $g^2$ is checked according to the Cutkosky rule using the diagrams: (a) massive vector boson, (b) ghost–antighost, (c) NL field.

Figure 8: In the massive case $M \neq 0$, incomplete mode cancellations violates the physical unitarity for the one-particle amplitude to the order $g^2$.

Figure 9: In the massive case $M \neq 0$, incomplete mode cancellations violates the physical unitarity for the one-particle amplitude to the order $g^2$. 
Perturbative treatment

The inverse transformation is obtained by iteration procedures. We obtain The CF Lagrangian is rewritten order by order of $g$:

$$L_{CF} = L_0 + L_1 + O(g^2),$$  \hfill (37)

$$L_0 = -\frac{1}{4}(\partial_\mu \mathcal{K}_\nu - \partial_\nu \mathcal{K}_\mu)^2 + \frac{1}{2}M^2 \mathcal{K}_\mu \cdot \mathcal{K}^\mu$$

$$- \frac{1}{2M^2}(\partial_\mu \mathcal{N})^2 + \frac{\beta}{2} \mathcal{N} \cdot \mathcal{N} - i\partial_\mu \bar{\mathcal{C}} \cdot \partial_\mu \mathcal{C} + \beta M^2 i\bar{\mathcal{C}} \cdot \mathcal{C},$$  \hfill (38)

$$L_1 = -\frac{g}{2}(\partial_\mu \mathcal{K}_\nu - \partial_\nu \mathcal{K}_\mu) \cdot (\mathcal{K}^\mu \times \mathcal{K}^\nu)$$

$$+ \frac{g}{2M^4}(\partial_\mu \mathcal{K}_\nu - \partial_\nu \mathcal{K}_\mu) \cdot (\partial_\mu \mathcal{N} \times \partial_\nu \mathcal{N}) - \frac{g}{M^2} \mathcal{K}_\mu \cdot (\mathcal{N} \times \partial_\mu \mathcal{N})$$

$$+ \frac{g}{2M^2}(\partial_\mu \mathcal{K}_\nu - \partial_\nu \mathcal{K}_\mu) \cdot (i\partial_\mu \bar{\mathcal{C}} \times \partial_\nu \mathcal{C} - i\partial_\nu \bar{\mathcal{C}} \times \partial_\mu \mathcal{C})$$

$$- g\mathcal{K}_\mu \cdot (i\bar{\mathcal{C}} \times \partial_\mu \mathcal{C}) + g\mathcal{K}_\mu \cdot (i\partial_\mu \bar{\mathcal{C}} \times \mathcal{C})$$

$$+ \frac{g}{M^2} \partial_\mu \mathcal{N} \cdot (i\partial_\mu \bar{\mathcal{C}} \times \mathcal{C}) - g\frac{\beta}{2} \mathcal{N} \cdot (i\bar{\mathcal{C}} \times \mathcal{C}).$$  \hfill (39)
Feynman rules:

Propagators

(a) massive vector propagator

\[
\langle \mathcal{K}_\mu^A(k) \mathcal{K}_\nu^B(-k) \rangle = -\frac{i}{k^2 - M^2 + i\varepsilon} \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2} \right) \delta^{AB} = \Delta^{AB}_{\mu\nu}(k). \tag{40}
\]

(b) FP ghost propagator

\[
\langle \mathcal{C}^A(k) \overline{\mathcal{C}}^B(-k) \rangle = -\frac{i}{k^2 - \beta M^2 + i\varepsilon} \delta^{AB} = \Delta^{AB}(k). \tag{41}
\]

(c) NL (auxiliary) field propagator

\[
\langle \mathcal{N}^A(k) \mathcal{N}^B(-k) \rangle = -\frac{iM^2}{k^2 - \beta M^2 + i\varepsilon} \delta^{AB}. \tag{42}
\]
Three-point vertices

(a) Three-vectors $\langle \mathcal{K}_\mu^A(k_1)\mathcal{K}_\nu^B(k_2)\mathcal{K}_\rho^C(k_3) \rangle$

$$= -ig f^{ABC} V_{\mu\nu\rho}(k_1, k_2, k_3),$$

$$V_{\mu\nu\rho}(k_1, k_2, k_3) := (k_1 - k_2)_\rho g_{\mu\nu} + (k_2 - k_3)_\mu g_{\nu\rho} + (k_3 - k_1)_\nu g_{\rho\mu}. \quad (44)$$

(b) One vector, one ghost and one anti-ghost $\langle \bar{\mathcal{C}}^A(k_1)\mathcal{C}^B(k_2)\mathcal{K}_\rho^C(k_3) \rangle$

$$= -ig f^{ABC} \left\{ M^{-2}[k_1\rho(k_2 \cdot k_3) - k_2\rho(k_1 \cdot k_3)] + k_1\rho - k_2\rho \right\}. \quad (45)$$

(c) One vector and two NL fields $\langle \mathcal{N}^A(k_1)\mathcal{N}^B(k_2)\mathcal{K}_\rho^C(k_3) \rangle$

$$= ig M^{-2} f^{ABC} \left\{ M^{-2}[k_1\rho(k_2 \cdot k_3) - k_2\rho(k_1 \cdot k_3)] + k_1\rho - k_2\rho \right\}. \quad (46)$$

(d) One NL field, one ghost and one anti-ghost $\langle \bar{\mathcal{C}}^A(k_1)\mathcal{C}^B(k_2)\mathcal{N}^C(k_3) \rangle$

$$= ig f_{ABC} [M^{-2}k_1 \cdot k_3 - \beta/2]. \quad (47)$$
Conclusion

In order to understand color confinement in a “massive” Yang-Mills model without the Higgs field, i.e., the Curci-Ferrari (CF) model, which is regarded as a low-energy effective theory of QCD (which is simpler than the refined Gribov-Zwanziger model).

• For of all, we have constructed the field $K^A_{\mu}(x)$ with the following properties:
  (i) $K_{\mu}$ is invariant under an extended BRST transformation, $\delta' K_{\mu}(x) = 0$ (off shell).
  (ii) $K_{\mu}$ is divergenceless (transverse), $\partial^\mu K_{\mu}(x) = 0$ (on shell).
  (iii) $K_{\mu}$ transforms according to the adjoint representation under color rotation.
  $K_{\mu}(x) \rightarrow U K_{\mu}(x) U^{-1}$
  (iv) $K$ is invariant under the FP conjugation. $K_{\mu}(x) \rightarrow K_{\mu}(x)$

Thus, we have identified $K^A_{\mu}$ with a physical and massive vector field with correct degrees of freedom as spin-1 massive boson. ($K$ is obtained by a nonlinear but local transformation from the original fields in the CF model.)

• We can construct a modified BRST invariant mass term for the non-Abelian vector field $K^A_{\mu}$:

$$\frac{1}{2} M^2 K^A_{\mu} K^A_{\mu}$$
We have examined the physical unitarity of the CF model.

- The perturbative analysis for the physical unitarity impose a restriction on the valid energy from the parameter of the CF model:

  \[ E^2 < 4 \beta M^2 \]

  \((\beta = 0 \text{ is not allowed})\) in order to confine unphysical modes (ghost, antighost, scalar mode).

  However,

  - The nonperturbative analysis can modify this conclusion.

  The physical unitarity can be recovered in the CF model respecting the FP conjugation invariance. If a set of bound states exists, the physical unitarity can be recovered in a non-perturbative way, as suggested from the structure of the extended and modified BRST algebra of the CF model.

  To show the existence of such bound states of ghost and antighost, the Nambu-Bethe-Salpeter equation is to be solved. This is a topic to be tackled in near future.
Thank you very much for your attention!