Decade of hydrodynamics - what have we learnt?

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there are rescatterings
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EoS has large # of d.o.f.
Summary

- there are rescatterings
- EoS has large # of d.o.f.
- $\eta/s(T)$ has low minimum
The space-time picture:

\[ r = \sqrt{t^2 - \varepsilon^2} = \text{const.} \]

freeze-out

hadrons

QGP

pre-equilibrium

\( \varepsilon \)
Ideal hydrodynamics

local conservation of energy, momentum and baryon number:

\[ \partial_\mu T^{\mu\nu}(x) = 0 \quad \text{and} \quad \partial_\mu N^\mu(x) = 0 \]

local equilibrium, no dissipation:

\[ T^{\mu\nu} = (e + p)u^\mu u^\nu - P g^{\mu\nu} \]
\[ N^\mu = nu^\mu \]

local, macroscopic variables:

- energy density \( e(x) \)
- pressure \( P(x) \)
- flow velocity \( u^\mu(x) \)

matter characterized by: equation of state \( P = P(e, n) \)

Unknowns: initial state, final state, equation of state
Elliptic flow

spatial anisotropy \rightarrow final azimuthal momentum anisotropy

\[ \epsilon_2 \equiv \frac{\int r dr d\phi r^2 \cos(2\phi) e(r,\phi)}{\int r dr d\phi r^2 e(r,\phi)} \]

\[ \nu_2 \equiv \frac{\int d\phi \cos(2\phi) \frac{dN}{d\phi}}{\int d\phi \frac{dn}{d\phi}} \]

sensitive to speed of sound \( c_s^2 = \frac{\partial p}{\partial e} \) and shear viscosity \( \eta \)

- \( \nu_2 \propto \epsilon_2 \)
$p_T$-averaged $v_2$ of charged hadrons:

- Large value observed $\Rightarrow$ there must be rescatterings
- Hydrodynamics seems to work
Pion gas EoS

Sollfrank et al, PRC55, 392 (1997)

• ideal pion gas EoS
• too stiff
• must have “many” hadronic d.o.f’s in the EoS
difficult to say anything about EoS

\[ \pi \text{ and } p v_2(p_T) \text{ in min.bias } \text{Au+Au collisions at RHIC} \]

- **HRG:** no phase transition as close to data as lattice EoS
- **HRG+QGP:** Maxwell construction, closest to data
difficult to say anything about EoS

\[ \pi \text{ and } p \ \nu_2(p_T) \text{ in } b=7 \text{ fm Au+Au collisions at RHIC} \]

- comparison of various lattice parametrizations
- uncertainties of the model . . .
Dissipative hydrodynamics

\[ T^{\mu\nu} = e u^\mu u^\nu - (P + \Pi) \Delta^{\mu\nu} + W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu} \]

**relativistic Navier-Stokes:** \( \pi^{\mu\nu} = 2 \eta \nabla \langle u^\mu u^\nu \rangle \)

- **unstable and acausal**

**Israel-Stewart:** \( \pi^{\mu\nu} \) independent dynamical variable

\[ \langle u^\lambda \partial_\lambda \pi^{\mu\nu} \rangle = \frac{\pi^{\mu\nu}}{\tau_\pi} - \frac{4}{3} \pi^{\mu\nu} \partial_\lambda u^\lambda \]

- \( \eta \): shear viscosity coefficient
- \( \tau_\pi \): shear relaxation time
- \( \pi^{\mu\nu}(t = 0) \)?
\[ \frac{\eta}{s} \]

**Ideal:**

\[ (\epsilon + P) D u^\mu = \nabla^\mu P \]

where \( D = u^\mu \partial_\mu, \quad \nabla^\mu = \Delta^{\mu\nu} \partial_\nu, \quad \Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu \)

**Viscous:**

\[ (\epsilon + P) D u^\mu = \nabla^\mu P - \Delta^\mu_\alpha \partial_\beta \pi^{\alpha\beta} \]

\[ D u^\mu = \frac{1}{\epsilon + P} \nabla^\mu P - \frac{2\eta}{\epsilon + P} \Delta^\mu_\alpha \partial_\beta \left[ \nabla^{\langle \alpha}_\beta u^{\rangle} + \cdots \right] + \cdots \]

\[ \mu = 0 \implies T_s = \epsilon + P : \]

\[ D u^\mu = \frac{1}{\epsilon + P} \nabla^\mu P - \frac{2\eta}{T_s} \Delta^\mu_\alpha \partial_\beta \left[ \nabla^{\langle \alpha}_\beta u^{\rangle} + \cdots \right] + \cdots \]
Comparison with observed $v_2$


- $\eta/s = 0.08$ or $\eta/s = 0.16$ depending on initialization
- $\eta/s = \text{const.}$
modeling the initial state of fluid

**MC-Glauber:** pure phenomenology
density $\propto$ # participants or # binary collisions

**MC-KLN:** Color Glass Condensate using $k_T$-factorization

**IP-Glasma:** Color Glass based, classical Yang-Mills evolution

- all initialize nucleon positions by sampling Woods-Saxon distribution
event-by-event

shape fluctuates event-by-event

all coefficients $v_n$ finite

$$\frac{dN}{dyd\phi} = \frac{dN}{dy} \left[ 1 + \sum_n 2v_n \cos(2(\phi - \Psi_n)) \right]$$


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Sensitivity to $\eta/s$


- higher coefficients are suppressed more by dissipation

- $v_n(\frac{\eta}{s}=0.08)/v_n(\text{ideal})$
- $v_n(\frac{\eta}{s}=0.16)/v_n(\text{ideal})$

$20-30\%$
• models can be distinguished

• MC-Glauber slightly favoured
Relative fluctuations

\begin{itemize}
  \item $\delta v_n \approx \delta \epsilon_n$ independent of $\eta/s$
  \item measurement of initial state?
\end{itemize}
vs. IP-Glasma

Schenke et al. arXiv:1209.6330

\[ \langle v_n^2 \rangle^{1/2} \]

\( p_T [GeV] \)

\( \eta/s = 0.2 \)

- works even better
$\eta/s(T)$ at RHIC


- $v_2(p_T)$ (almost) insensitive to plasma viscosity!
- $v_2(p_T)$ dominated by hadronic viscosity
\( \eta/s(T) \) at LHC (\( \sqrt{s_{NN}} = 2.76 \text{ TeV} \))


- Both plasma and hadronic \( \eta/s \) affect \( v_2(p_T) \)

\[
\begin{align*}
\text{charged hadrons} \\
\text{LHC 2760 AGeV}
\end{align*}
\]

\begin{itemize}
  \item \( \tau_\pi = c_\tau \eta/(e + P) \)
  \item \( v_2 \) sensitive to both \( \eta \) and \( \tau_\pi \)
  \item how to disentangle?
\end{itemize}
What we know

- there are rescatterings

- EoS has large # of d.o.f.

- $\eta/s(T)$ has low minimum
What we are working with

- extracting $\eta/s(T)$ difficult
- $\tau_\pi$?
- initial state? (there is progress)
- fluctuations provide more constraints