Confinement viewed with dyon and dimeron ensembles

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5. Summary
1. Introduction, motivation

Reinvestigation of an old issue: semiclassical approach to QCD path integral

[’t Hooft, ’76; Callan, Dashen, Gross, ’78; Gross, Pisarski, Yaffe, ’81;
Ilgenfritz, M.-P., ’81; Shuryak, ’82; Diakonov, Petrov, ’84]

\[ T = 0: \text{Belavin-Polyakov-Shvarts-Tyupkin (BPST) instantons} \]
\[ T > 0: \text{Harrington-Shepard (HS) calorons} \]

BPST: \[ A_{a,\mu}^{\text{inst}}(x) = R_{a\alpha} \bar{\eta}_{\mu\nu}^{\alpha} \frac{x_{\nu}}{x^{2} + \rho^{2}}x^{\mu}, \quad A_{a,\mu}^{\text{antiinst}} : \bar{\eta} \leftrightarrow \eta \]

superpositions of “pseudo-particles”

\[ A_{\text{class}}(z = \{\rho^{(i)}, x^{(i)}, R^{(i)}\}) = \sum_{i=1}^{N_{+} + N_{-}} A_{a,\mu}^{(i)}(x - x^{(i)}, \rho^{(i)}, R^{(i)}), \]

in order to approximate the functional integral by \[ A = A_{\text{class}}(z) + \varphi \]

\[ \int DA \exp(-S[A]) \simeq \sum_{\text{class}} \int [dz] \exp(-S[A_{\text{class}}]) \int D\varphi \exp \left( -\frac{1}{2!} \int \varphi \frac{\delta^{2}S}{\delta A^{2}}|_{A_{\text{class}}\varphi} \right) + \cdots \]

\[ \int [dz] \text{ – modular space integration (“collective coordinates”).} \]

Turns path integral into statstistical mechanics partition function.
• Powerful approach for non-perturbative phenomena like chiral symmetry breaking and $U_A(1)$ problem $\iff$ confinement hard to explain
  [Reviews by Schäfer, Shuryak, ’98; Forkel, ’00; Dyakonov, ’03;...]

• (Anti)selfdual solutions seen on the lattice with “cooling”, “smoothing”,...
  [Teper, ’86; Ilgenfritz, Laursen, M.-P., Schierholz, Schiller, ’86; Polikarpov, Veselov, ’88; ...]

Old idea to implement confinement $\implies$ increase entropy by “dissociation” of instantons into constituents (“instanton quarks”).

• $T = 0$: “meron” mechanism [Callan, Dashen, Gross, ’77 - ’79].
  [Lenz, Negele, Thies, ’03-’04; M. Wagner, ’06]

• $T > 0$: KvBLL (multi-) calorons with non-trivial holonomy – “dyons”

Here simulate caloron, dyon ensembles – for $0 < T < T_c$, meron pair (“dimeron”) ensembles – for $T = 0$. 
2. Instantons at \( T > 0 \): calorons with non-trivial holonomy

Partition function

\[
Z_{YM}(T, V) \equiv \text{Tr} \, e^{-\frac{\hat{H}}{T}} \propto \int DA \, e^{-S_{YM}[A]} \quad \text{with} \quad A(\vec{x}, x_{4}+b) = A(\vec{x}, x_{4}), \; b = 1/T. 
\]

Old treatment with HS caloron solutions

\( \equiv \) \( x_{4} \)-periodic instanton chains

\[ A^{a_{\text{HS}}} = \tilde{\eta}_{a}^{\mu} \partial_{\nu} \log(\Phi(x)) \]

\[
\Phi(x) = 1 + \sum_{k \in \mathbb{Z}} \frac{\rho^{2}}{(\vec{x} - \vec{z})^{2} + (x_{4} - z_{4} - kb)^{2}} \\
= 1 + \frac{\pi \rho^{2}}{b|\vec{x} - \vec{z}|} \frac{\sinh \left( \frac{2\pi}{b} |\vec{x} - \vec{z}| \right)}{\cosh \left( \frac{2\pi}{b} |\vec{x} - \vec{z}| \right) - \cos \left( \frac{2\pi}{b} (x_{4} - z_{4}) \right)}
\]

\( ^{t} \text{Hooft symbols:} \quad \eta_{a\mu\nu} = \varepsilon_{a\mu\nu}, \quad \eta_{a\mu4} = -\eta_{a4\mu} = \delta_{a\mu} \quad \text{for} \quad \mu, \nu = 1, 2, 3, \quad \eta_{a44} = 0; \quad \tilde{\eta}_{a\mu\nu} = (-1)^{\delta_{\mu4} + \delta_{\nu4}} \eta_{a\mu\nu}. \)

HS caloron exhibits trivial holonomy, i.e. Polyakov loop behaves as:

\[
\frac{1}{2} \text{tr} \mathbf{P} \exp \left( i \int_{0}^{b=1/T} A_{4}(\vec{x}, t) \, dt \right) \xrightarrow{\|\vec{x}\| \rightarrow \infty} \pm 1.
\]
Kraan - van Baal - Lee - Lu solutions (KvBLL)  
= (multi-) calorons with non-trivial asymptotic holonomy ($SU(2)$)  

\[ P(\vec{x}) = P \exp \left( i \int_0^{b=1/T} A_4(\vec{x}, t) \, dt \right) \bigg|_{|\vec{x}| \to \infty} \quad P_\infty = e^{2\pi i \omega \tau_3} \not\in \mathbb{Z}(2) \]

Holonomy parameter:  \( 0 \leq \omega \leq \frac{1}{2}, \quad \omega = \frac{1}{4} \) – maximally non-trivial holonomy.

Action density of an $SU(3)$ caloron (van Baal, ’99)  
\[ \implies \text{not a simple } SU(2) \text{ embedding into } SU(3) !! \]
SU(2) KvBLL caloron with non-trivial holonomy: $0 < \omega < \frac{1}{2}$


- (anti)selfdual with topological charge $Q_t = \pm 1$,
- has two centers at $\vec{x}_1, \vec{x}_2$ ⇒ “instanton quarks”,
  carry opposite magnetic charges (visible in maximally Abelian gauge),
- limiting cases:
  - $\omega \to 0$ ⇒ ‘old’ HS caloron,
  - $|\vec{x}_1 - \vec{x}_2|$ large ⇒ two static BPS monopoles or “dyon pair” (DD) with topological charges ($\sim$ masses)
    $q^{dyon}_t = 2\omega, \quad 1 - 2\omega$,
  - $|\vec{x}_1 - \vec{x}_2|$ small ⇒ non-static single caloron (CAL).

- $L(\vec{x}) = \frac{1}{2} \text{tr} P(\vec{x}) \to \pm 1$ close to $\vec{x} \simeq \vec{x}_{1,2}$ ⇒ “dipole” structure.
- dyons localizable from zero-modes of the fermion Dirac operator.
KvBLL caloron portrait
Action density

Polyakov loop

CAL

DD

Seen on the lattice with various filter techniques.
[V. Bornyakov, E.-M. Ilgenfritz, B. Martemyanov, M. M.-P., ..., '02 - '09]
Simulating caloron ensembles

[Gerhold, Ilgenfritz, M.-P., NPB 760, 1 (2007)]

Random randomly localized superpositions of KvBLL calorons in a 3d box.

– Superpone in algebraic gauge ⇒ $A_4$-components fall off.
– Gauge rotation into periodic gauge

$$A_{\mu}^{per}(x) = e^{-2\pi i x_4 \vec{\omega} \cdot \vec{\tau}} \cdot \sum_i A_{\mu}^{(i), alg}(x) \cdot e^{+2\pi i x_4 \vec{\omega} \cdot \vec{\tau}} + 2\pi \vec{\omega} \cdot \delta_{\mu, 4}.$$ 

Study the influence of the holonomy

• same fixed holonomy for all (anti)calorons: $\mathcal{P}_\infty = \exp 2\pi i \omega \tau_3$,
  compare: $\omega = 0$ – trivial versus $\omega = 1/4$ – maximally non-trivial,

• Fix parameters:
  temperature: $T = 1$ fm$^{-1} \simeq T_c$,  
density: $n = 1$ fm$^{-4}$,  
scale size: fixed $\rho = 0.33$ fm
Polyakov loop correlator $\rightarrow$ quark-antiquark free energy

\[ F(R) = -T \log \langle L(\vec{x})L(\vec{y}) \rangle, \quad R = |\vec{x} - \vec{y}| \]

$\Rightarrow$ Non-trivial (trivial) holonomy creates long-distance coherence (incoherence) and (de)confines for standard instanton or caloron liquid model parameters.
3. Dyon gas ensembles and confinement

Working hypothesis (cf. Polyakov, ’77):
Confinement evolves from magnetic monopoles effectively in 3D.
Here: monopoles = dyons (KvBLL caloron constituents) for $0 < T < T_c$.

Assume:
integration measure over KvBLL caloron moduli space
rewritten in terms of dyon degrees of freedom,
⇒ difficult task for (still unknown) general multi-caloron solutions.

Diakonov, Petrov, ’07:
proposed integration measure for all kind dyons
(Abelian fields; no antidyons, i.e. CP is violated).
Dyon ensemble statistics analytically solved ⇒ confinement.

However, observation from numerical simulation:
Moduli space metric satisfies positivity only for a small fraction
of dyon configurations and only for low density ⇒ inconsistent metric.
Simplify the model:

- Far-field limit, i.e. purely Abelian monopole fields, non-trivial holonomy.
- Neglect moduli space metric and describe random monopole gas.

[Bruckmann, Dinter, Ilgenfritz, Maier, M.-P., Wagner, PRD 85, 034502 (2012)]

Monopole field:

\[ a_0(r; q) = \frac{q}{r}, \quad a_1(r; q) = -\frac{qy}{r(r - z)}, \quad a_2(r; q) = +\frac{qx}{r(r - z)}, \quad a_3(r; q) = 0, \]

\( r = (x, y, z), \quad r = |r| - 3d \) distance to the dyon center, \( q - \) magnetic charge.

Holonomy to be added to superponed monopole fields:

\[ A_0 \rightarrow 2\pi \omega T \tau_3, \]

\[ P(r) \equiv \frac{1}{2} \text{Tr} \left( \exp \left( i \int_0^{1/T} dx_0 \ A_0(x_0, r) \right) \right) \rightarrow \frac{1}{2} \text{Tr} \left( \exp \left( 2\pi i \omega \tau_3 \right) \right) = \cos(2\pi \omega), \]
Superposition of gauge fields of $2K$ dyons:

$$A_\mu(r) = \left( \delta_\mu_0 2\pi \omega T + \frac{1}{2} \sum_{i=1}^{K} \sum_{m=1}^{2} a_\mu(r - r_i^m; q_m) \right) \tau_3,$$

$r_i^m$ and $q_m = -(-1)^m$ – positions and magnetic charges of $i$-th dyon ($m = 1$), antidyon ($m = 2$), respectively.

Local Polyakov loop $P(r)$:

$$P(r) = \cos \left( 2\pi \omega + \frac{1}{2T} \Phi(r) \right), \quad P(r) \bigg|_{\omega=1/4} = -\sin \left( \frac{1}{2T} \Phi(r) \right),$$

where

$$\Phi(r) \equiv \sum_{i=1}^{K} \sum_{m=1}^{2} \frac{q_m}{|r - r_i^m|} = \sum_{i=1}^{K} \left[ \frac{1}{|r - r_1^i|} - \frac{1}{|r - r_2^i|} \right].$$

Compute free energy of a static $\bar{Q}Q$ pair from Polyakov loop correlator

$$F_{\bar{Q}Q}(d) = -T \ln \left\langle P(r) P^\dagger(r') \right\rangle, \quad d \equiv |r - r'|.$$
Expectation value:

\[ \langle O \rangle = \int \prod_{i=1}^{K} dr_i^1 dr_i^2 O \left( \{ r_i^1, r_i^2 \} \right) \bigg/ \int \prod_{i=1}^{K} dr_i^1 dr_i^2 \]

\[ = \int \prod_{i=1}^{K} dr_i^1 dr_i^2 O \left( \{ r_i^1, r_i^2 \} \right) / V^{2K}, \]

\( V \) – spatial volume \( \implies \rho = \frac{2K}{V} \) – density of monopole gas.
\( \omega, T, \) and \( \rho \) – basic parameters of the model.

Expectation values can be computed numerically generating random distributions of the dyon positions in the volume \( V \).

Monopole fields have infinite range \( \implies \) strong finite size effects (well-known e.g. in plasma physics)

– use infinite identical replica of volumes,
– separate short-distance and long-distance contributions.
\( \implies \) talk by B. Maier – this section
Fortunately, model can be solved analytically:

Infinite-volume limit \( V \to \infty , \quad d = |\mathbf{r} - \mathbf{r}'| \) large

\[
\langle P(\mathbf{r})P(\mathbf{r}') \rangle = \frac{1}{2} \exp \left( - \frac{\pi d \rho}{2 T^2} + \text{const.} \right)
\]

\[\Rightarrow \text{free energy} \quad F_{\bar{Q}Q}(d) = \sigma d + \text{const.}\]

\[\Rightarrow \text{string tension} \quad \sigma = \frac{\pi \rho}{T}.
\]

At arbitrary finite distance \( d \):

Polyakov loop correlator can be obtained by numerical integrations.
Comparison with Ewald’s method:

Ewald’s result extrapolated to $V \to \infty$  

$F_{\bar{Q}Q}(d)/T$ vs. $d$ for $V \to \infty$

$\implies$ excellent agreement !!

$\implies$ simplest dyon gas model provides strict confinement.
4. $T = 0$: Simulating dimeron ensembles


– KvBLL-like solutions in Euclid. 4d Yang-Mills theory unknown.
– Possibility: Dimeron (D) configurations [De Alfaro, Fubini, Furlan, '76 - '77]

single (anti)merons: $Q_t = \pm 1/2, \quad S \to \infty$. 

(anti)dimeron = (anti)meron pair ($r \equiv$ regular gauge, $SU(2)$): $Q_t = \pm 1$

$$A_{\mu}^{(D,r)}(x; \{x_0, a, u\}) = \left[ \frac{(x - x_0 + a)_\nu}{(x - x_0 + a)^2} + \frac{(x - x_0 - a)_\nu}{(x - x_0 - a)^2} \right] u^\dagger \sigma_{\mu\nu} u,$$

$$\sigma_{\mu\nu} := \eta_{a\mu\nu} \tau_a/2, \quad \text{for } D \leftrightarrow \overline{D} \ \text{replace } \eta \leftrightarrow \bar{\eta}, \quad u - \text{colour rotations}.$$  

Limiting cases:

$a \to 0 \quad \Rightarrow \quad$ regular gauge instanton,

$a \to \infty \quad \Rightarrow \quad$ well-separated merons with ‘locked’ colour orientation.

D has 11 collective coordinates $z \equiv \{x_0, a, u\}$

(instead of 8 for one instanton, 14 for two single, colour-unlocked merons).

$\Rightarrow \quad$ corresponding increase of entropy in the path integral.
Before superpone (anti)dimerons put them into singular gauge ("s")

\[ A^{(D,s)}_\mu \sim \frac{1}{x^3} \text{ for } |x| \gg |a|, \text{ i.e. better localized.} \]

For numerical integrations (action, top. charge, parallel transporters etc.) meron and gauge singularities have to be regularized.

**Superpositions:** (anti)dimerons superponed (neglect meron-antimeron pairs)

\[
A_\mu(x, \{z_I, \bar{z}_{\bar{I}}\}) = \sum_I N_D A^{(D,s)}_\mu(x, \{z_I\}) + \sum_{\bar{I}} N_{\bar{D}} \bar{A}^{(\bar{D},s)}_\mu(x, \{\bar{z}_{\bar{I}}\}).
\]

**Partition function:**

\[
Z = \int N_D N_{\bar{D}} \prod_{I, \bar{I}} dz_I d\bar{z}_{\bar{I}} \exp \left\{ -S[A_\mu(x, \{z_I, \bar{z}_{\bar{I}}\})] \right\},
\]

with

\[
S[A] = \frac{1}{2 g^2} \int d^4 x \text{ tr } \{F_{\mu\nu} F_{\mu\nu}\} =: \int d^4 x \ s(x),
\]

\[
F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - i [A_\mu(x), A_\nu(x)].
\]

In the following study \( g^2 \) dependence ("temperature").
Simulating the dimeron gas:

- Take statistical weight of dimeron confs. $\exp -S[A]$ into account.

Multi-layered multigrid to control boundary + discretization effects.

Inner measurement volume $\Leftrightarrow$ approximate constant action density.
• MC method: Metropolis.
• Ensemble: $N_D = 243$, $N_{\overline{D}} = 244$ in whole volume.
• $g^2 \in \{1, 25, 100, 1000, \infty\}$.

Measurements:
• Ensemble parameters: intermeron distances, neighbour densities, colour correlations,...
• Topology: spatial top. charge distribution, topological susceptibility.
• Wilson loops: static $\bar{Q}Q$-potential, string tension.
Results:

Average meron distance within dimerons $2\langle |a| \rangle$ versus $g^2$.

$\rightarrow$ Dimerons dissociate into their constituents with increasing $g^2$. 
Spatial topological order / disorder:

average radial density of neighbour pseudoparticles vs. distance $d$
equal (black), opposite (red) sign topological charges

$g^2 = 1$

$g^2 = 100$
Nearest neighbour dimeron colour-angle distribution $\langle f_\alpha \rangle$

black squares: equal top. charge,
red bullets: opposite top. charge,
blue crosses: random distribution ($g^2 = \infty$).

$g^2 = 1$

$\Rightarrow \alpha \simeq \pi/2$ dominant for DD pairs, $D\bar{D}$ pairs randomly mutually orientated.

$\Rightarrow$ qualitatively understood from two-dimeron ($\simeq$ two-instanton) interactions.
Wilson loops $\log < W >$ vs. area $A$

\[ g^2 = 1, 25, 100, 1000, \infty \]

static potential $V(R)$:

\[ g^2 = 1, 25, 100 \]
Dimensionless ratio: string tension / top. susceptibility

\[ \frac{\sigma^{1/2}}{\chi^{1/4}} \text{ vs. } g^2 \]

\[ \frac{\chi^{1/4}}{\sigma^{1/2}} \approx 0.30, \ldots, 0.55. \]

\[ \Rightarrow \text{ Compatible with lattice } (SU(2)): \frac{\chi^{1/4}}{\sigma^{1/2}} = 0.483 \pm 0.006 \] [Lucini, Teper, 01]

\[ \Rightarrow \text{ Compatible also with simulations of single meron and regular instanton ensembles } [Lenz, Negele, Thies, 08] \]
5. Summary

- Standard instanton gas/liquid remains phenomenologically important: chiral symmetry breaking, solution of $U_A(1)$, ..., but fails to explain confinement.

- $0 < T < T_c$: KvBLL caloron gas model with non-trivial holonomy very encouraging for description of confinement. Model can be improved.

- Non-interacting Abelian dyon gas model provides strict confinement. Ewald’s method allows to keep finite-size effects under control and provides same infinite volume result. Full modular space metric (?) should be taken into account.

- $T = 0$: Dimerons play similar role as KvBLL calorons for $0 < T < T_c$. Shows Callen-Dashen-Gross mechanism of meron disorder at strong coupling. Reasonable results for topological susceptibility in units of the string tension obtained.