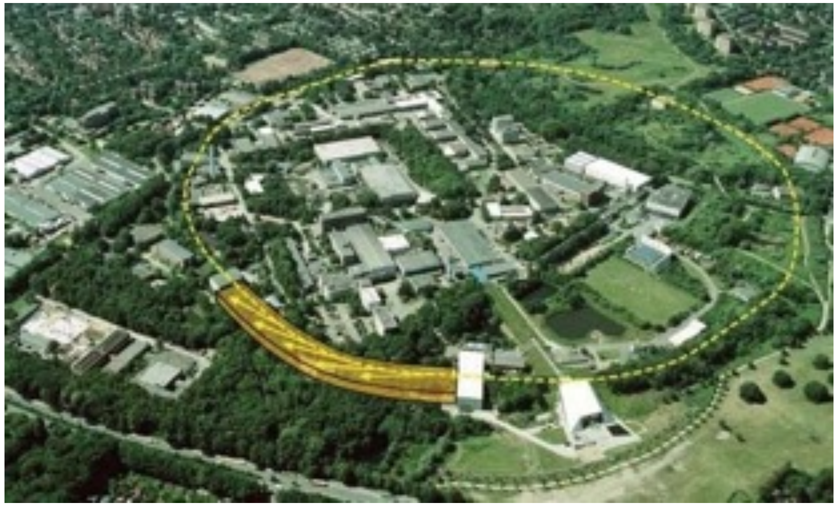
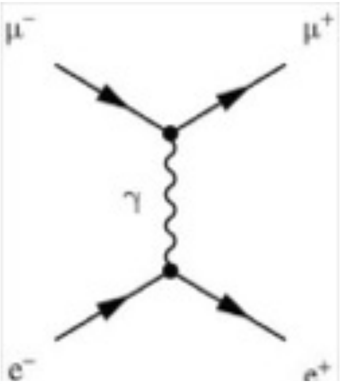
$e^+ e^-$ Collider (DESY)



How Colour is Found

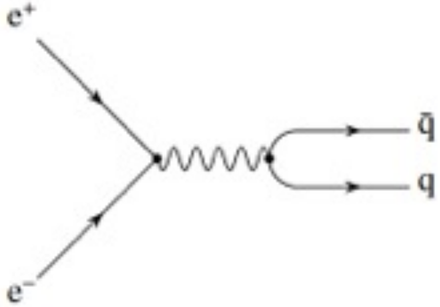
Relate to "standard" cross section

$$\sigma_{\mathrm{tot}}(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3Q^2}$$



Production of quarks:

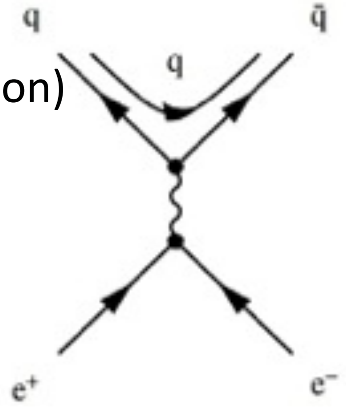
photon couples to electric charge Z_q



$$\sigma_{\mathrm{tot}}(e^+e^- \to q_i\bar{q}_i) = Z_q^2\,\sigma_{\mathrm{tot}}(e^+e^- \to \mu^+\mu^-)$$

Specialty: gluons cannot propagate freely

- Photon "dissociates" into $u\bar{u}$, $d\bar{d}$, $s\bar{s}$.. Pair (no "open" flavour")
  intermediate resonance - vector meson (same quantum numbers as photon)
  $\sqrt{s} = M_V$ $E_{e^+} + E_{e^-} = \sqrt{s}$
- Quarks carry kinetic energy →creation of new $q\bar{q}$-pairs →mesons
  $\sqrt{s} \geq M_1 + M_2$



How Colour is Found

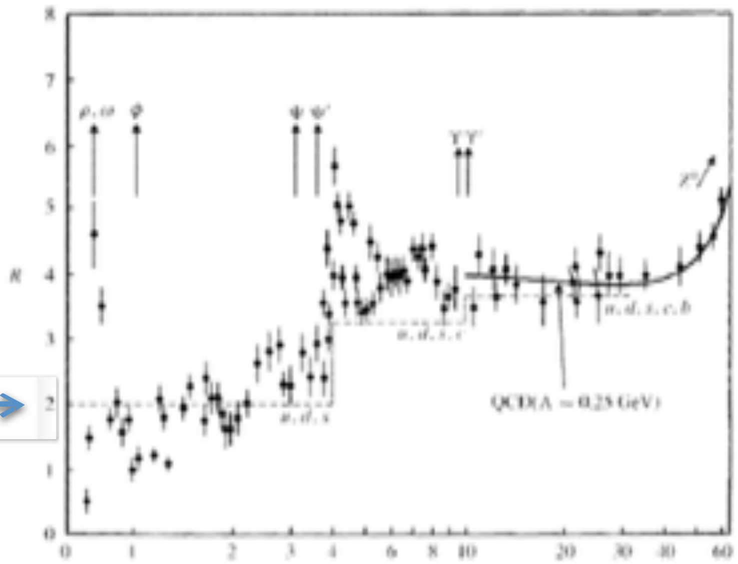
Lets explore the energy regime:

$$R = \frac{\sigma_{\text{tot}}(e^+e^- \to \text{Hadronen})}{\sigma_{\text{tot}}(e^+e^- \to \mu^+\mu^-)} = \underbrace{\sum_{j=1}^{N_x}}_{\text{unbekannte Eigenschaften}} \frac{\sum_{i=1}^{N_{\text{Flavour}}} Z_{q_i}^2\, \sigma_{\text{tot}}^{\mu^+\mu^-}}{\sigma_{\text{tot}}^{\mu^+\mu^-}}$$

measure $R(\sqrt{s}\,)$

only u,d,s quarks for

$\sqrt{s} < 3\,\text{GeV}$



$$\mathbf{R} = \sum_{j=1}^{N}\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) = \sum_{j=1}^{N}\frac{2}{3}$$

As R = 2 → N = 3

Inner degrees of freedom of

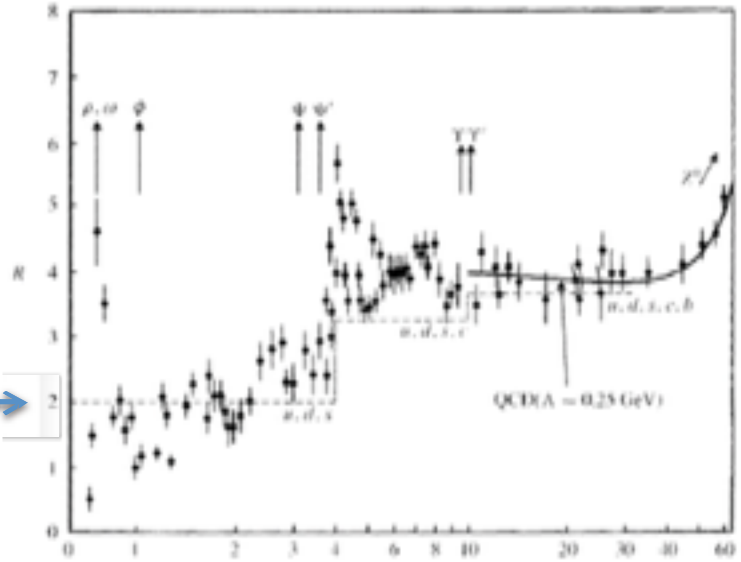
Quarks: N= 3 = colours

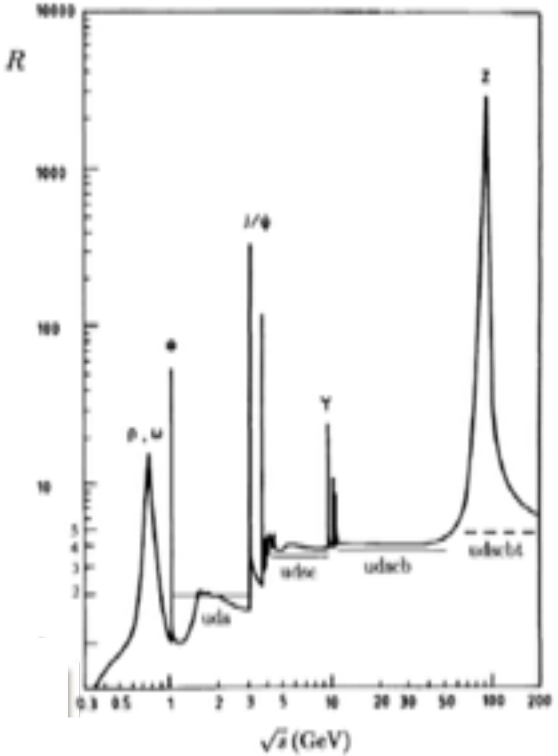
How Colour is Found

Lets explore the energy regime:

$$R = \frac{\sigma_{\text{tot}}(e^+e^- \to \text{Hadronen})}{\sigma_{\text{tot}}(e^+e^- \to \mu^+\mu^-)} = \underbrace{\sum_{j=1}^{N_x}}_{\text{unbekannte Eigenschaften}} \frac{\sum_{i=1}^{N_{\text{Flavour}}} Z_{q_i}^2 \sigma_{\text{tot}}^{\mu^+\mu^-}}{\sigma_{\text{tot}}^{\mu^+\mu^-}}$$

measure $R(\sqrt{s})$





only u,d,s quarks for

$$\sqrt{s} < 3\,\text{GeV}$$

$$\mathbf{R} = \sum_{j=1}^{N}\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) = \sum_{j=1}^{N}\frac{2}{3}$$

As R = 2 $\rightarrow$ N = 3

Inner degrees of freedom of

Quarks: N= 3 = colours

Colour and New Flavour

Search for new quarks:

- Look for “step-like” structure $\sqrt{s} < 3.7\,\mathrm{GeV}$ $\sqrt{s} < 10\,\mathrm{GeV}$

  Production of new quark flavours of charm z = 2/3 and beauty/bottom z = -1/3

$$R = 3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}\right)$$

$$\underbrace{\underbrace{\underbrace{u\quad d\quad s}_{3\cdot\frac{6}{9}=2}\quad c}_{3\cdot\frac{10}{9}=3.3}\quad b}_{3.\frac{11}{9}=3.7}$$

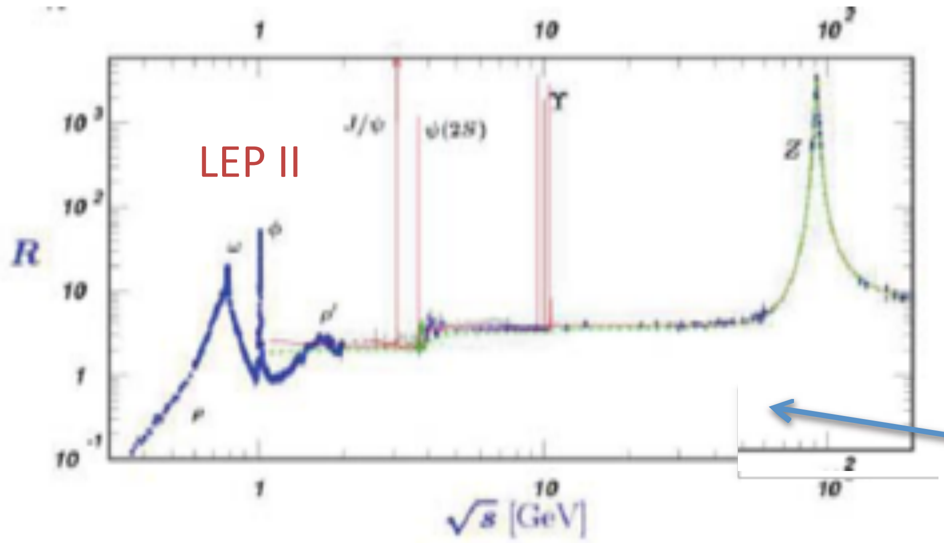
$$\sqrt{s} \sim 4\,\mathrm{GeV} \quad e^+e^- \rightarrow c\bar{c} \rightarrow hadrons$$

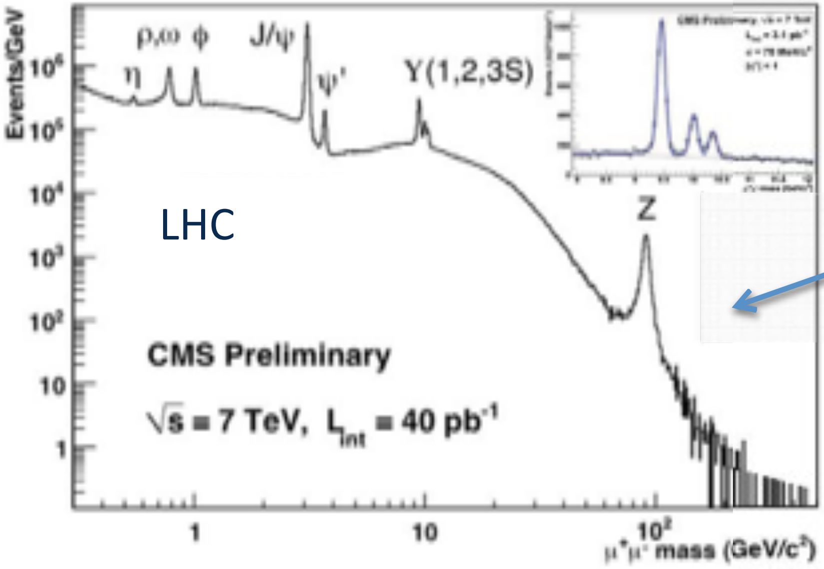
$$e^+e^- \rightarrow \tau^+\tau^- \rightarrow hadrons$$

$$\sqrt{s} \sim 10\,\mathrm{GeV} \quad e^+e^- \rightarrow b\bar{b} \rightarrow hadrons$$

- Look for appearance of new “vector meson” resonance

R-plot and Vectormeson/Boson





$$\sqrt{s} > 55\,\mathrm{GeV}$$

Rise of R towards maximum at 90 GeV:
Z^0 Boson production

$$e^{+}e^{-} \rightarrow Z^{0} \rightarrow q\bar{q} \rightarrow hadrons$$

$$\mu^{+}\mu^{-} \leftarrow X^{0} \leftarrow q\bar{q} \leftarrow hadrons$$

How Gluons are Found

Bremsstrahlung:

Acceleration of electric charge:

bremsstrahlung: $e^- \rightarrow e^- + \gamma$

cross section: $\sigma \approx \alpha_{em}$

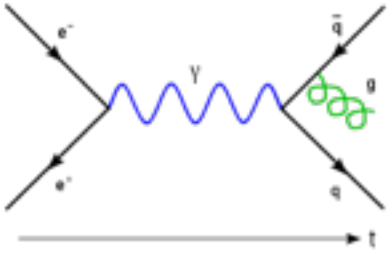
Acceleration of colour charge:

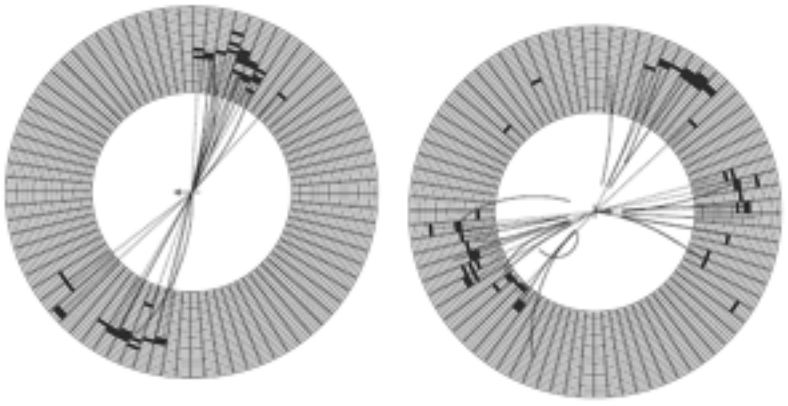
gluon bremsstrahlung: $q \rightarrow q + g$

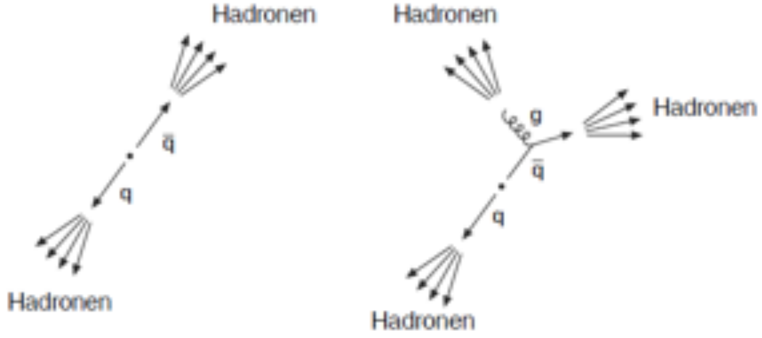
cross section: $\sigma \approx \alpha_s$

Measurement of ratio: $\dfrac{\sigma(3Jet)}{\sigma(2Jet)} \approx \alpha_s$

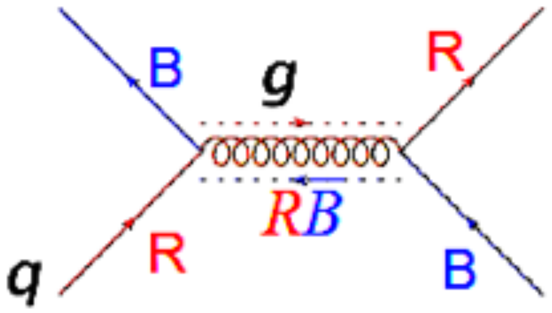
First observation at DESY 1978

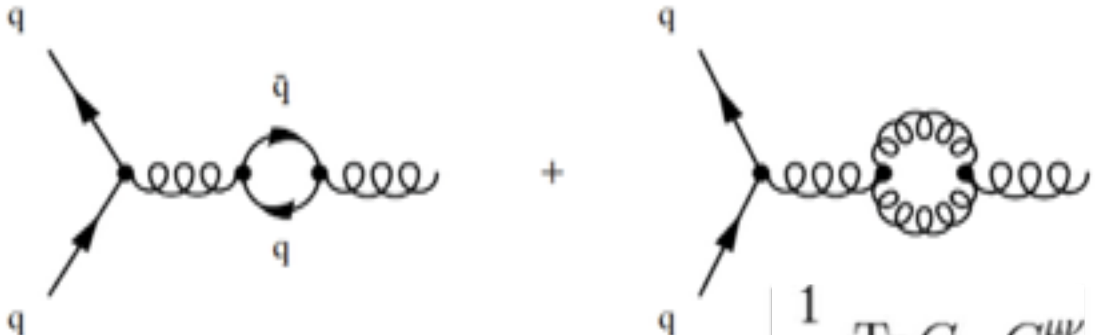






Strong Interaction : Coupling

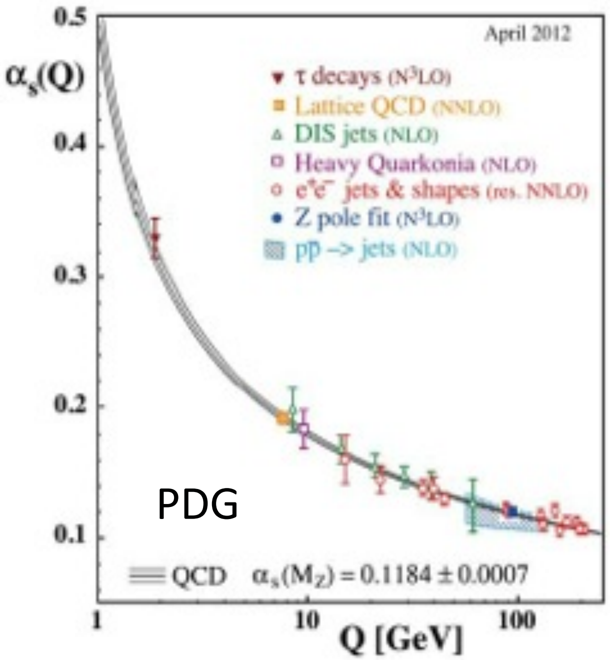




Vacuum polarization

$$\frac{1}{4g^2}\operatorname{Tr} G_{\mu\nu}G^{\mu\nu}$$

$$\alpha_s(k^2) \overset{\text{def}}{=} \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{\beta_0 \ln(k^2/\Lambda^2)},$$



PDG

What about the QCD Potential ?

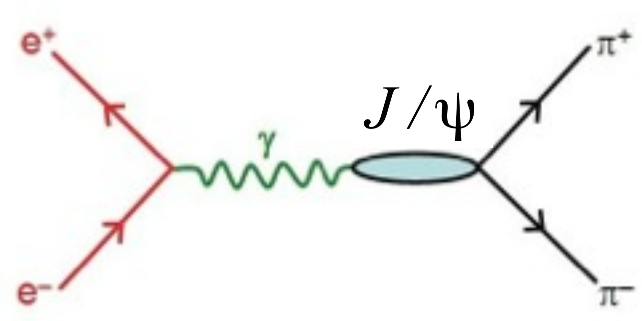
What is the equivalent of the Coulomb potential in QCD ?

Atomic physics: study hydrogen atom and spectroscopy

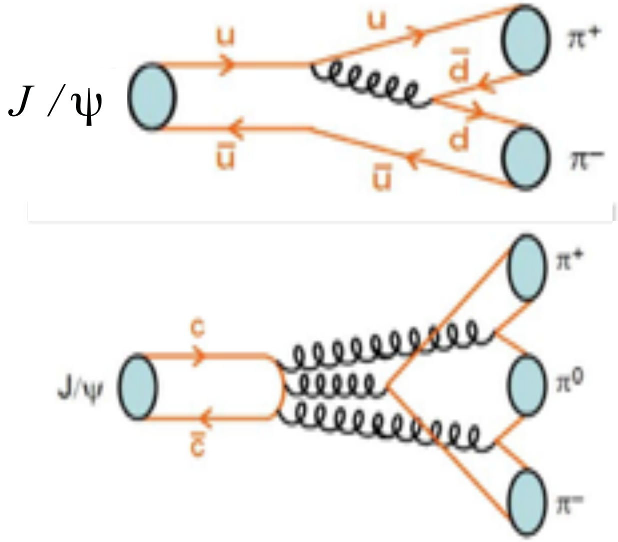
e$^-$ and p move non-relativistically in central potential

QCD: study hydrogen-like system

study $q\bar{q}$-system (quarkonium) with heavy quarks (c, b) with $m_q = M_{quarkonium}/2$



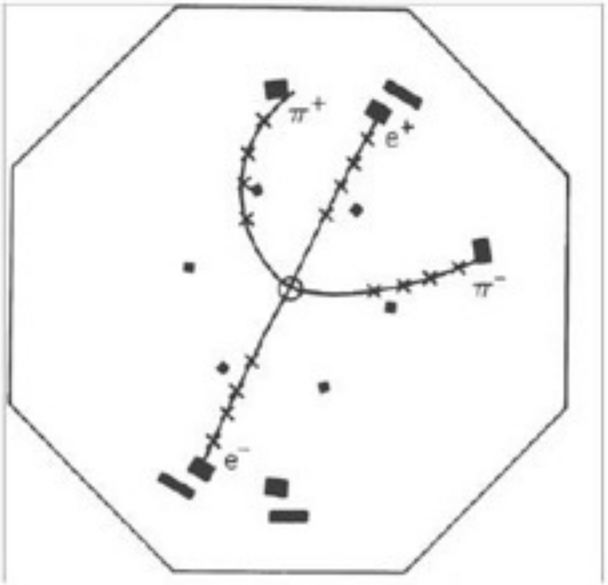
Resonance production



Discovery of Charm Quark I

Logbuch 1974
November Revolution

B. Richter
Nobelpreis 1976



ψ

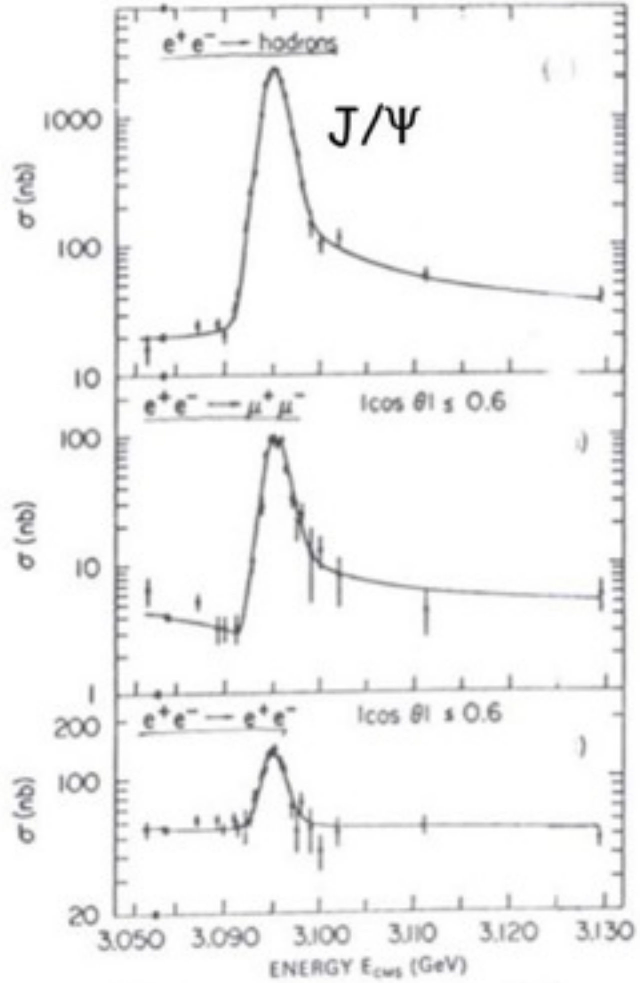
Discovery of Charm Quark II

SLAC, SPEAR

$$e^+e^- \rightarrow \psi \rightarrow hadrons$$

$$e^+e^- \rightarrow \psi \rightarrow \mu^+\mu^-$$

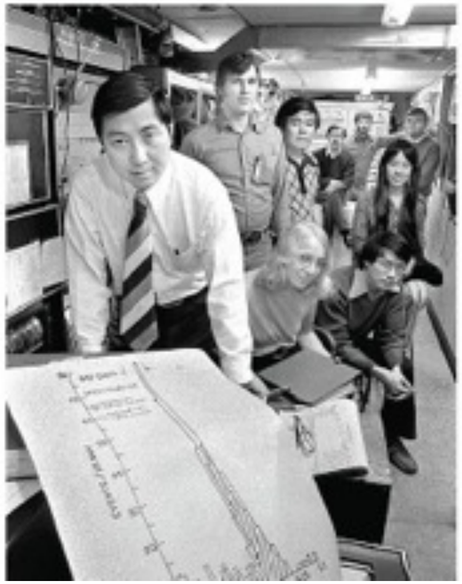
$$e^+e^- \rightarrow \psi \rightarrow e^+e^-$$

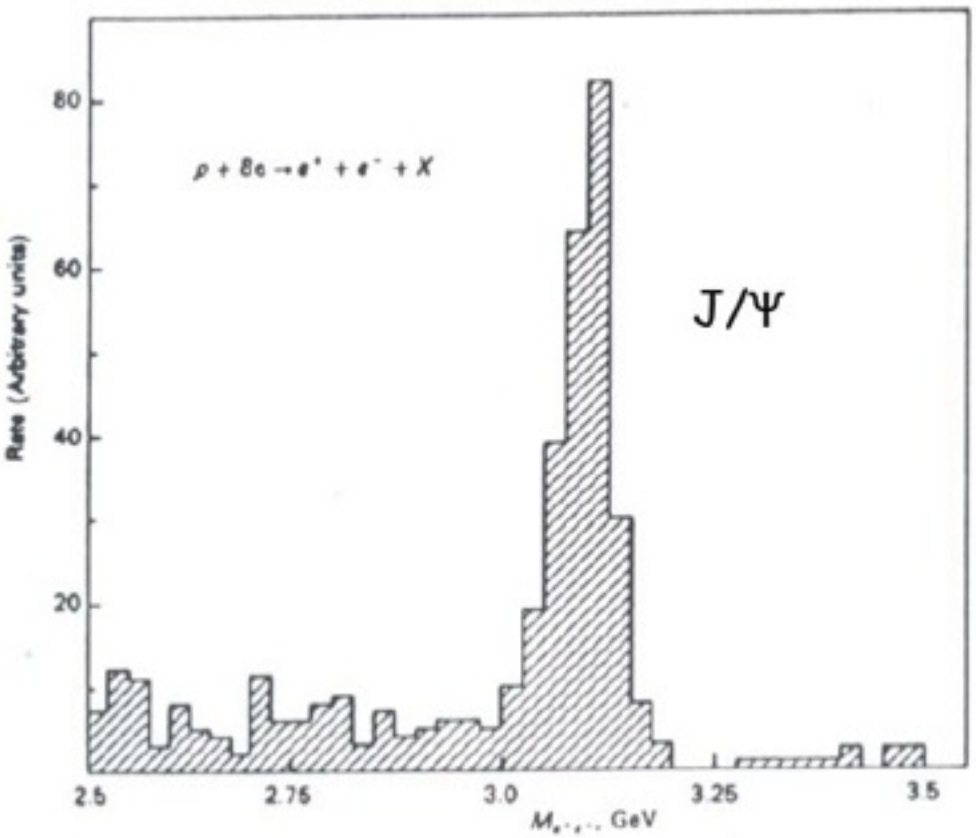


Discovery of Charm Quark III (1974)

Brookhaven BNL

$$p + Be \rightarrow \underbrace{e^{+} + e^{-}}_{J} + X$$

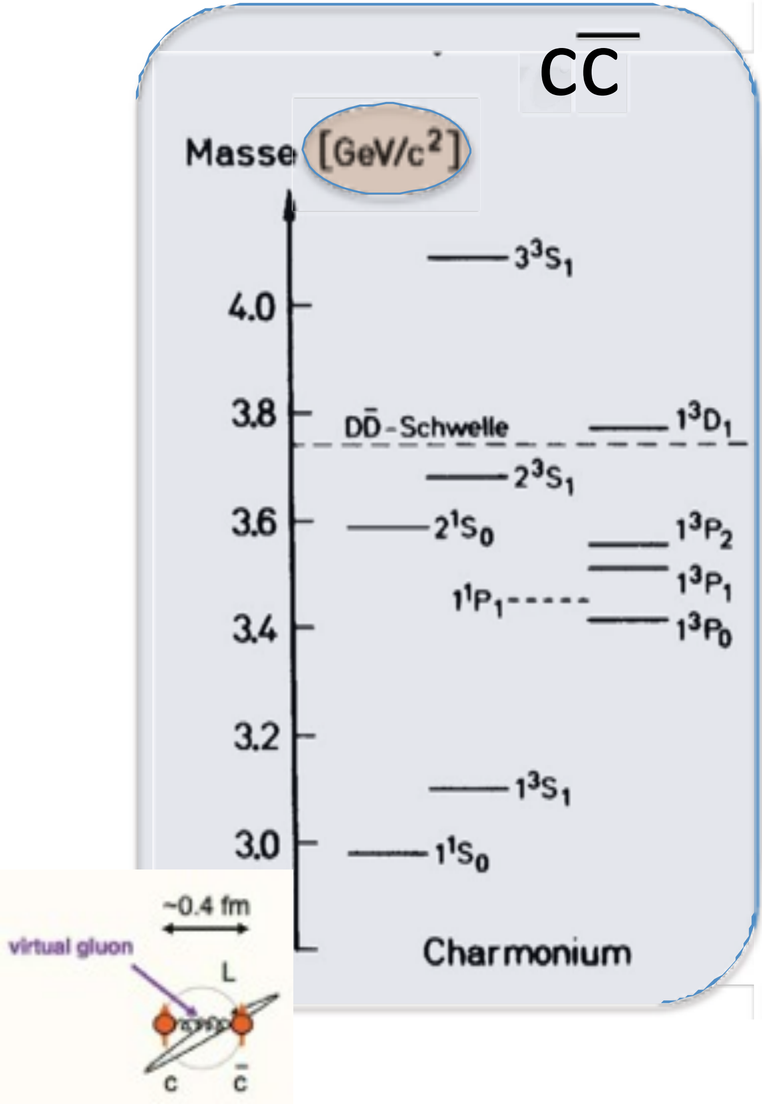




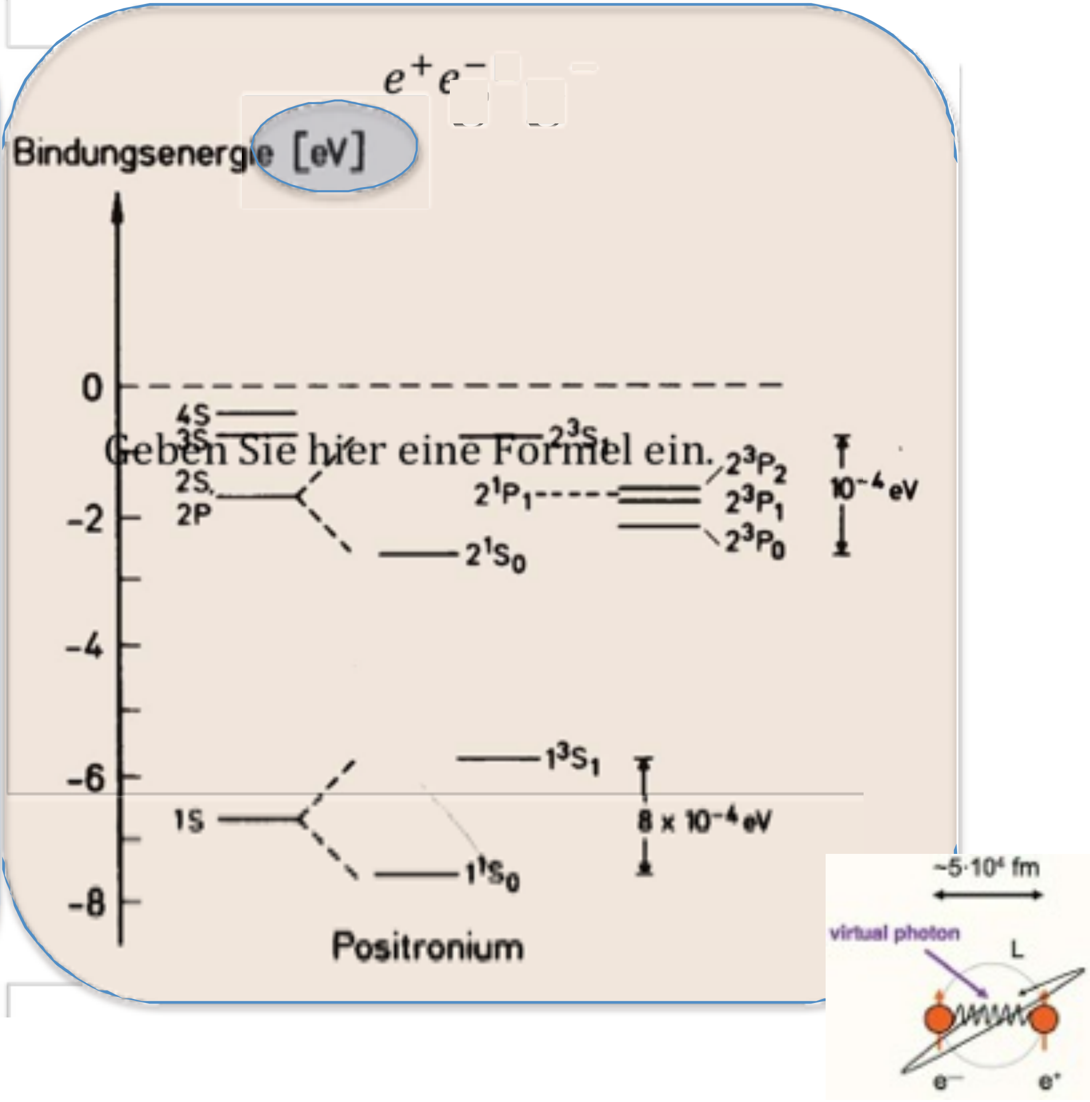
S. Ting
Nobelpreis 1976

Compare: Charmonium - Positronium

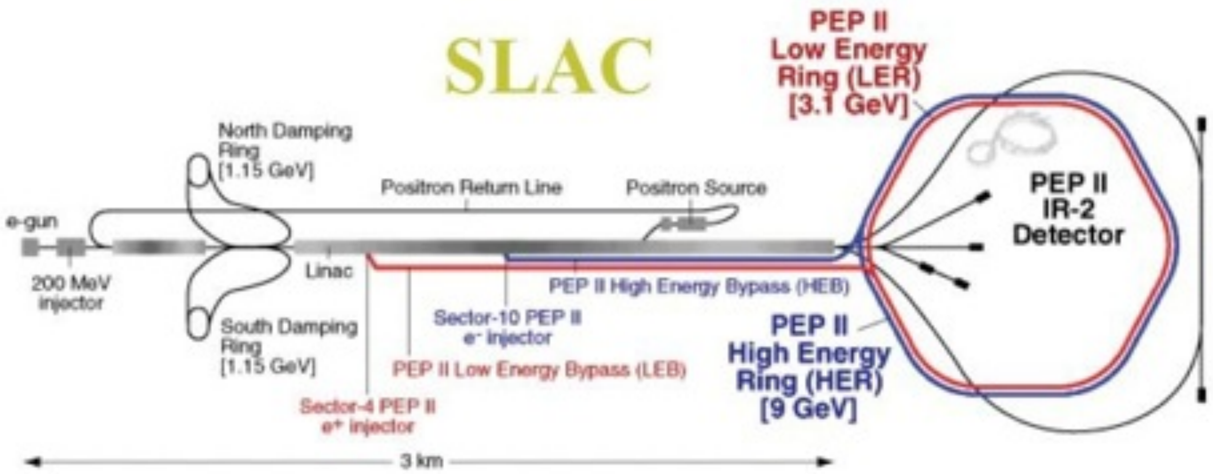
$c\overline{c}$

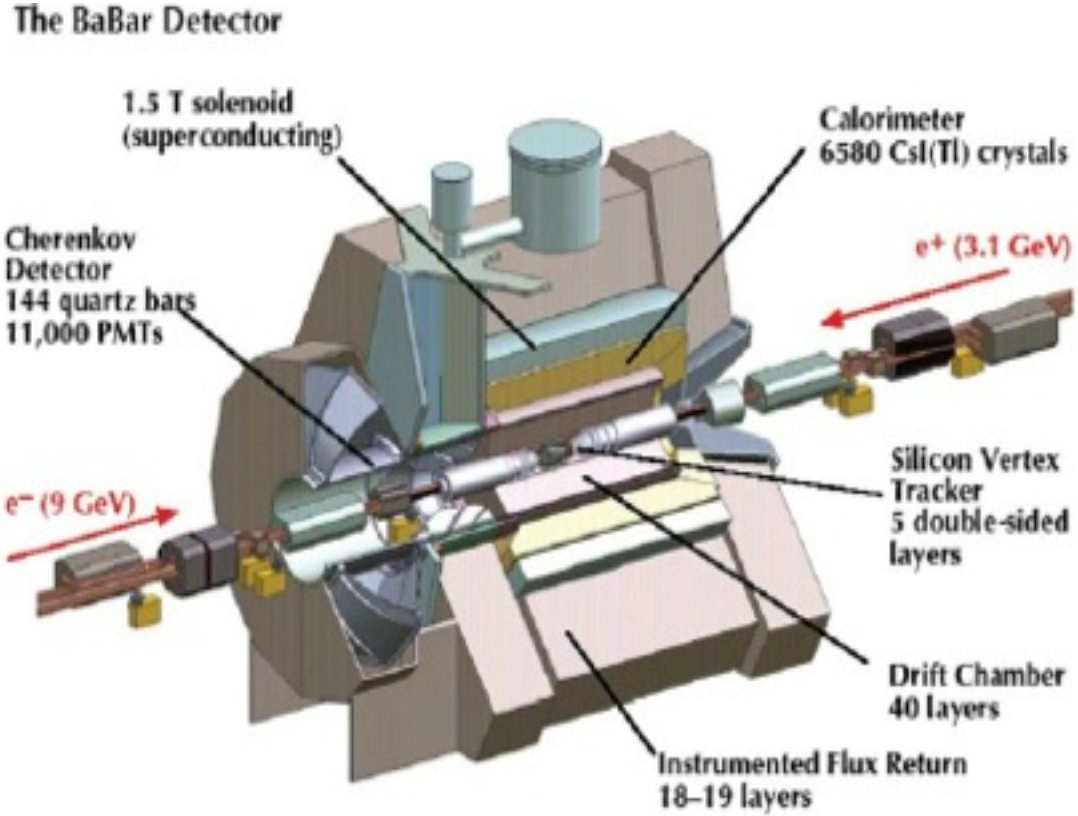


e^+e^-

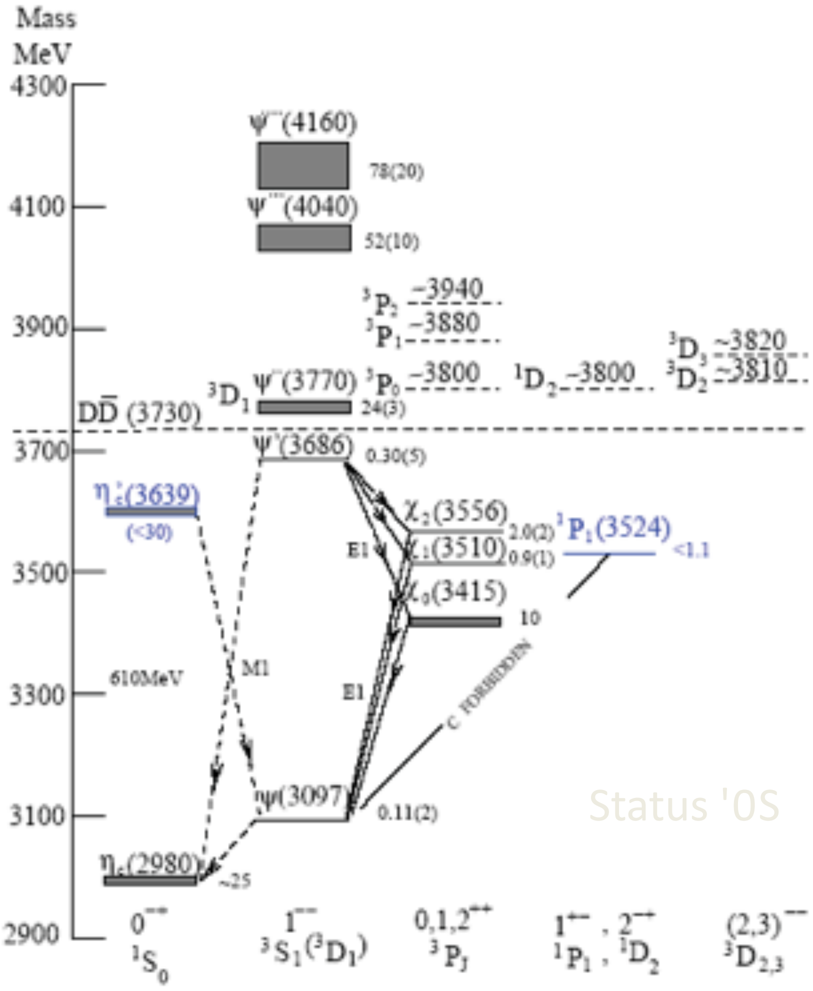


B-Factory

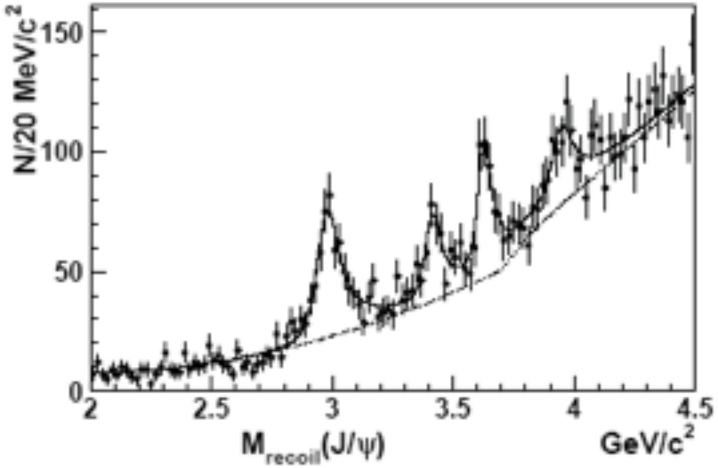

SLAC
PEP II
Low Energy
Ring (LER)
[3.1 GeV]
North Damping
Ring
[1.15 GeV]
Positron Return Line
Positron Source
PEP II
IR-2
Detector
e-gun
200 MeV
injector
Linac
PEP II High Energy Bypass (HEB)
South Damping
Ring
[1.15 GeV]
Sector-10 PEP II
e- injector
PEP II
High Energy
Ring (HER)
[9 GeV]
PEP II Low Energy Bypass (LEB)
Sector-4 PEP II
e+ injector
3 km


The BaBar Detector
1.5 T solenoid
(superconducting)
Calorimeter
6580 CsI(Tl) crystals
Cherenkov
Detector
144 quartz bars
11,000 PMTs
e+ (3.1 GeV)
Silicon Vertex
Tracker
5 double-sided
layers
e− (9 GeV)
Drift Chamber
40 layers
Instrumented Flux Return
18–19 layers

Charmonium Spectroscopical Results

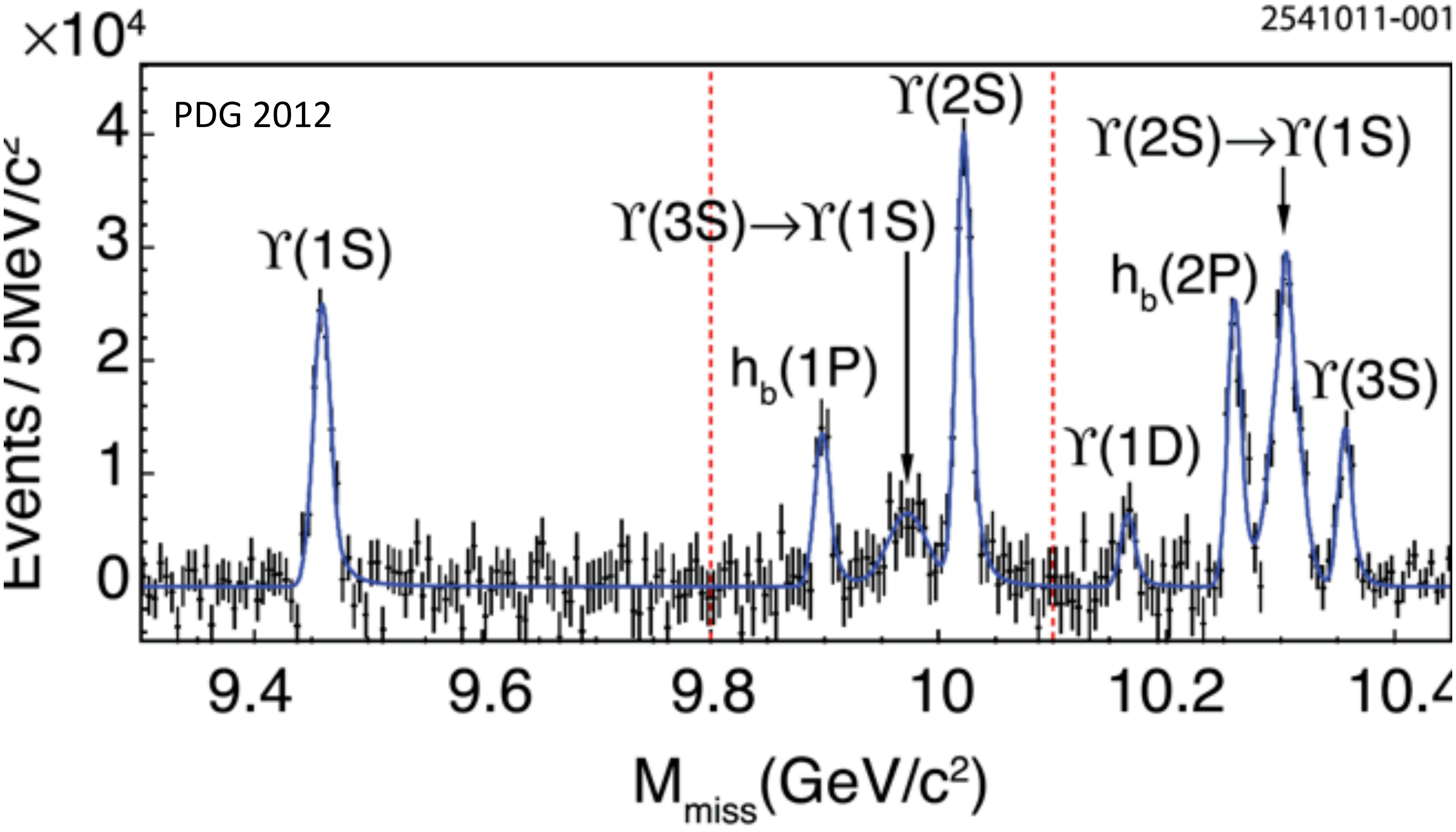


Status '0S



(a) X(3943)

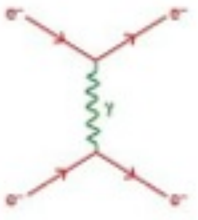
Ypsilonium Spectroscopy (bb)



What about the QCD potential II

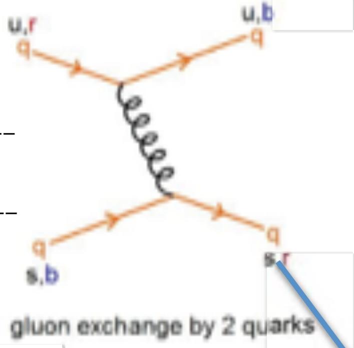
Gluon interaction:

- Weak at small distances
  - One-boson exchange
  - Asymtotic freedom



Photon: $m_\gamma = 0, \ J^{PC} = 1^{--}$

Gluon: $m_G = 0, \ J^{PC} = 1^{--}$

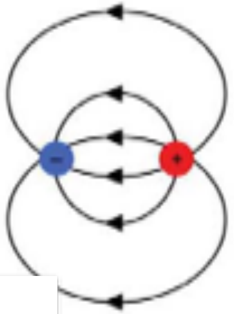


gluon exchange by 2 quarks

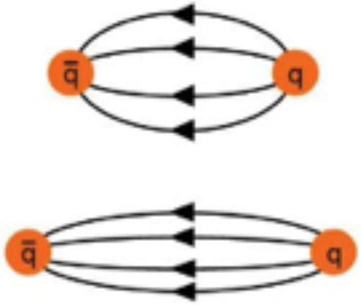
- Strong at large distances
  - Confinement
  - String breaking
  - Hadronisation

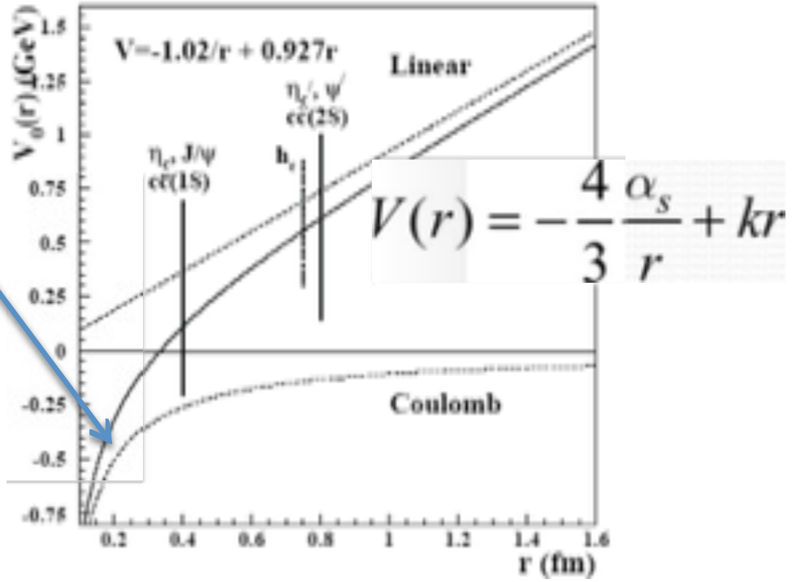
gluon-gluon scattering

electromagnetic



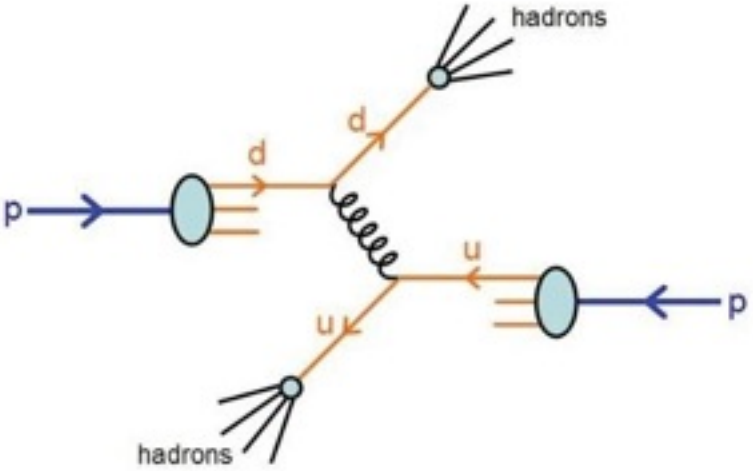
strong

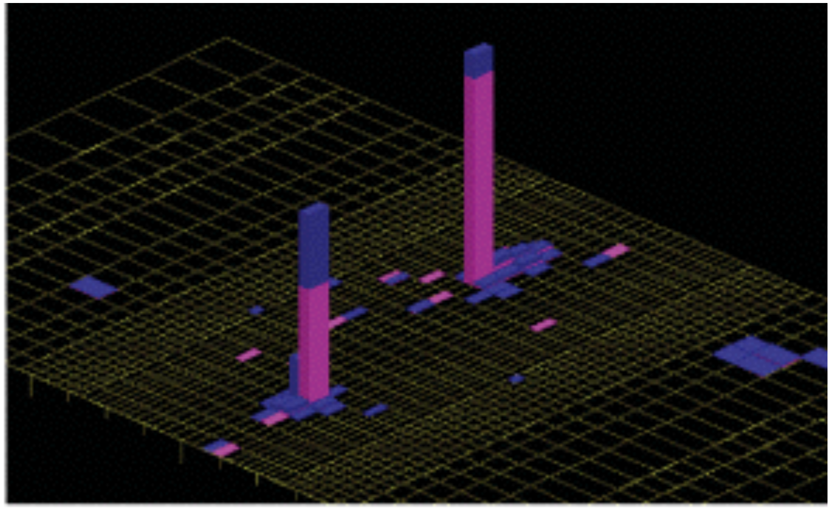




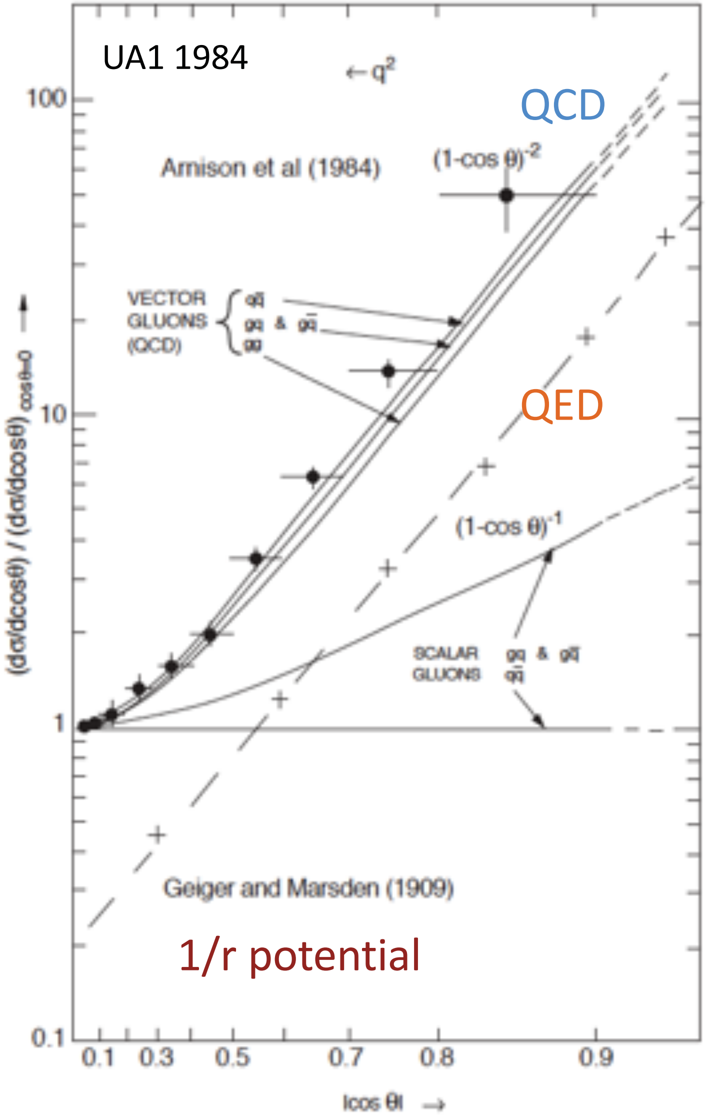
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

Example for Asymptotic Freedom





2 Jets at CDF (Fermilab) $M_{jetjet} = 1.364\ TeV / c^2$



From Quarks to Hadrons

Quarks:

- Flavour: (u,d,s,c,b)
- Colour: (r,g,b)
- Spin: 1/2

Build bound systems from quarks

- Colour-less
- Wave function must obey symmetry:
  - Bose symmetry for mesons (integer spin)
  - Fermi symmetry for baryons (half integer spin)

Hadron wave function product of

$$\psi = \varphi_{colour}\psi_{flavour}\phi_{spin}\psi_{spatial}$$

Group theory easily solves problem forming wave function

- Not treated here..

From Quarks to Baryons

Example: baryons (qqq)

$$\phi^{\text{Baryon}}_{\text{Farbe}} = \frac{1}{\sqrt{6}} \sum_{\alpha=\mathrm{R,G,B}} \sum_{\beta=\mathrm{R,G,B}} \sum_{\gamma=\mathrm{R,G,B}} \varepsilon_{\alpha\beta\gamma} |\alpha\rangle|\beta\rangle|\gamma\rangle$$

$$= \frac{1}{\sqrt{6}} \big(|\mathrm{R}\rangle|\mathrm{G}\rangle|\mathrm{B}\rangle + |\mathrm{G}\rangle|\mathrm{B}\rangle|\mathrm{R}\rangle + |\mathrm{B}\rangle|\mathrm{R}\rangle|\mathrm{G}\rangle - |\mathrm{G}\rangle|\mathrm{R}\rangle|\mathrm{B}\rangle - |\mathrm{B}\rangle|\mathrm{G}\rangle|\mathrm{R}\rangle - |\mathrm{R}\rangle|\mathrm{B}\rangle|\mathrm{G}\rangle \big)$$

changes sign under exchange of any 2-particles
→ completely "anti-symmetric"

Only way to build colour "singlet" (white colour)

- open colour has not been observed so far !

$$|\mathrm{B}\rangle = |\mathrm{qqq}\rangle_A = |\mathrm{Farbe}\rangle_A \otimes |\mathrm{Ort, Spin, Flavour}\rangle_S$$

- Consider L=0 spatial wave function
  → symmetric under exchange of position of 2-particles
  homogeneous spatial distribution
- Check ISpin, Flavour$>_S$ part
- Example: proton

$$|\mathrm{p}\uparrow\rangle = \frac{1}{\sqrt{6}} \Big(2|\mathrm{u}\uparrow \mathrm{u}\uparrow \mathrm{d}\downarrow\rangle - |\mathrm{u}\uparrow \mathrm{u}\downarrow \mathrm{d}\uparrow\rangle - |\mathrm{u}\downarrow \mathrm{u}\uparrow \mathrm{d}\uparrow\rangle \Big)_{\text{sym}}$$

From Quarks to Baryons

Consider spin S=3/2 baryons:

- Spin: $\left|\frac{3}{2},+\frac{3}{2}\right\rangle = |\uparrow\rangle|\uparrow\rangle|\uparrow\rangle$ Completely symmetric in spin

  S, S_z

- Spatial: L = 0 Completely symmetric in space
- Flavour: must be Completely symmetric in flavour

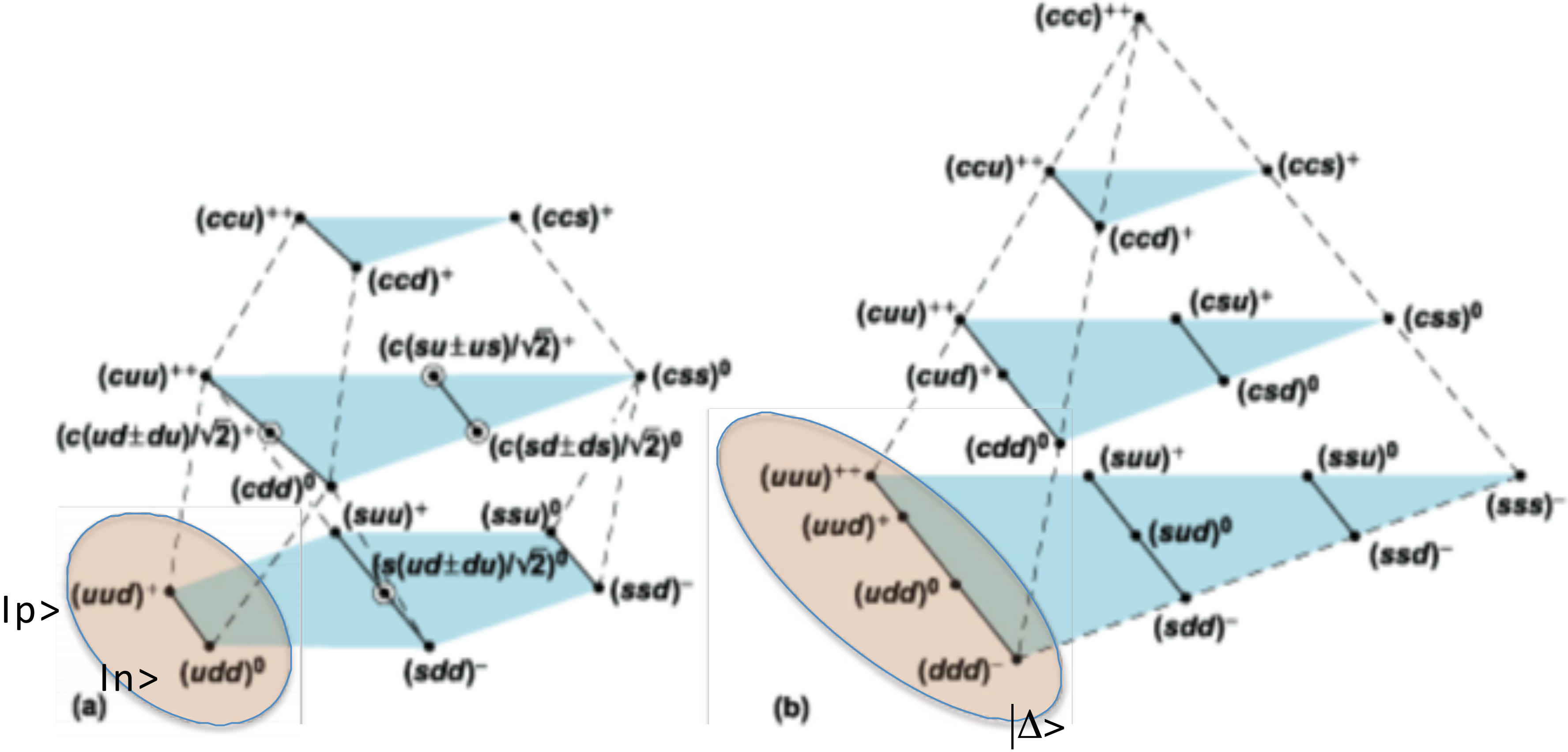
First hint for colour quantum number

$$
1232\,\text{MeV}\begin{cases}
|\Delta^{++}\rangle = |uuu\rangle \\
|\Delta^{+}\rangle = \frac{1}{\sqrt{3}}\left(|uud\rangle + |udu\rangle + |duu\rangle\right) \\
|\Delta^{0}\rangle = \frac{1}{\sqrt{3}}\left(|ddu\rangle + |dud\rangle + |udd\rangle\right) \\
|\Delta^{-}\rangle = |ddd\rangle
\end{cases}
$$

$$
1385\,\text{MeV}\begin{cases}
|\Sigma^{*+}\rangle = \frac{1}{\sqrt{3}}\left(|uus\rangle + |usu\rangle + |suu\rangle\right) \\
|\Sigma^{*0}\rangle = \frac{1}{\sqrt{6}}\left(|uds\rangle + |dsu\rangle + |sdu\rangle + |usd\rangle + |dus\rangle + |sud\rangle\right) \\
|\Sigma^{*-}\rangle = \frac{1}{\sqrt{3}}\left(|dds\rangle + |dsd\rangle + |sdd\rangle\right)
\end{cases}
$$

Symmetries and Hadron Spectrum

Baryons



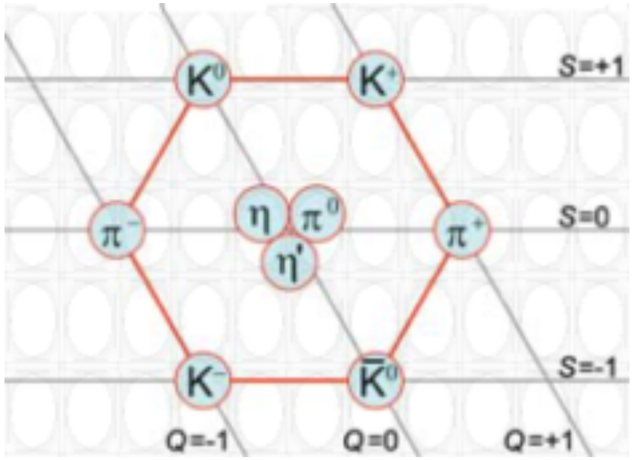
S = 1/2

S = 3/2

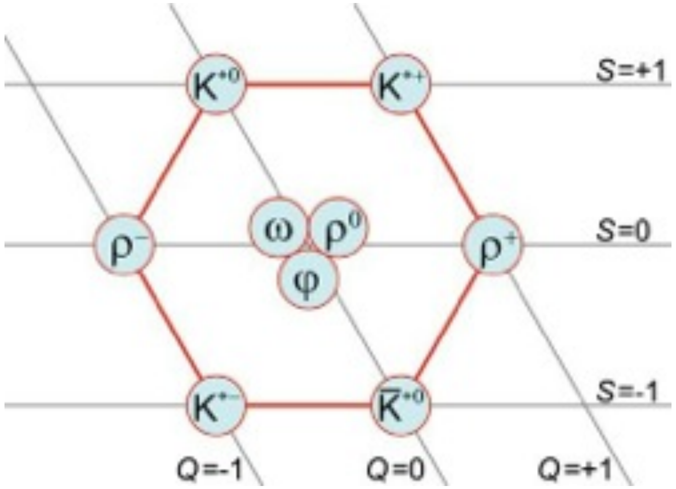
Symmetries and Hadron Spectrum

Mesons (flavour: u,d,s)

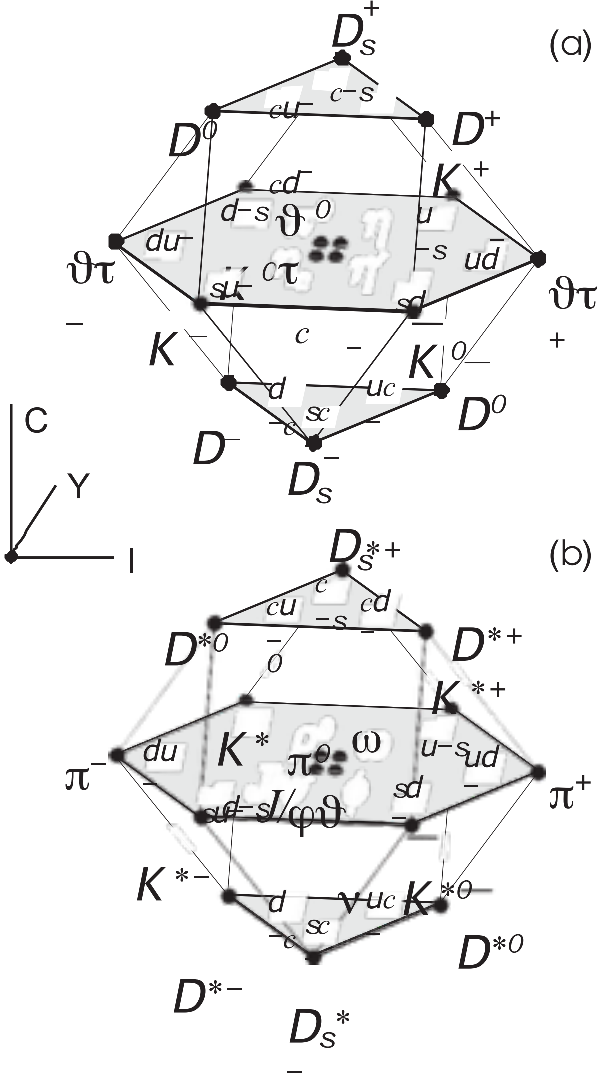
Mesons (flavour: u,d,s,c)



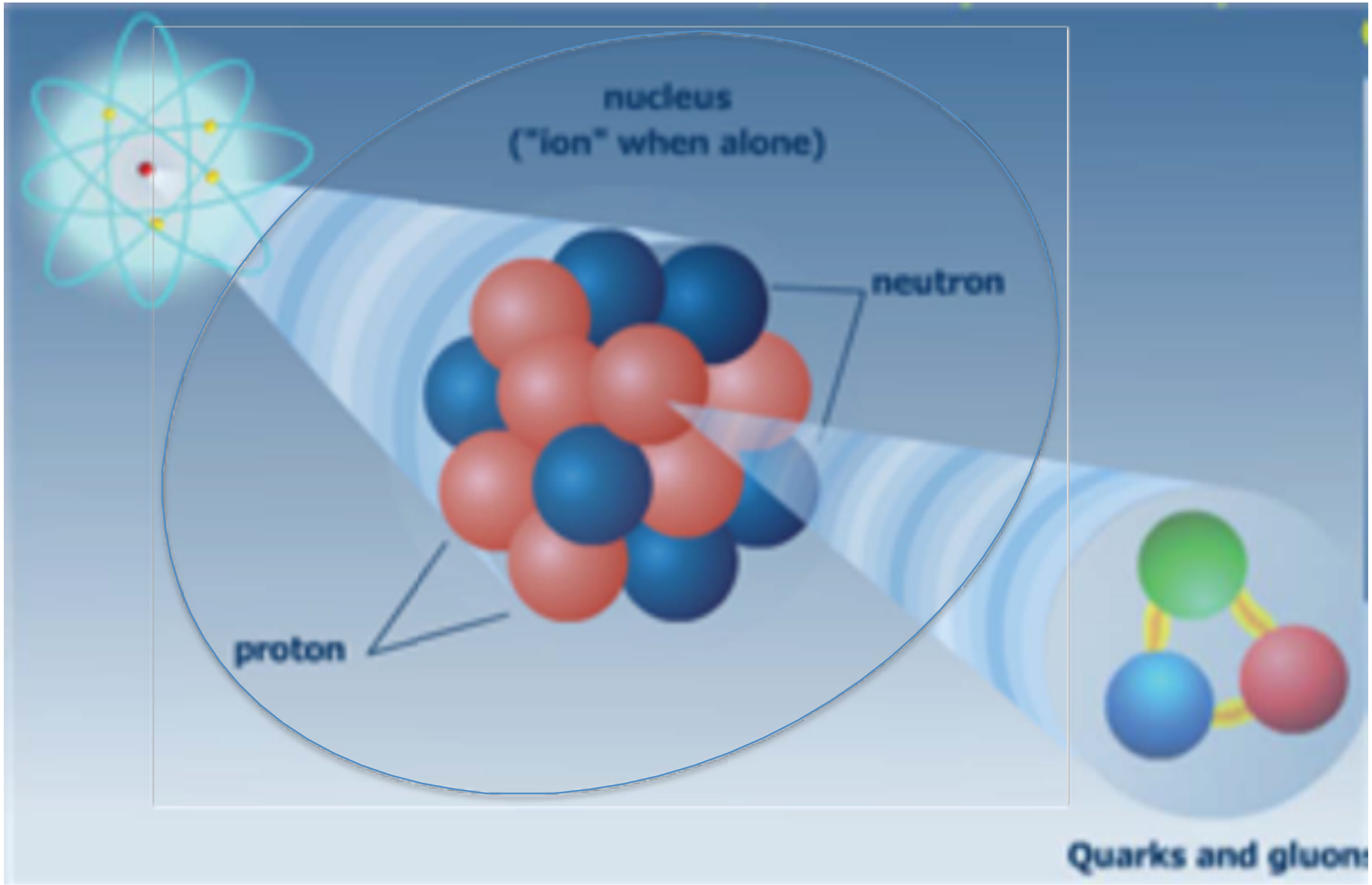
Spin = 0



Spin = 1

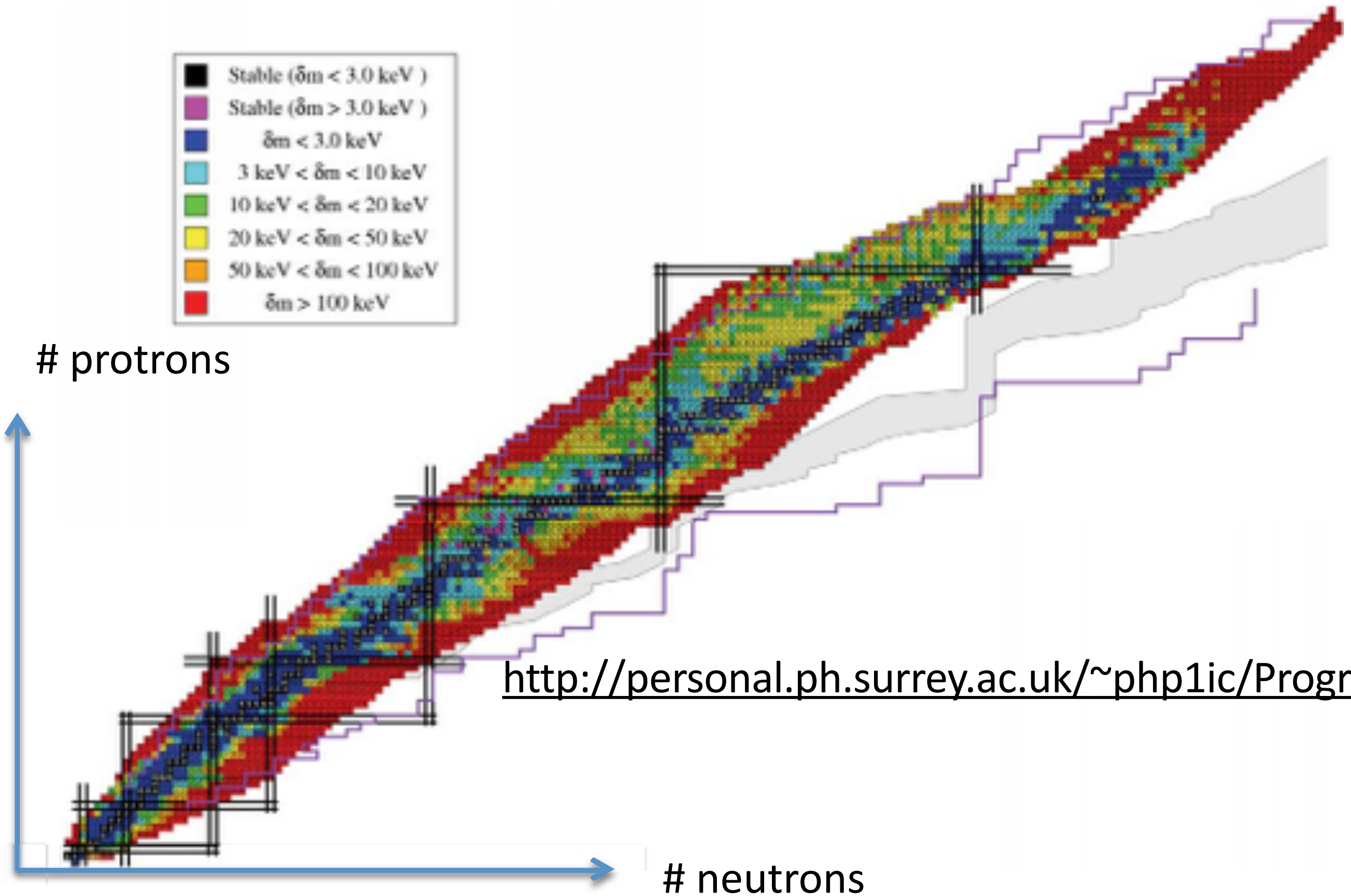


Nuclei



Nuclei

Nuclei built from protons and neutrons



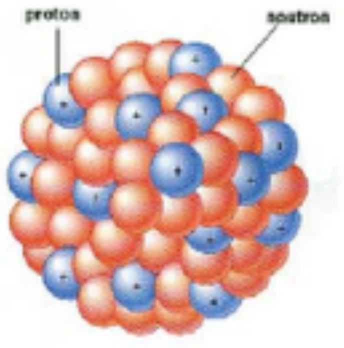
http://personal.ph.surrey.ac.uk/~php1ic/Programs/Inch/chart.png

Nuclei – Radii and Masses

Nuclear radii:

From electron scattering

A = N(neutrons) + Z(protons)



$$R = R_0 \cdot A^{\frac{1}{3}} = 1.21\,\mathrm{fm} \cdot A^{\frac{1}{3}}$$

$$\frac{V}{A} = \mathrm{const.} = 0.17 \frac{\mathrm{Nukleonen}}{\mathrm{fm}^3} \hat{=} 3 \cdot 10^{14} \frac{\mathrm{g}}{\mathrm{cm}^3}$$

Volume scales with A : constant nuclear density

nuclei are not easily compressible

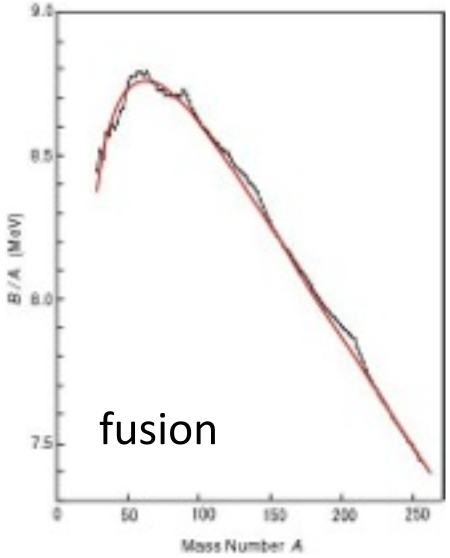
nucleons do not really overlap much ("hard" spheres")

Nuclear masses:

Binding energy: $B = \Big(Z \cdot (M_\mathrm{p} + m_\mathrm{e}) + N \cdot M_\mathrm{n} - M(A,Z)\Big) \cdot \mathrm{c}^2$

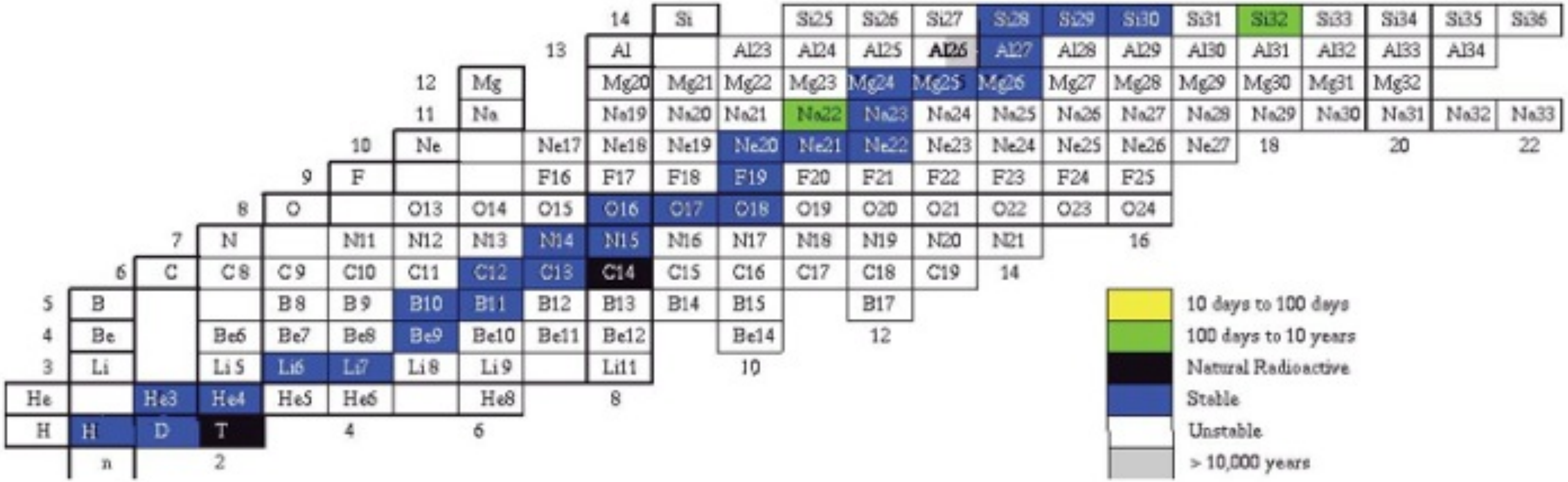
Weizsäcker mass formula:

$$\frac{B}{A} = a_\mathrm{V} - a_\mathrm{S} A^{-1/3} - a_\mathrm{Sym} \frac{(Z-N)^2}{A^2} - a_\mathrm{Coul} Z^2 A^{-4/3} \pm a_\mathrm{P} A^{-3/2}$$

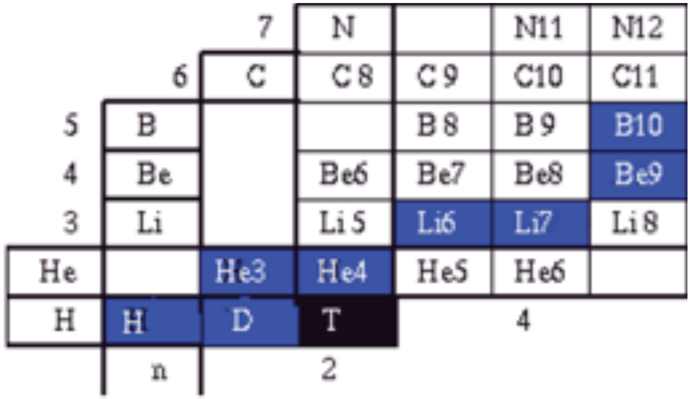


fusion

Nuclear Chart

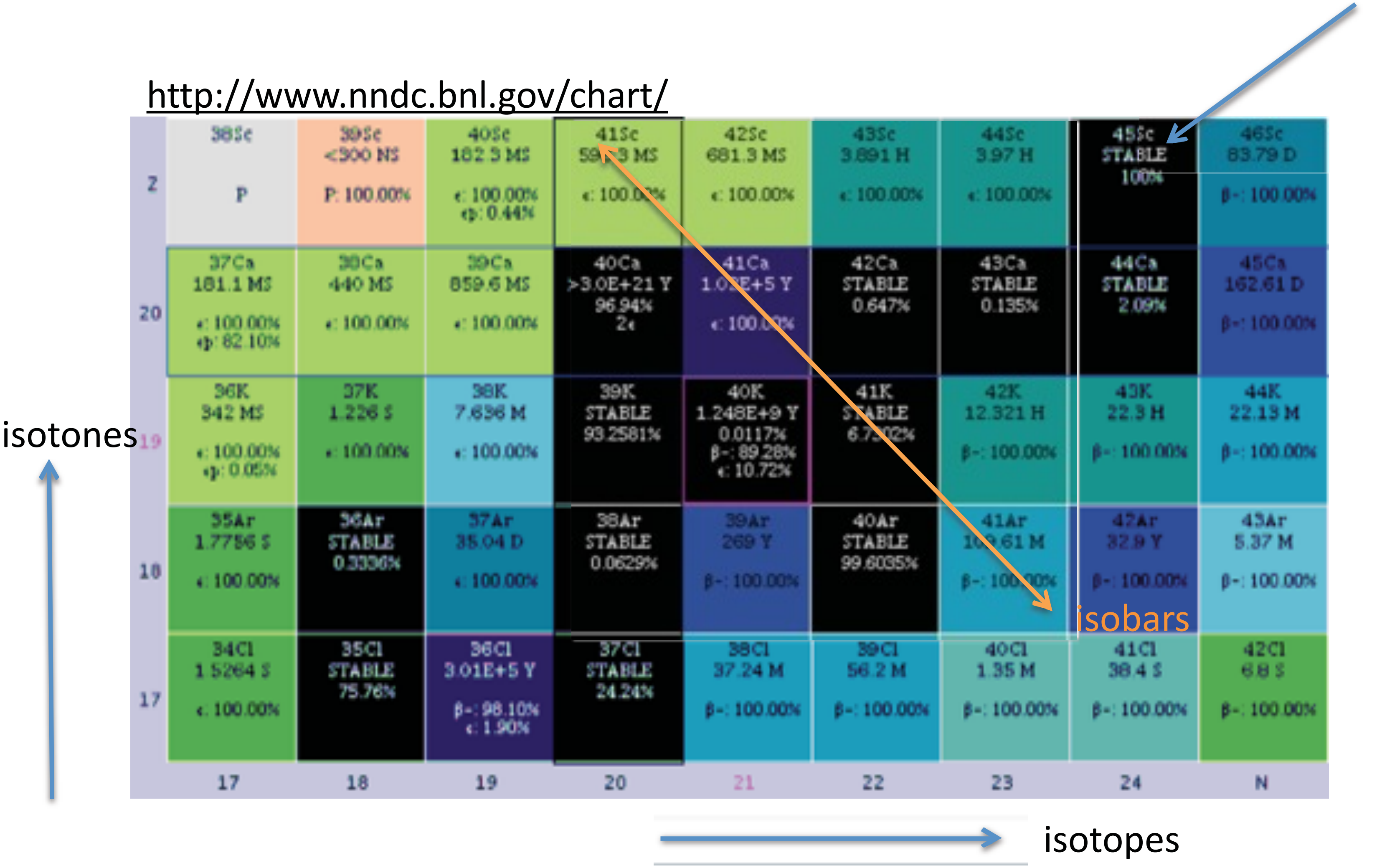


No stable elements with A=5, 8



Nuclei built from protons and neutrons

http://www.nndc.bnl.gov/chart/



	17	18	19	20	21	22	23	24	N
Z	38Sc P	39Sc <300 NS P: 100.00%	40Sc 182.3 MS ε: 100.00% εp: 0.44%	41Sc 596.3 MS ε: 100.00%	42Sc 681.3 MS ε: 100.00%	43Sc 3.891 H ε: 100.00%	44Sc 3.97 H ε: 100.00%	45Sc STABLE 100%	46Sc 83.79 D β-: 100.00%
20	37Ca 181.1 MS ε: 100.00% εp: 82.10%	38Ca 440 MS ε: 100.00%	39Ca 859.6 MS ε: 100.00%	40Ca >3.0E+21 Y 96.94% 2ε	41Ca 1.02E+5 Y ε: 100.00%	42Ca STABLE 0.647%	43Ca STABLE 0.135%	44Ca STABLE 2.09%	45Ca 162.61 D β-: 100.00%
19	36K 342 MS ε: 100.00% εp: 0.05%	37K 1.226 S ε: 100.00%	38K 7.636 M ε: 100.00%	39K STABLE 93.2581%	40K 1.248E+9 Y 0.0117% β-: 89.28% ε: 10.72%	41K STABLE 6.7302%	42K 12.321 H β-: 100.00%	43K 22.3 H β-: 100.00%	44K 22.13 M β-: 100.00%
18	35Ar 1.7756 S ε: 100.00%	36Ar STABLE 0.3336%	37Ar 35.04 D ε: 100.00%	38Ar STABLE 0.0629%	39Ar 269 Y β-: 100.00%	40Ar STABLE 99.6035%	41Ar 109.61 M β-: 100.00%	42Ar 32.9 Y β-: 100.00%	43Ar 5.37 M β-: 100.00%
17	34Cl 1.5264 S ε: 100.00%	35Cl STABLE 75.76%	36Cl 3.01E+5 Y β-: 98.10% ε: 1.90%	37Cl STABLE 24.24%	38Cl 37.24 M β-: 100.00%	39Cl 56.2 M β-: 100.00%	40Cl 1.35 M β-: 100.00%	41Cl 38.4 S β-: 100.00%	42Cl 6.8 S β-: 100.00%

Binding

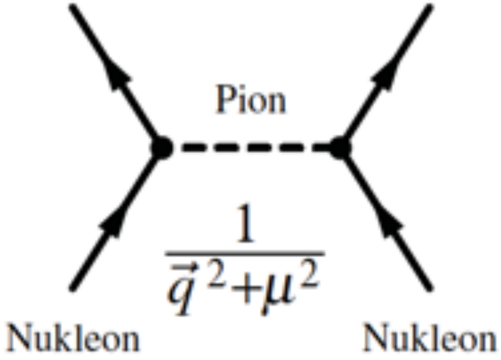
Effective interaction mediated by π

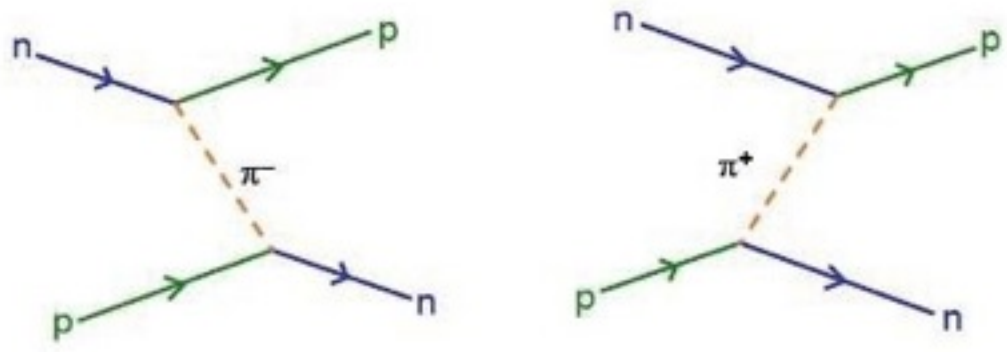
"long" range nuclear force

Yukawa exchange force

$$U(r) = \frac{\lambda}{r} e^{-\mu r}$$

μ^{-1} Compton wave length

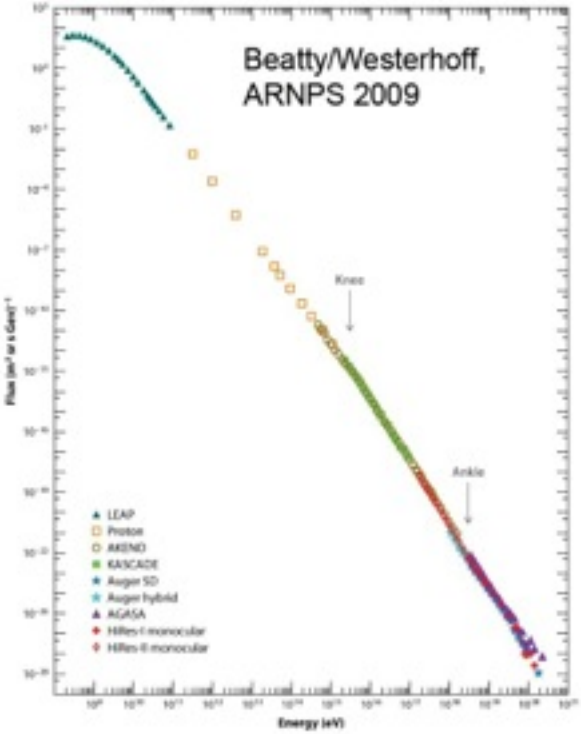




Strong Interaction in Astrophysics (Example)

- Element formation in stars and supernovae
- Limitations for highest energy cosmic rays

$$p + \gamma\,(CMB) \rightarrow \Delta \rightarrow p + \pi$$



Beatty/Westerhoff, ARNPS 2009

Greisen-Zatsepin-Kuzmin limit

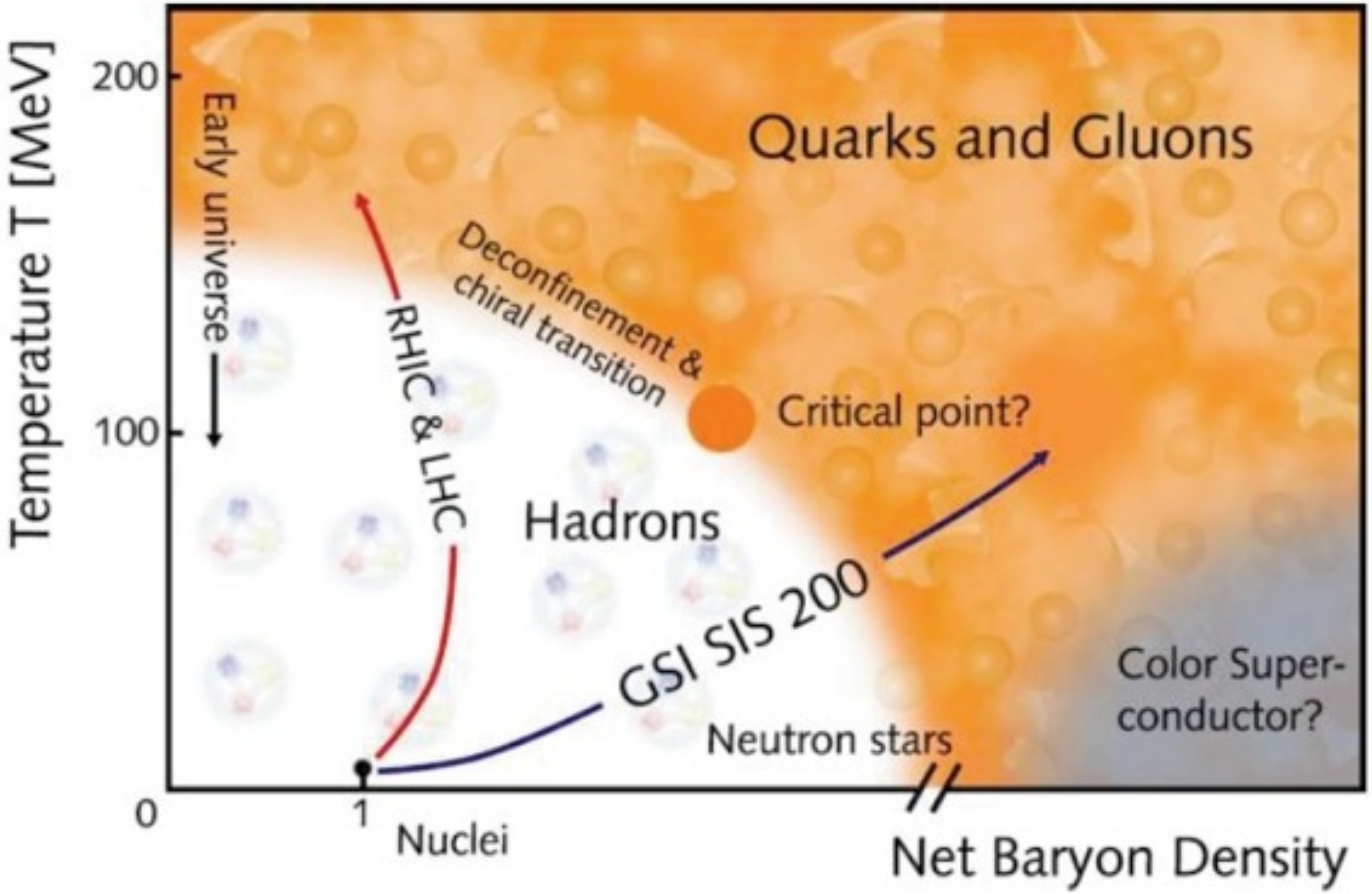
(GZK-cutoff)

$$E(p) < 10^{19}\,eV = 10^{10}\,GeV$$

- Phase transition to "nucleonic Universe"

Quark Gluon Plasma

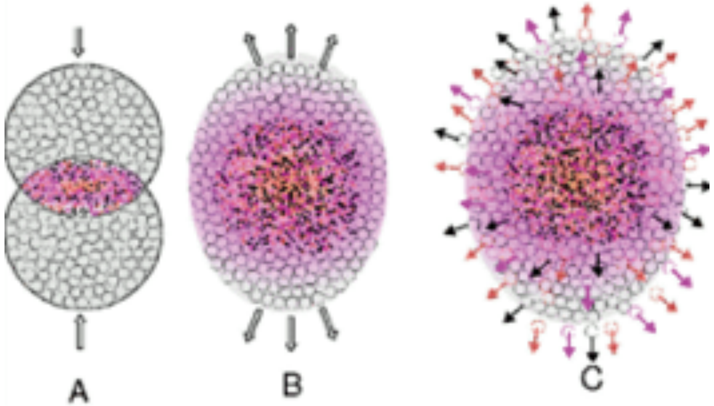
The Phase Diagram of QCD



Heavy Ion Collisions

Assume: collision of Au + Au with E_{kin} = 1 TeV/A

Kinetic energy >> binding energy of nucleons



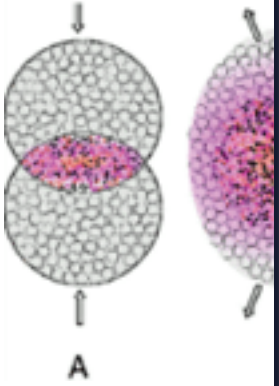
Central collision creates soup of nucleons

- being heated up
- melting of nucleons creates soup of quarks and gluons (quark-gluon plasma)
- Gluons fluctuate into quark-antiquark pairs
- Soup expands → cooling → condensation (hadronisation)
  - New nucleons form
  - Pions evaporate from soup

Heavy Ion

Assume: col

Kinetic energy



Central collisio
- being heate
- melting of n
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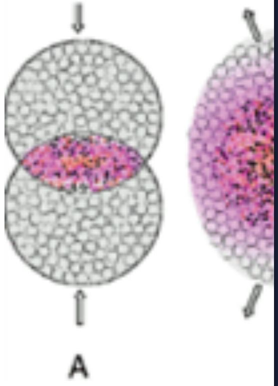
Heavy Ion

Assume: col

Kinetic energy



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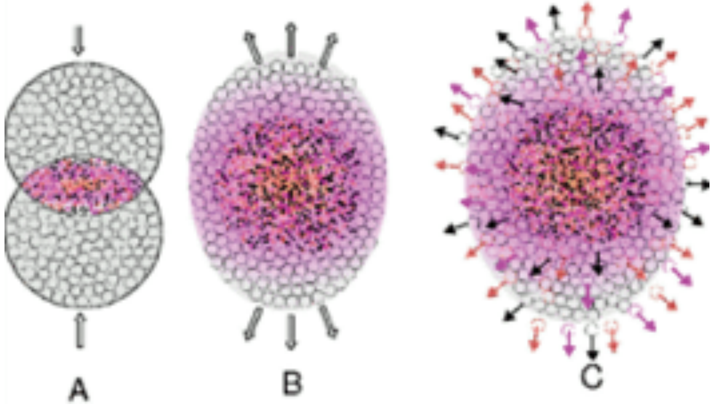

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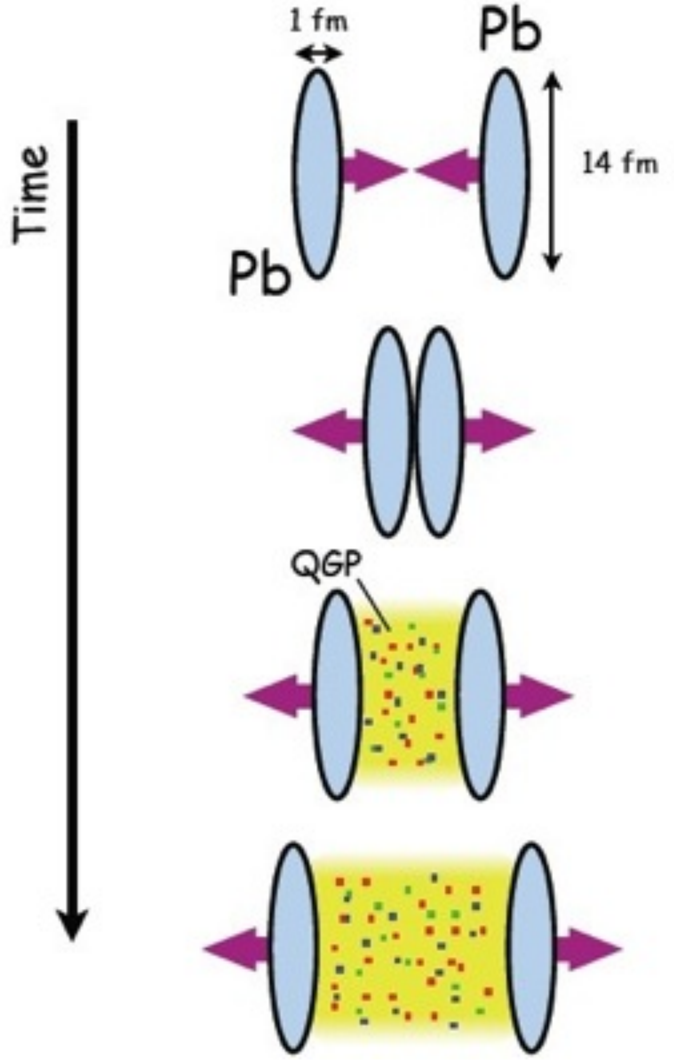
Stages of Heavy Ion collision



Heavy Ion Collision:

Ideal to get conditions at high T and ρ

Formation time τ_0 = 1 fm/c

[1 fm/c = 3.3×10^{-24} s]

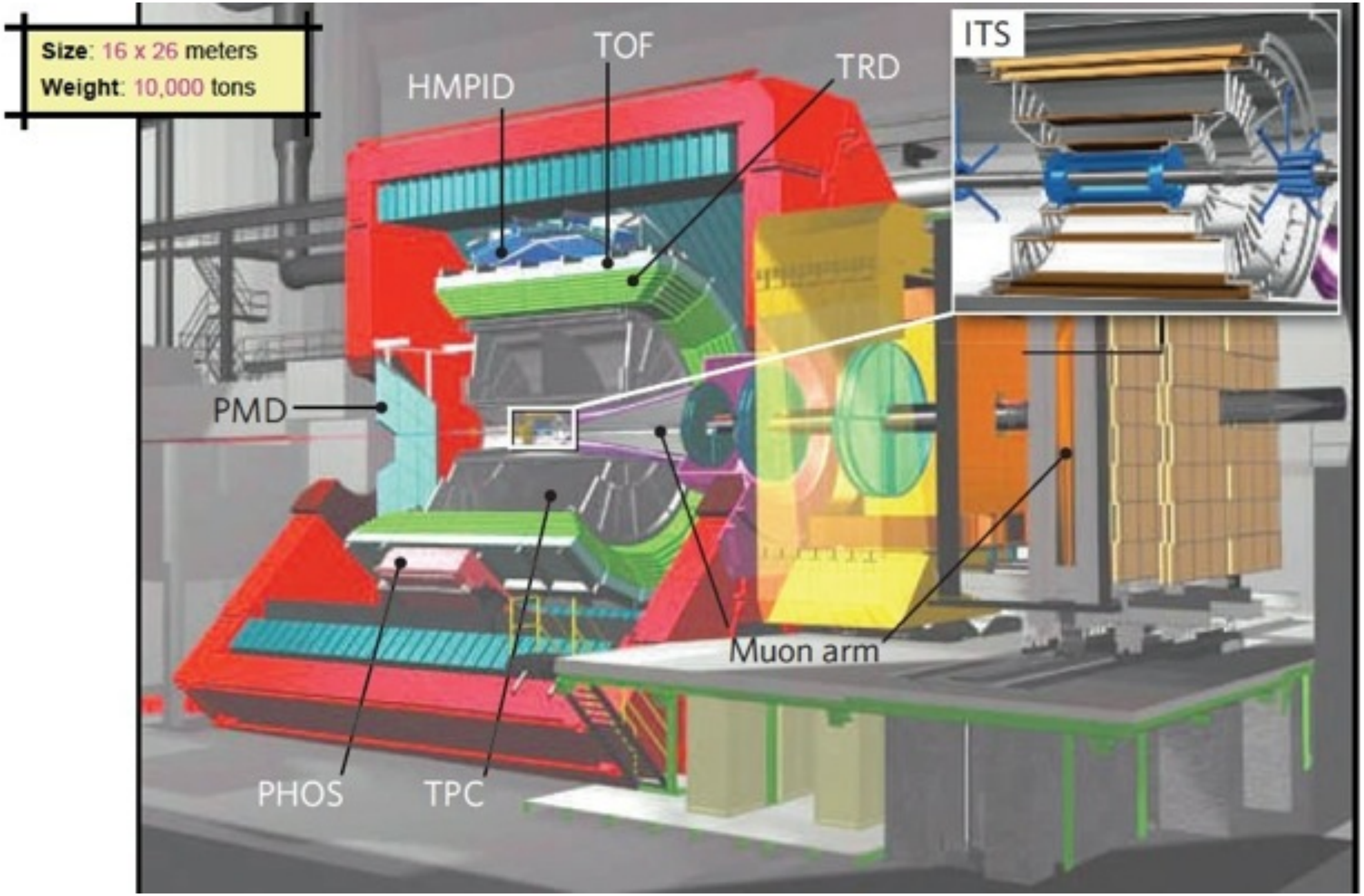
Temperature of O(10^{12})

Lifetime: 10 fm/c

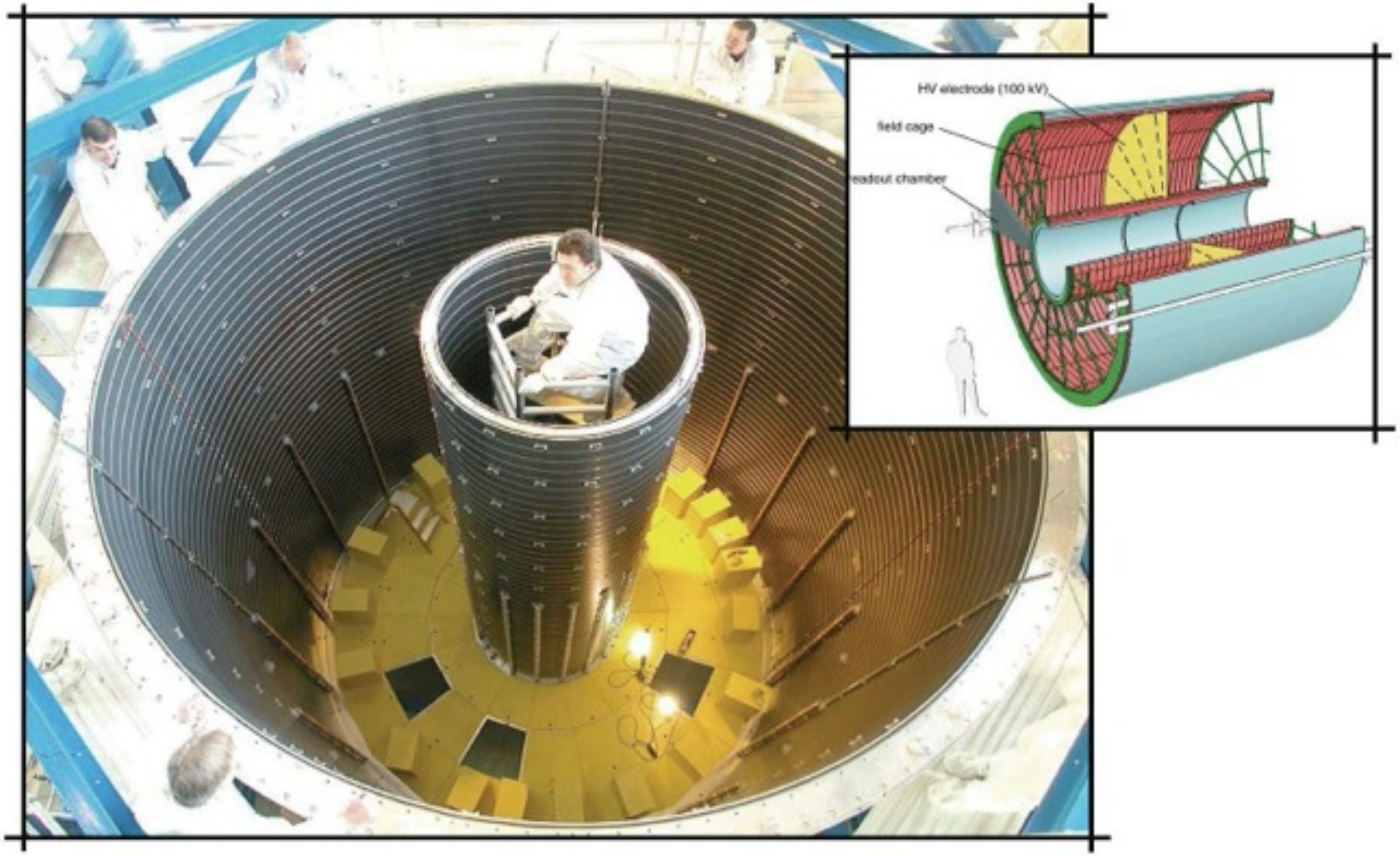
QGP in equilibrium (!?)

Cool down: hadronization

ALICE @LHC

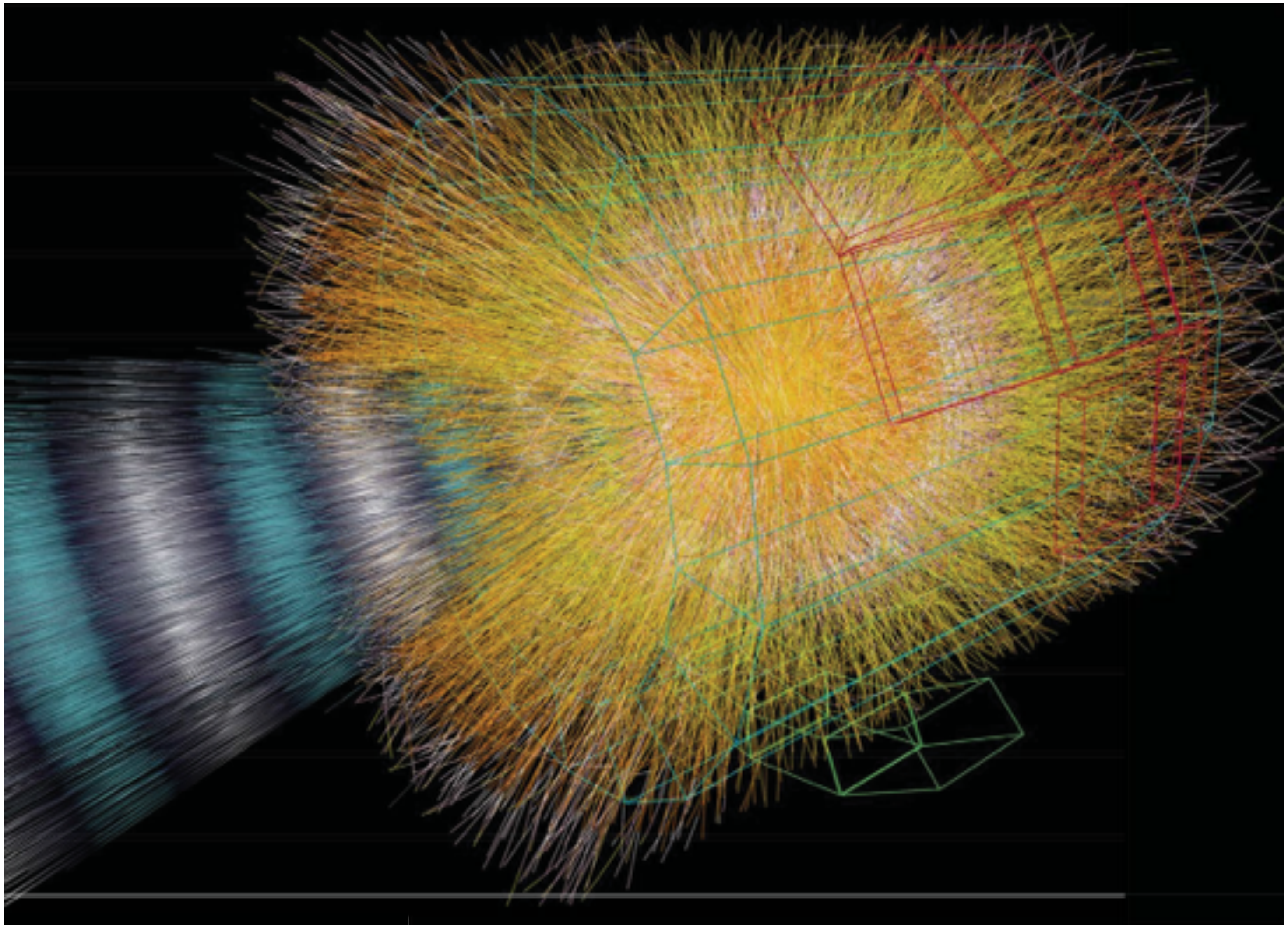

Size: 16 x 26 meters
Weight: 10,000 tons
TOF
HMPID
TRD
ITS
PMD
Muon arm
PHOS
TPC

ALICE TPC



Heavy Ion Collision

ALICE @ LHC

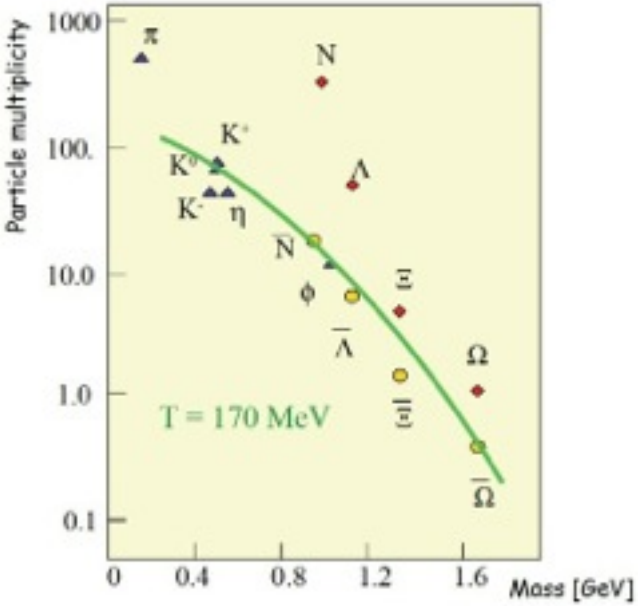


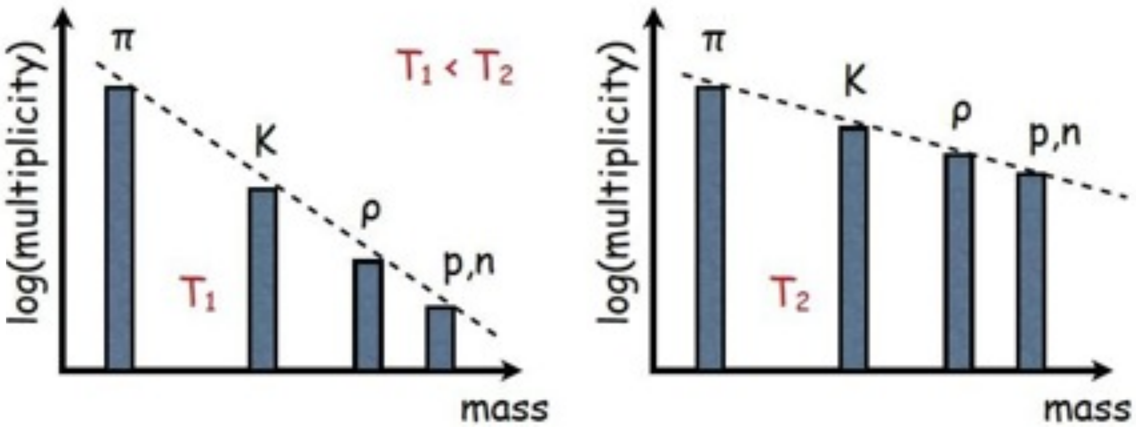
Quark Gluon Plasma II

Measure temperature from "particle multiplicities" momenta

Relative yield of particles of different species (mass) depends on temperature

$$Multiplicity: M \approx e^{-\frac{m}{T}}$$



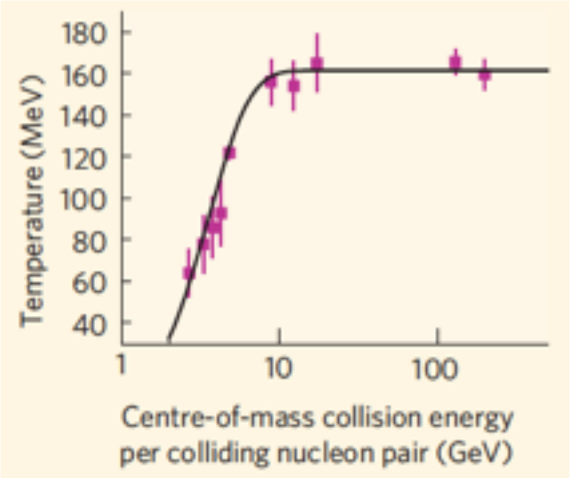


Measured distribution consistent with T = 170 MeV

What do we know about the QGP ?

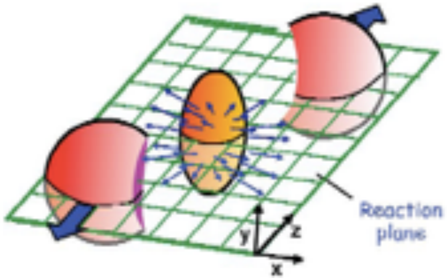
Temperature:

- Heating of nucleon soup stops at T=170 MeV



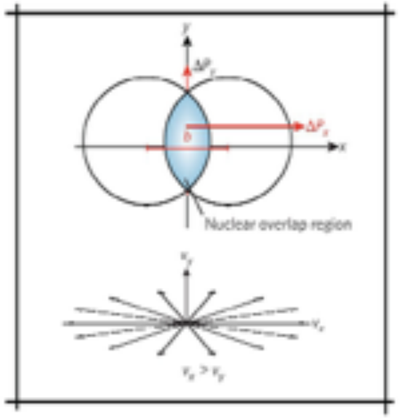
Fluid Properties:

- Measuring characteristics of “QGP-decay”:
  - Flow direction of particles (elliptic flow): QGP is perfect fluid
  - Expected: "perfect gas" - no interactions
  - Viscosity lower the superfluid helium
  - Close to theoretical limit (film)

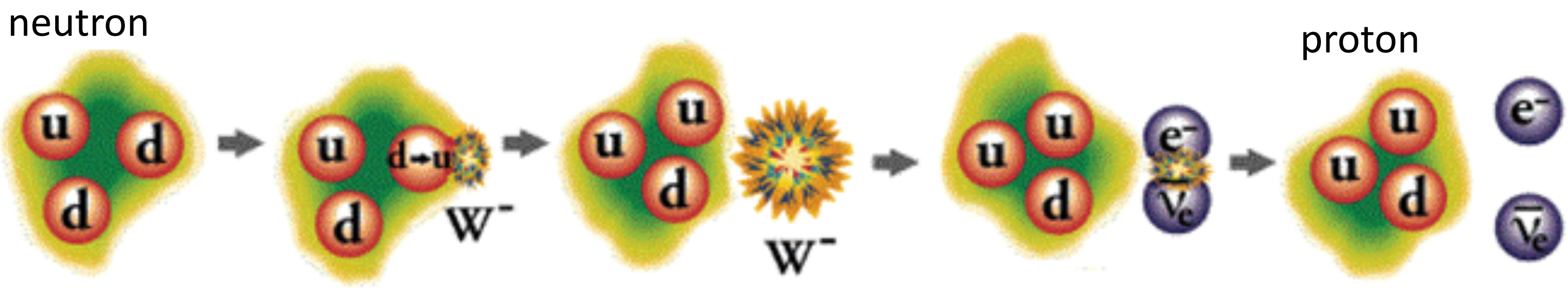


Phase transition:

- Transition from QGP to “normal” hadronic phase proceeds slowly
  - Phase transition is "cross-over"
  - Order parameter is "chiral condensate" $\langle q\bar{q}\rangle$ ($q\bar{q}$-correlations)

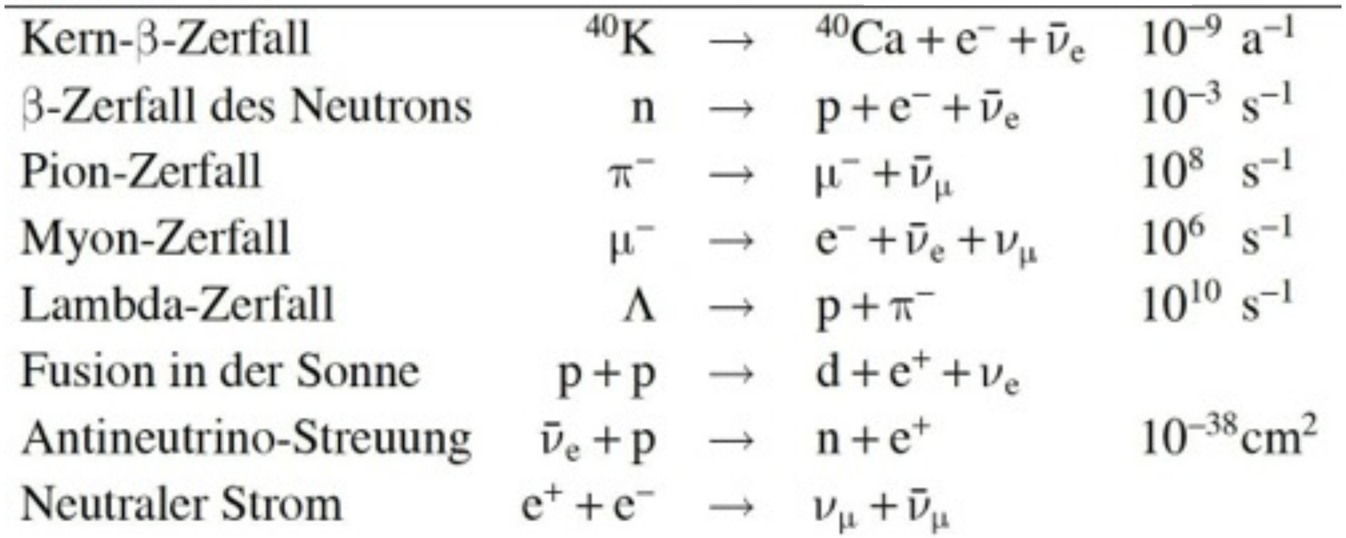


Weak Interaction



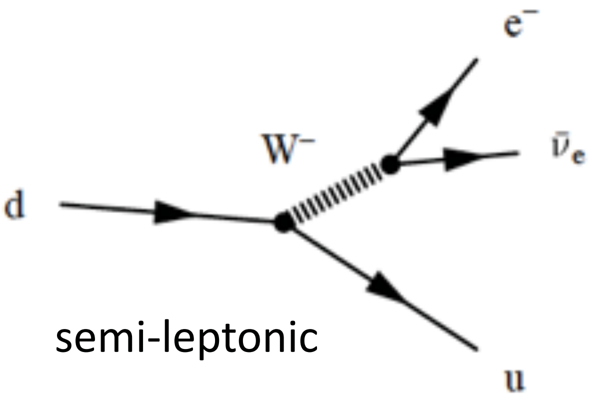
Typical Processes

Strong variation in transition rate

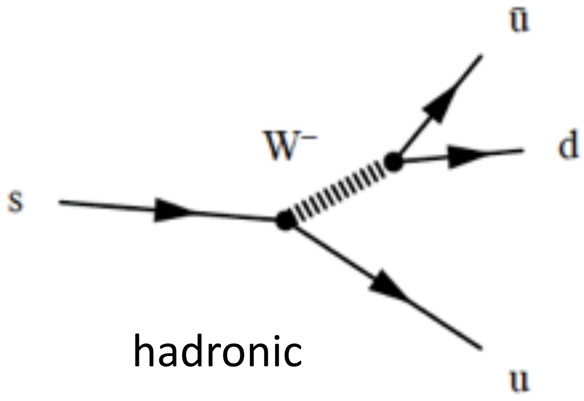


Process				Transition rate
Kern-β-Zerfall	$^{40}\mathrm{K}$	$\rightarrow$	$^{40}\mathrm{Ca} + e^- + \bar{\nu}_e$	$10^{-9}\ \mathrm{a}^{-1}$
β-Zerfall des Neutrons	n	$\rightarrow$	$p + e^- + \bar{\nu}_e$	$10^{-3}\ \mathrm{s}^{-1}$
Pion-Zerfall	π^-	$\rightarrow$	$\mu^- + \bar{\nu}_\mu$	$10^{8}\ \mathrm{s}^{-1}$
Myon-Zerfall	μ^-	$\rightarrow$	$e^- + \bar{\nu}_e + \nu_\mu$	$10^{6}\ \mathrm{s}^{-1}$
Lambda-Zerfall	Λ	$\rightarrow$	$p + \pi^-$	$10^{10}\ \mathrm{s}^{-1}$
Fusion in der Sonne	$p + p$	$\rightarrow$	$d + e^+ + \nu_e$	
Antineutrino-Streuung	$\bar{\nu}_e + p$	$\rightarrow$	$n + e^+$	$10^{-38}\mathrm{cm}^2$
Neutraler Strom	$e^+ + e^-$	$\rightarrow$	$\nu_\mu + \bar{\nu}_\mu$	

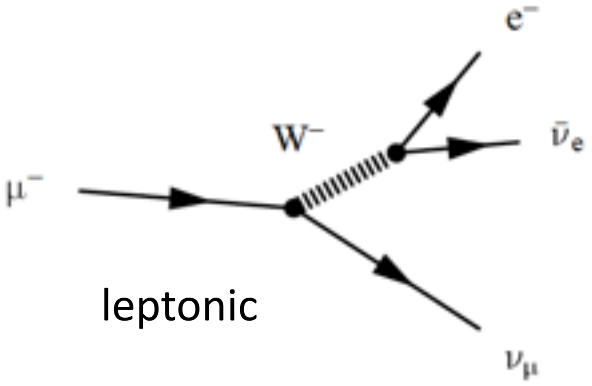
Typical processes



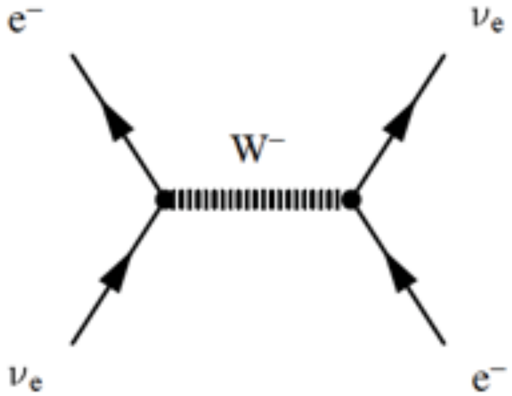
semi-leptonic



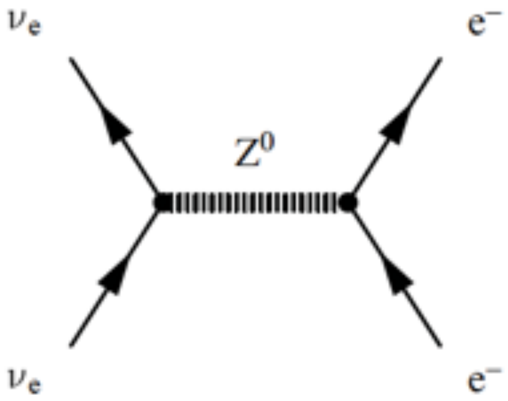
hadronic



leptonic



charged



neutral

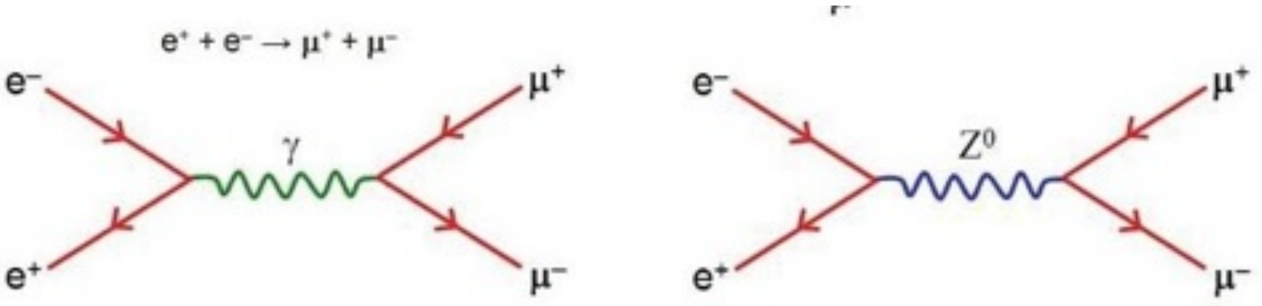
currents

Characteristics

Weak interaction:

- Violates parity, time and charge conjugation
- Violates CP, CT, PT
- Conserves CPT (so far)
- Violates quark flavour conservation
- Violates Lepton flavour conservation
- Conserves Lepton number and baryon number (so far)

Similarities to electromagnetic interaction



$m_\gamma = 0 \quad m_{Z^0} = 91.1\ GeV/c^2$ Weak interaction very short ranged

Weak Interaction Characteristics II

Propagator: $\approx \frac{1}{Q^2 + M_{boson}^2}$ with M_{boson} = 80 GeV/c^2 (91 GeV/c^2)

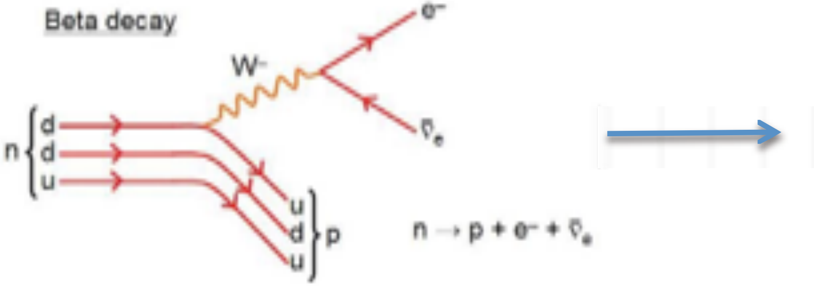
a) for $Q^2 >> M^2$ Propagator: $\approx \frac{1}{Q^2} =$ Propagator (γ)

b) for $Q^2 << M^2$ Propagator: $\approx \frac{1}{M_{boson}^2} \approx const.$

Interaction (matrix element) is constant with energy

→ replacement of matrix element with "effective coupling constant" G_F [GeV/c^2]

→ interaction: point like (Fermi-interaction)



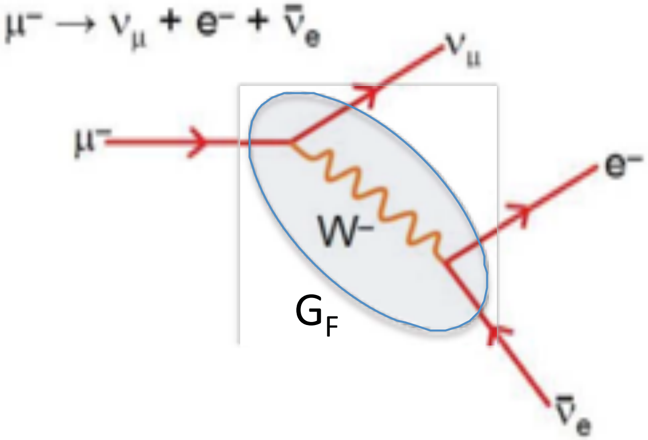
Characteristics of β-Decay

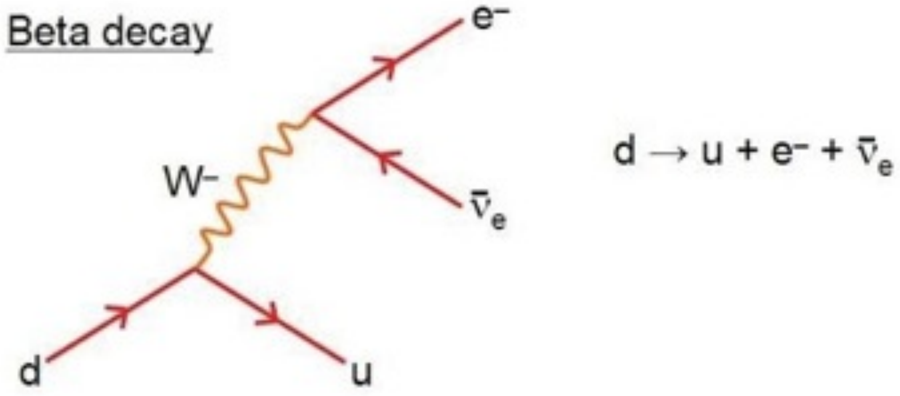
phase space

β-decay:

3−body decay: decay rate: $\Gamma \approx G_F (\Delta m)^5$

"Sargent rule" responsible for large variation of particle lifetimes !





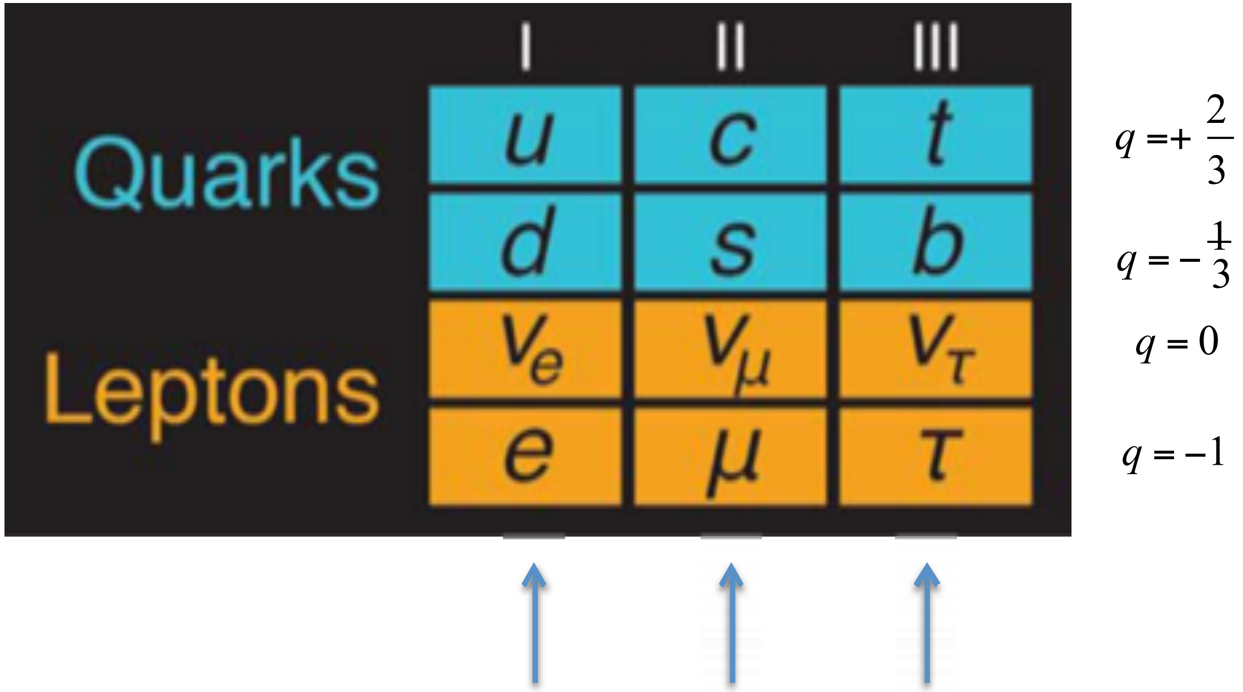
Lepton and quark decay (almost) identical (universality of weak coupling)

Quark decay: price to pay for change in quark-flavour !! → CKM matrix

Families

Strong & electromagnetic interaction "conserves"particle flavour

only "pair-creation" possible as $(f\bar{f})$



	I	II	III
Quarks	u	c	t
	d	s	b
Leptons	ν_e	ν_μ	ν_τ
	e	μ	τ

$q = +\frac{2}{3}$

$q = -\frac{1}{3}$

$q = 0$

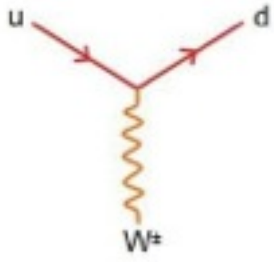
$q = -1$

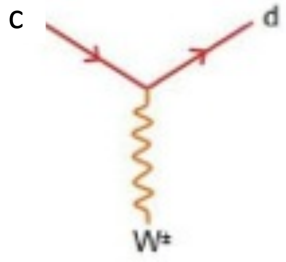
Weak interaction acts within "family"

charged current induces transitions: e.g. d⟶u or $e^- \longrightarrow \nu_e$

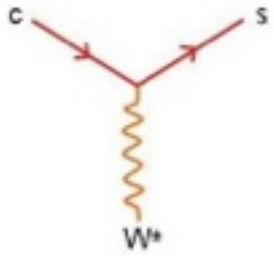
CKM I

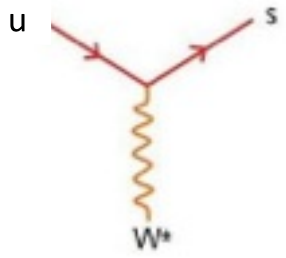
However:





and





Thus:

- change between families
- only in quark sector

Solution:

- Quarks are "eigenstates" of strong interaction (mass eigenstates)
- not identical to "eigenstates" of weak interaction
- Unitary rotation matrix V_{CKM}
- Weak interaction acts on rotated quarks q'

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM Matrix II

Diagonal elements are dominating

$$(|V_{ij}|) = \begin{bmatrix} 0{,}97459 & 0{,}2257 & 0{,}00359 \\ 0{,}2256 & 0{,}97334 & 0{,}0415 \\ 0{,}00874 & 0{,}0407 & 0{,}999133 \end{bmatrix}$$

- Transition rates: $\Gamma \approx \left|M_{if}\right|^2 \approx \left|V_{ij}^{CKM}\right|^2 \longrightarrow \text{decay rate:}\,\Gamma \approx G_F^2 m^5 \left|V_{ij}^{CKM}\right|^2$
- Transitions within "family" (mass eigenstates) dominate
- Unitary 3x3 matrix has
  - 3 “rotation angles” (Euler angles)
  - 1 phase

CKM III

Build rotation matrices (in flavour space)

$$R_1(\theta_{12}) = \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Mischung zw. Generation } 1\leftrightarrow 2} \qquad R_2(\theta_{23}) = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Mischung zw. Generation } 2\leftrightarrow 3} \qquad R_3(\theta_{13}) = \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}}_{\text{Mischung zw. Generation } 1\leftrightarrow 3}$$

$$c_{12} = \cos\theta_{12} \text{ and } s_{12} = \sin\theta_{12}$$

$$V_{CKM} = R_1 R_2 R_3 = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-I\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

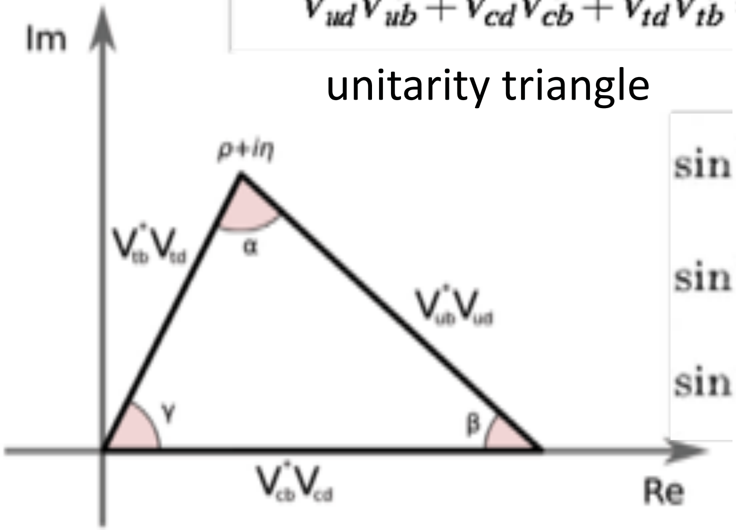
CKM III

In flavour physics often "Wolfenstein" parametrization used:

$$\hat{V}_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,.$$

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

unitarity triangle



$$\sin 2\beta = \frac{2\overline{\eta}(1-\overline{\rho})}{\overline{\eta}^2 + (1-\overline{\rho})^2},$$

$$\sin 2\alpha = \frac{2\overline{\eta}(\overline{\eta}^2 + \overline{\rho}(\overline{\rho}-1))}{(\overline{\eta}^2 + (1-\overline{\rho})^2)(\overline{\eta}^2 + \overline{\rho}^2)},$$

$$\sin 2\gamma = \frac{2\overline{\rho}\overline{\eta}}{\overline{\rho}^2 + \overline{\eta}^2}.$$

angles measured in decays of K, B and B_s

Neutrinos

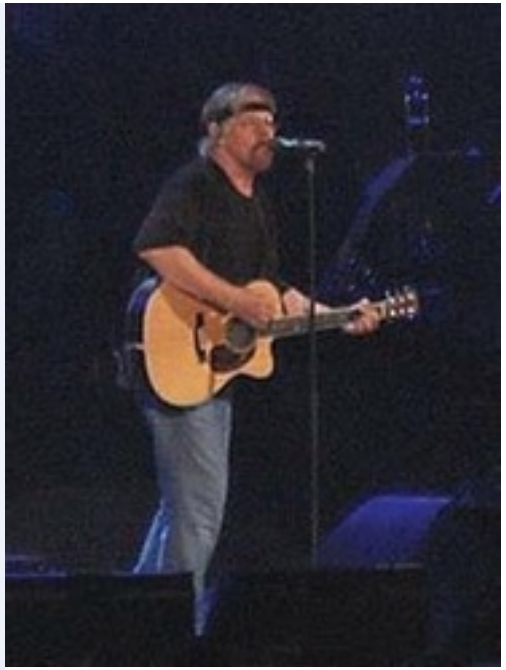
Obviously.. Biggest question for many:

What is the mass of neutrinos ?

Lyrics: Bob Seger (2003)

"Tomorrow"

from Greatest Hits Vol 2
. . .
Let me see a show of hands
Tell me the truth now
What happens if
Neutrinos have mass
I can't tell you about tomorrow
I'm as lost as yesterday
. . .



Rock and Roll Hall of Fame
> 60 million albums sold !!

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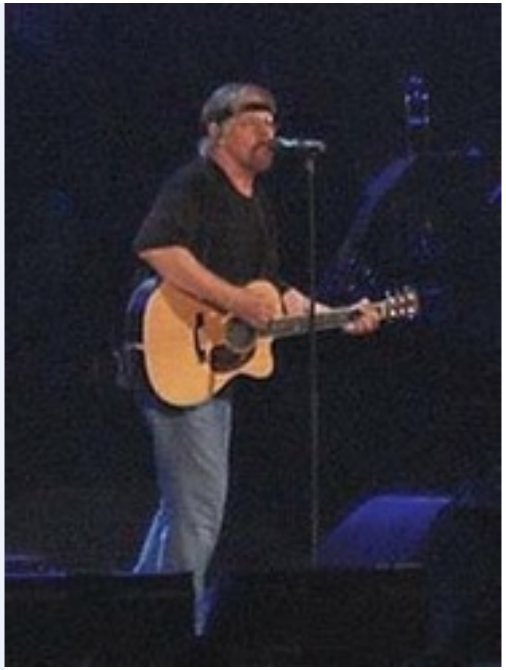
from Greatest Hits Vol 2
. . .
Let me see a show of hands
Tell me the truth now
What happens if
Neutrinos have mass
I can't tell you about tomorrow
I'm as lost as yesterday
. . .



Rock and Roll Hall of Fame
> 60 million albums sold !!

Some Neutrino properties

Neutrinos are (quasi) massless

- all reactions involving neutrinos infer: $m_\nu=0$ (valid for e, μ, τ)
- direct mass measurements for neutrinos:

$$\begin{aligned} m_{\nu_e} &< 3 \quad \mathrm{eV} \\ m_{\nu_\mu} &< 190 \quad \mathrm{keV} \\ m_{\nu_\tau} &< 18,2\,\mathrm{MeV} \end{aligned}$$

- only hint for masses: neutrino oscillations
- In the following assume: neutrinos are massless !
- neutrinos and anti-neutrinos are different particles

$$\begin{aligned} \mathrm{n} &\rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e \\ \bar{\nu}_e + \mathrm{p} &\rightarrow \mathrm{n} + \mathrm{e}^+ \\ \bar{\nu}_e + \mathrm{n} &\not\rightarrow \mathrm{p} + \mathrm{e}^- \end{aligned}$$

- Neutrinos know about families

$$\begin{aligned} \bar{\nu}_\mu + \mathrm{p} &\rightarrow \mathrm{n} + \mu^+ \\ \nu_\mu + \mathrm{p} &\not\rightarrow \mathrm{n} + \mu^+ \\ \bar{\nu}_\mu + \mathrm{p} &\not\rightarrow \mathrm{n} + \mathrm{e}^+ \end{aligned}$$

Lyrics: Bob Seger (2003)

"Tomorrow"

from Best Hits Vol 2
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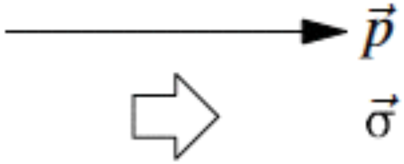
Nobel prize 1988:
Lederman, Schwarz
Steinberger

Parity Violation in Weak interaction

Neutrinos have definite helicity



$$H = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{\sigma}|\,|\vec{p}|}$$

right-handed

H = +1

For massless particle: $J_z = \pm 1$ $H = \pm 1$

Helicity is "good" quantum number - valid in all frames of inertia ($v_{particle}$ = c)

- Helicity couples "spin" to direction in space
- Photons have: $H = \pm 1$ - emission from atom equal for both helicity states
  - Emission in all directions : no space direction is preferred
  - Parity is conserved

Experimentally found:

- Neutrinos come in only one helicity state

$$\boxed{H_{\upsilon} = -1 \qquad H_{\bar{\upsilon}} = +1}$$

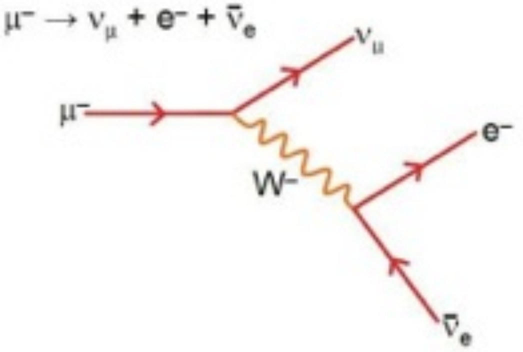
Examples for Parity Violating Processes

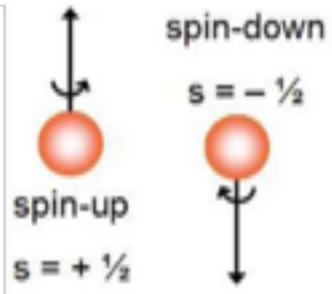
All process involving weak interaction violate parity

not only those involving neutrinos

- Charged currents
  - have maximal parity violation
  - Only couple to left handed fermions (right-handed anti-fermions)
- Neutral currents
  - Degree of parity violation depends on electric charge of particles involved (see later)

Example: μ-decay





The two possible spin polarisations of the electron

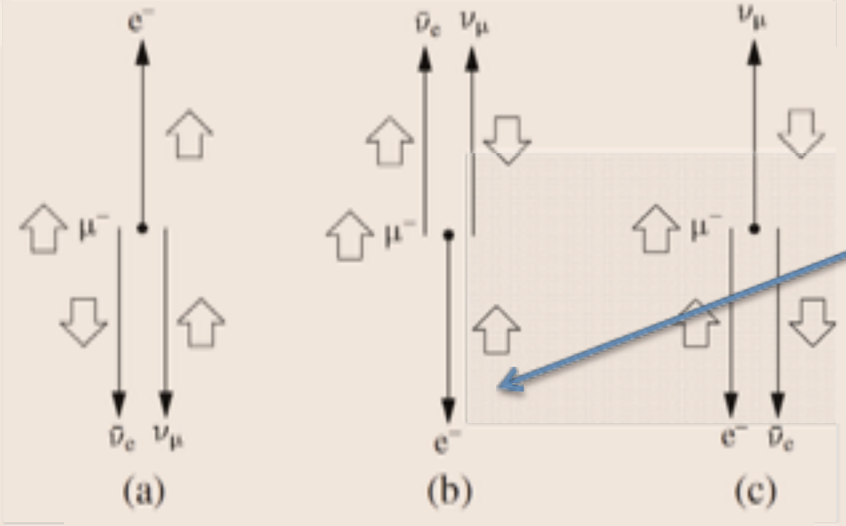
$$H = \frac{\vec{\sigma} \cdot \vec{p}c}{|\vec{\sigma}||E|} = \frac{\vec{\sigma}}{|\vec{\sigma}|} \cdot \vec{\beta} = \frac{\vec{\sigma}}{|\vec{\sigma}|} \cdot \frac{\vec{v}}{c}$$

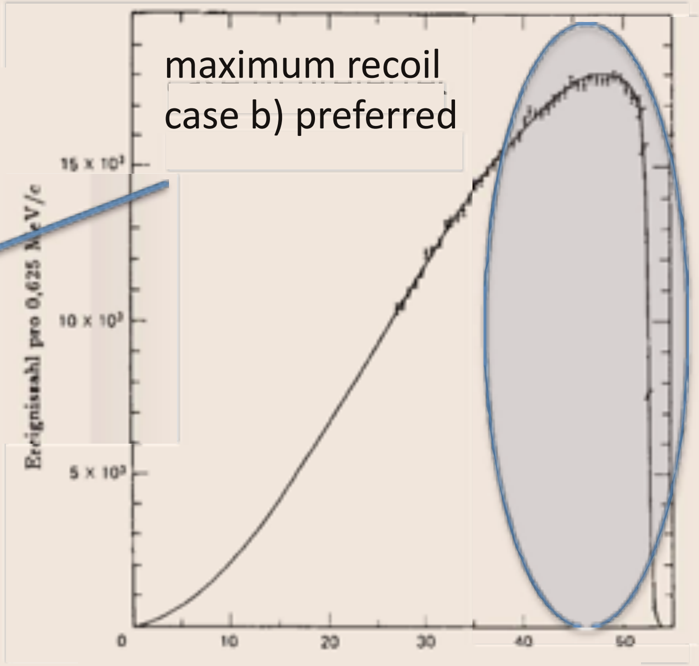
for particles with m = 0 :

helicity not frame independent !

electrons: $E_{kin} >> m_e$ and $v = c$

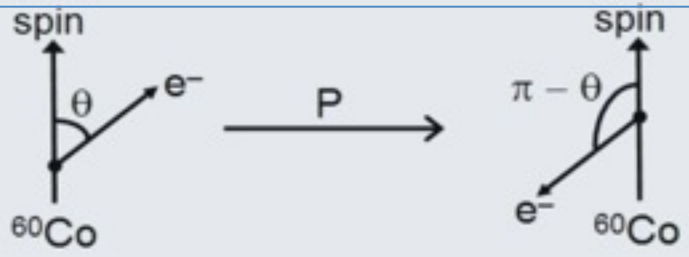
Examples for Parity Violation





maximum recoil
case b) preferred

preferred

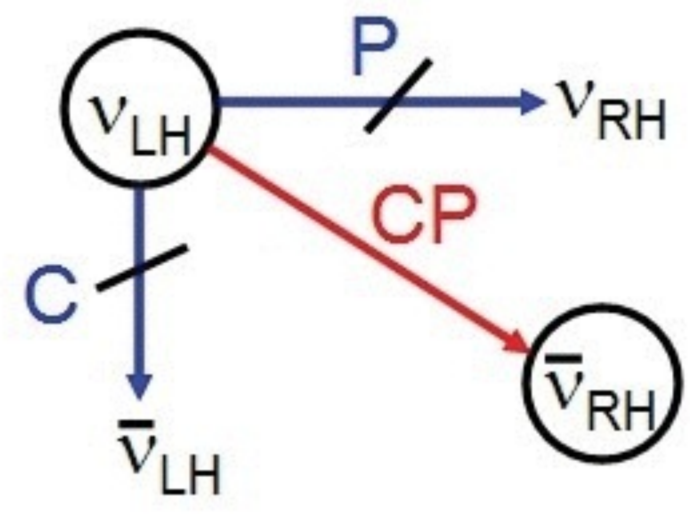


Beta-decay of cobalt-60

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni}^{*} + e^{-} + \bar{\nu}_{e}$$

P-Violation III

Note: CP-Symmetry still conserved !



Wait for later...

V-A Structure

How to describe parity-violating interaction ?

E. Fermi:

- Vector current (like e.m.): $\mathcal{L}_{\text{inf}}^{\text{eff.}} = \underbrace{i \cdot e\bar{\Psi}(x)\gamma_\mu \Psi(x)}_{\text{Vektorstrom}} \underbrace{A^\mu(x)}_{\text{Photonfeld}}$ has negative parity

  V^μ

- Axial-vector current (new): $A^\mu = \overline{\psi}\gamma^\mu \gamma_5 \psi$ has positive parity

  γ_S: projection operator for chirality $\gamma_5 \Psi(x) = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\Psi(x)$ $\gamma_5 \psi_{\mathrm{L,R}} = \mp \psi_{\mathrm{L,R}}$

  chirality = helicity for m=0

  $$P_R = \frac{1}{2}(1+\gamma_5) \quad P_L = \frac{1}{2}(1-\gamma_5) \quad \text{mit}\, P_{L,R}^2 = P_{L,R}$$

- Interference of both: violates parity: $V^\mu \pm A^\mu$
- "charged" interaction couples to left-handed fermions ⟶ V^μ - A^μ

  $\frac{G_F}{\sqrt{2}} \left(\bar{\nu}_\ell \gamma_\mu (1-\gamma_5)\ell\right)$ *example for leptons*

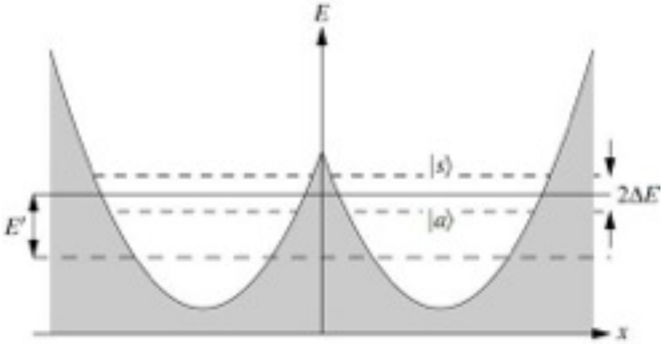
  effective coupling constant:
  "charge" * propagator

  $$G_F = \frac{\sqrt{2}\, g^2}{8M_W^2} = 1.16637 \cdot 10^{-5}\,\mathrm{GeV}^{-2} \approx 10^{-5} \cdot M_p^{-2}$$

- S, P, T seem not to be realized

Flavour Oscillations

Consider 2-state problem



$\mathcal{H}_0|L\rangle = E_0|L\rangle \quad \mathcal{H}_0|R\rangle = E_0|R\rangle$

no transitions for infinite wall

assume interaction allowing transitions

$$\mathcal{H}|\psi\rangle = (\mathcal{H}_0 + \mathcal{H}_{\text{int}})|\psi\rangle = E|\psi\rangle \qquad \mathcal{H}_{\text{int}} \ll \mathcal{H}_0$$

symmetry around $x = 0$

$$[\mathcal{H}, \mathcal{P}] = [\mathcal{H}_0 + \mathcal{H}_{\text{int}}, \mathcal{P}] = 0$$

$$\mathcal{P}|L\rangle = |R\rangle \qquad \mathcal{P}|R\rangle = |L\rangle$$

new Eigenstates with definite parity:

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle + |R\rangle\right) \qquad \mathcal{P}|S\rangle = |S\rangle$$

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle - |R\rangle\right) \qquad \mathcal{P}|A\rangle = -|A\rangle$$

orthonormal

$$\langle A|\mathcal{H}_0|S\rangle = 0$$

perturbation theory:

$$\langle S|\mathcal{H}_{\text{int}}|S\rangle = E' + \Delta E$$

$$\langle A|\mathcal{H}_{\text{int}}|A\rangle = E' - \Delta E$$

level shift

$$\langle L|\mathcal{H}_{\text{int}}|L\rangle = \langle R|\mathcal{H}_{\text{int}}|R\rangle = E'$$

degeneracy broken

$$\langle L|\mathcal{H}_{\text{int}}|R\rangle = \langle R|\mathcal{H}_{\text{int}}|L\rangle = \Delta E$$

Particle Oscillations II

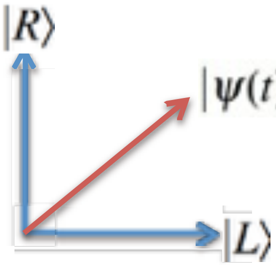
Gedanken-experiment

at t=0: through particle into left potential well: $|\psi(0)\rangle = |L\rangle = \frac{1}{\sqrt{2}}\left(|S\rangle + |A\rangle\right)$ no defined parity

switch in H_{int} and observe time evolution: $i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = (\mathcal{H}_0 + \mathcal{H}_{\mathrm{int}})|\psi(t)\rangle$

we can write: $|\psi(t)\rangle = \alpha(t)|L\rangle + \beta(t)|R\rangle$ $\qquad |\alpha(t)|^2 + |\beta(t)|^2 = 1$

$\alpha(0) = 1 \text{ und } \beta(0) = 0$



$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}(E_0+E')t/\hbar}\left(\cos\frac{\Delta Et}{\hbar}|L\rangle + \mathrm{i}\sin\frac{\Delta Et}{\hbar}|R\rangle\right)$$

Just like coupled pendulum !

$P(R) = \sin^2\frac{\Delta Et}{\hbar}$ probability to find system in R

$\omega = \frac{\Delta E}{\hbar} = \frac{1}{\hbar}\langle L|\mathcal{H}_{\mathrm{int}}|R\rangle$ oscillation frequency

The case of K^0-mesons

Production via strong interaction: Decay via weak interaction

$K^0 : \vert s\bar{d} >$ $Y(K^0) = 1$	$pA \to K^0 K^+ n + X$	$\Delta S_{strangeness} = -1$	$K^0 \to \pi^+\pi^-$ $CP = +1$	$K^0 \to \pi^+\pi^-\pi^0$ $CP = -1$
$\bar{K}^0 : \vert \bar{s}d >$ $Y(\bar{K}^0) = -1$	$pA \to \bar{K}^0 K^- p + X, \bar{K}^0 \Lambda + X$	$\Delta S_{strangeness} = +1$	$\bar{K}^0 \to \pi^+\pi^-$ $CP = +1$	$\bar{K}^0 \to \pi^+\pi^-\pi^0$ $CP = -1$

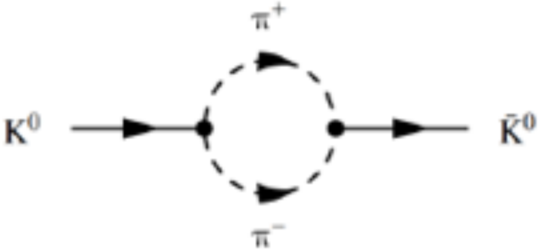
Different particles for strong interaction

Eigenstates towards H_0

$m(K^0) = m(\bar{K}^0)$ CPT – theorem

$$\mathcal{H}_0 = \mathcal{H}_{\text{stark}} + \mathcal{H}_{\text{em}}$$

Same particles for weak interaction



$$\mathcal{H}_{\text{int}} = \mathcal{H}_{\text{schwach}}$$

Particle Oscillations IV

Weak interaction leads to mixing: $|K(t)> \; = \; \alpha(t)\big|K^0 >+ \beta(t)\big|\bar{K}^0 >$

For weak interaction: build Eigenstates to CP-operator: $[\mathcal{H}\,,\mathcal{CP}] = 0$

$$\mathcal{C}\left|\mathrm{K}^0\right\rangle = -\left|\bar{\mathrm{K}}^0\right\rangle \qquad \mathcal{C}\left|\bar{\mathrm{K}}^0\right\rangle = -\left|\mathrm{K}^0\right\rangle \qquad \mathcal{P}\left|\mathrm{K}^0\right\rangle = -\left|\mathrm{K}^0\right\rangle$$

$$\mathcal{CP}\left|\mathrm{K}^0\right\rangle = \left|\bar{\mathrm{K}}^0\right\rangle \qquad \mathcal{CP}\left|\bar{\mathrm{K}}^0\right\rangle = \left|\mathrm{K}^0\right\rangle$$

Eigenstates to H_{int}

$$\left|\mathrm{K}_1^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\mathrm{K}^0\right\rangle + \left|\bar{\mathrm{K}}^0\right\rangle\right), \qquad \left|\mathrm{K}_2^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\mathrm{K}^0\right\rangle - \left|\bar{\mathrm{K}}^0\right\rangle\right)$$

$$\mathcal{CP}\left|\mathrm{K}_1^0\right\rangle = \left|\mathrm{K}_1^0\right\rangle, \qquad \mathcal{CP}\left|\mathrm{K}_2^0\right\rangle = -\left|\mathrm{K}_2^0\right\rangle$$

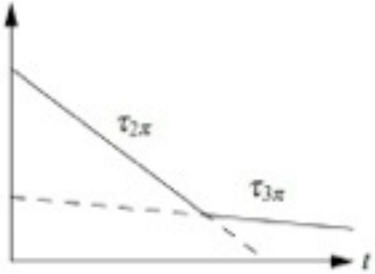
$\Delta m(\mathrm{K} \to 2\pi) = 220\ \mathrm{MeV}$

$\Delta m(\mathrm{K} \to 3\pi) = 80\ \mathrm{MeV}$

$\tau_{\mathrm{K}_1^0} \ll \tau_{\mathrm{K}_2^0}$ phase space

$\tau_{\mathrm{K}_2^0} = 0.517 \cdot 10^{-7}\ \mathrm{s} \approx 580 \cdot \tau_{\mathrm{K}_1^0}$

Thus there are "two lifetimes" for $\left|\mathrm{K}^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\mathrm{K}_1^0\right\rangle + \left|\mathrm{K}_2^0\right\rangle\right)$ →



Particle Oscillations V

Time evolutions:

$$|K_1^0(t)\rangle = |K_1^0(0)\rangle e^{-im_1 t} \underbrace{e^{-\Gamma_1 t/2}}_{\text{Zerfall}} = |K_1^0(0)\rangle e^{-(im_1+\Gamma_1/2)t} \qquad |K_2^0(t)\rangle = |K_2^0(0)\rangle e^{-(im_2+\Gamma_2/2)t}$$

Amplitude to find *K⁰* after time *t* if we start with a *K⁰* at *t=0*

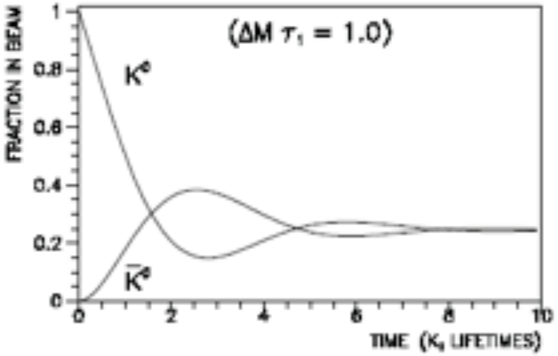
$$A_{K^0}(t) = \underbrace{\frac{1}{\sqrt{2}}\left(\langle K_1^0(0)| + \langle K_2^0(0)|\right)}_{\langle K^0|} \underbrace{\frac{1}{\sqrt{2}}\left(|K_1^0(t)\rangle + |K_2^0(t)\rangle\right)}_{|K^0(t)\rangle} =$$

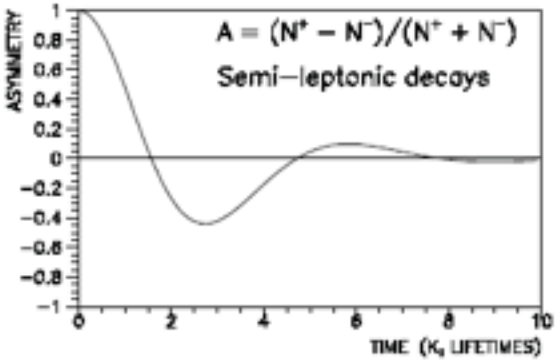
$$P_{K^0}(t) = \left|A_{K^0}(t)\right|^2 = \frac{1}{4}\left(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-(\Gamma_1+\Gamma_2)\frac{t}{2}} \cos\left((m_1 - m_2)t\right)\right)$$

$$\updownarrow$$

$$P_{\bar{K}^0}(t) = \frac{1}{4}\left(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_1+\Gamma_2)\frac{t}{2}} \cos\left((m_1 - m_2)t\right)\right)$$

$$K^0 \to l^+ + \nu + \pi^- \quad , \quad \bar{K}^0 \to l^- + \bar{\nu} + \pi^+$$

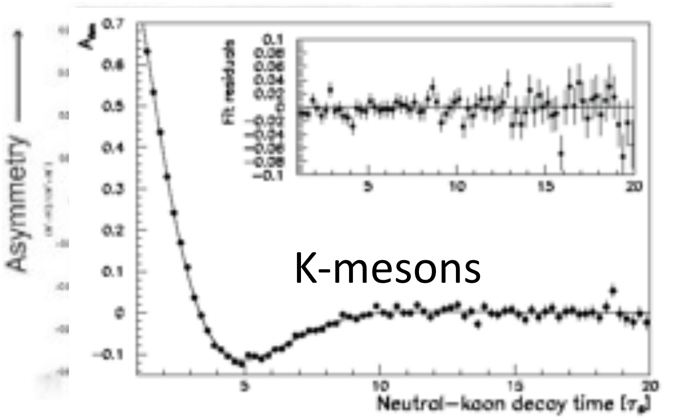




Holds for all particle oscillations !!

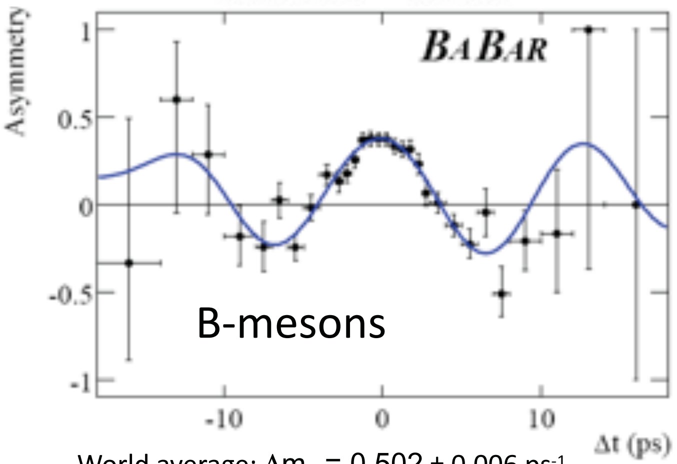
$$\underset{\bar{b}d}{B^0} \longleftrightarrow \underset{b\bar{d}}{\bar{B}^0} \qquad \underset{\bar{b}s}{B_s^0} \longleftrightarrow \underset{b\bar{s}}{\bar{B}_s^0}$$

Particle Oscillations V



$$\Delta m = (0.5295 \pm 0.0020_{\text{stat}} \pm 0.0003_{\text{syst}}) \times 10^{10}\ \hbar/s$$

$$\mathcal{A}_{\text{mix}}(t) = \frac{N(t)_{\text{unmix}} - N(t)_{\text{mixed}}}{N(t)_{\text{unmix}} + N(t)_{\text{mixed}}} = \cos(\Delta m\, t)$$

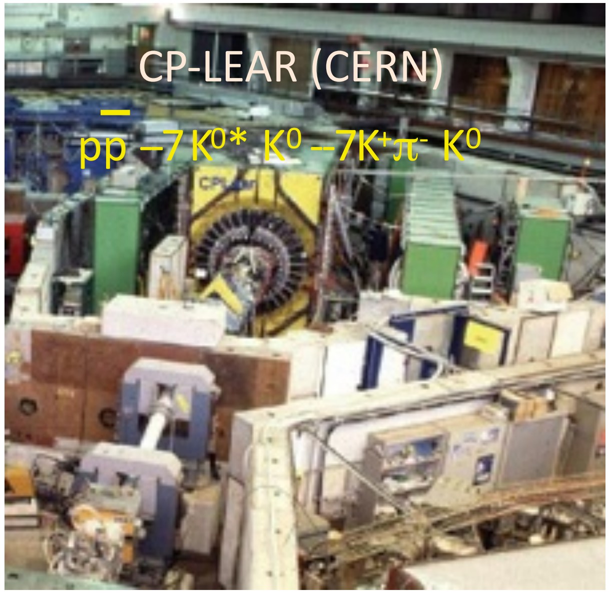


$$e^+ + e^- \to B^0 \bar{B}^0$$

$$B^0 \to l^- + X$$

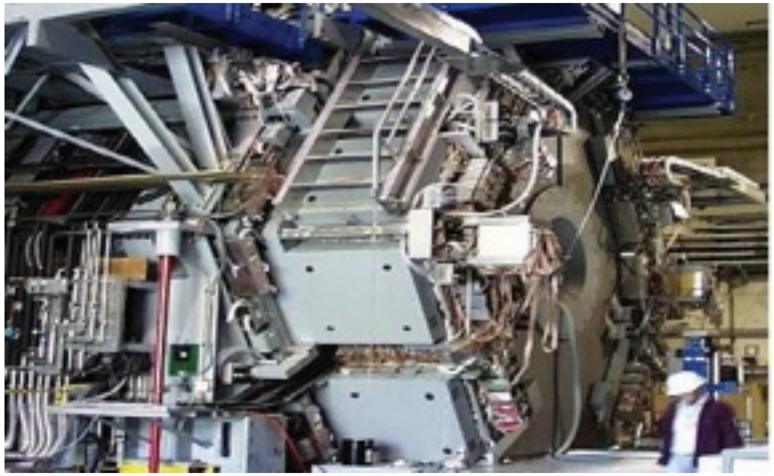
$$B^0 \to l^+ + X$$

World average: $\Delta m_d = 0.502 \pm 0.006\ \text{ps}^{-1}$



CP-LEAR (CERN)

$p\bar{p} \rightarrow K^{0*}\ K^0 \rightarrow K^+\pi^-\ K^0$



B-factories (BaBar, Belle)