Introductory Course in Particle Physics

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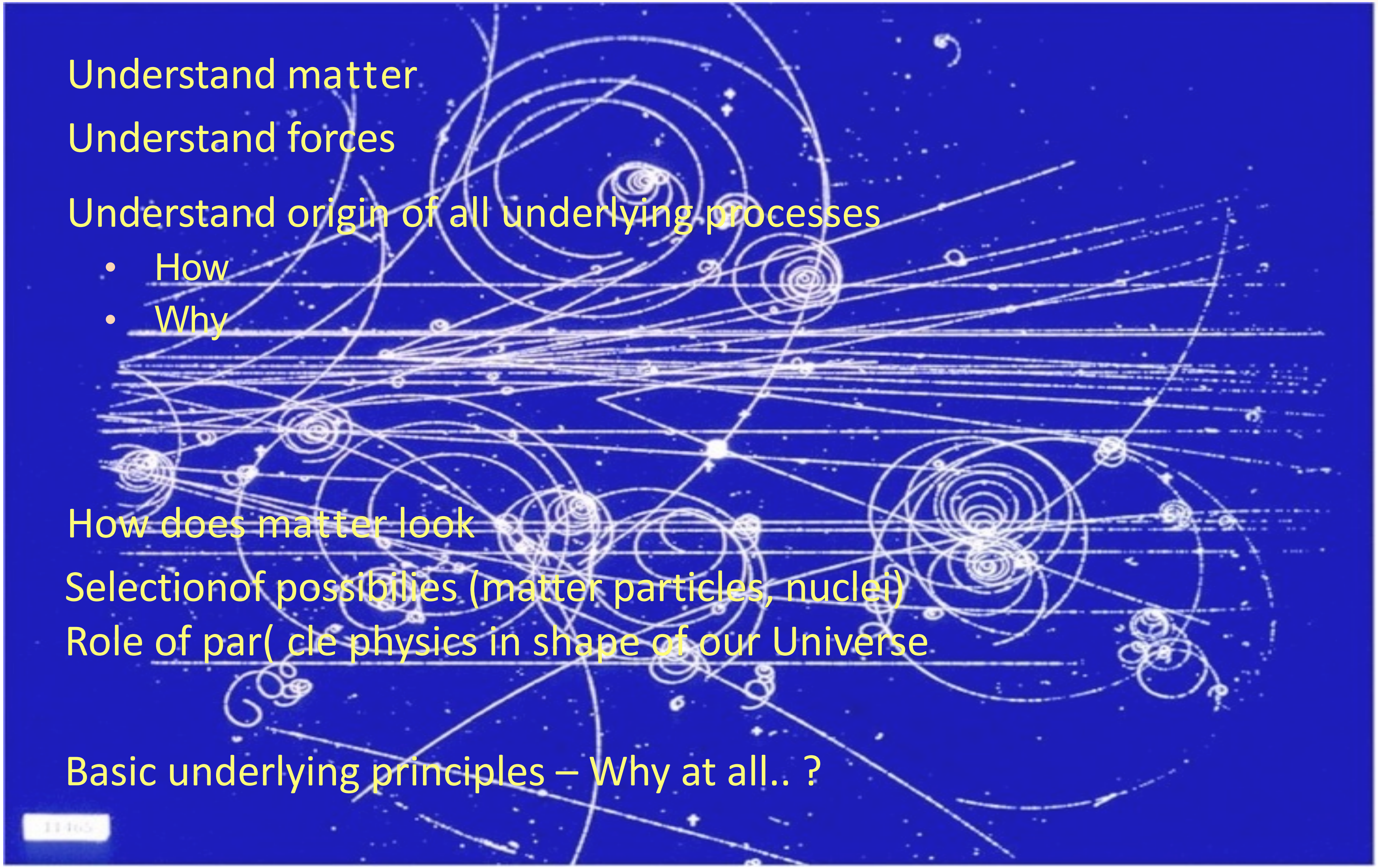
Course for astrophysicists

Role of particle and nuclear physics

Exzellenzcluster Universe



Understand matter

Understand forces

Understand origin of all underlying processes

- How
- Why

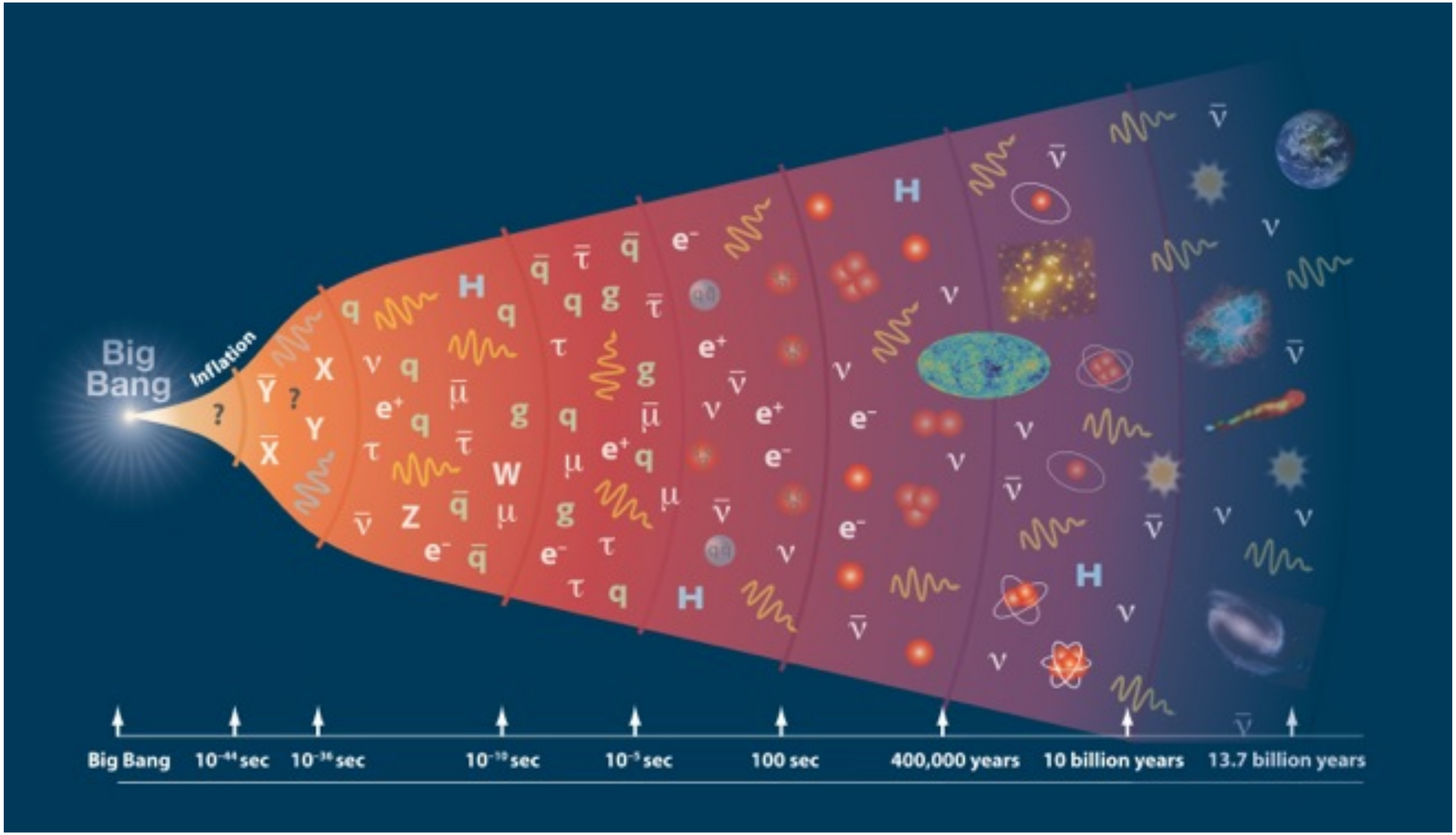
How does matter look

Selectionof possibilies (matter particles, nuclei)

Role of par(cle physics in shape of our Universe

Basic underlying principles – Why at all.. ?

History of the Universe



Building Blocks of Matter

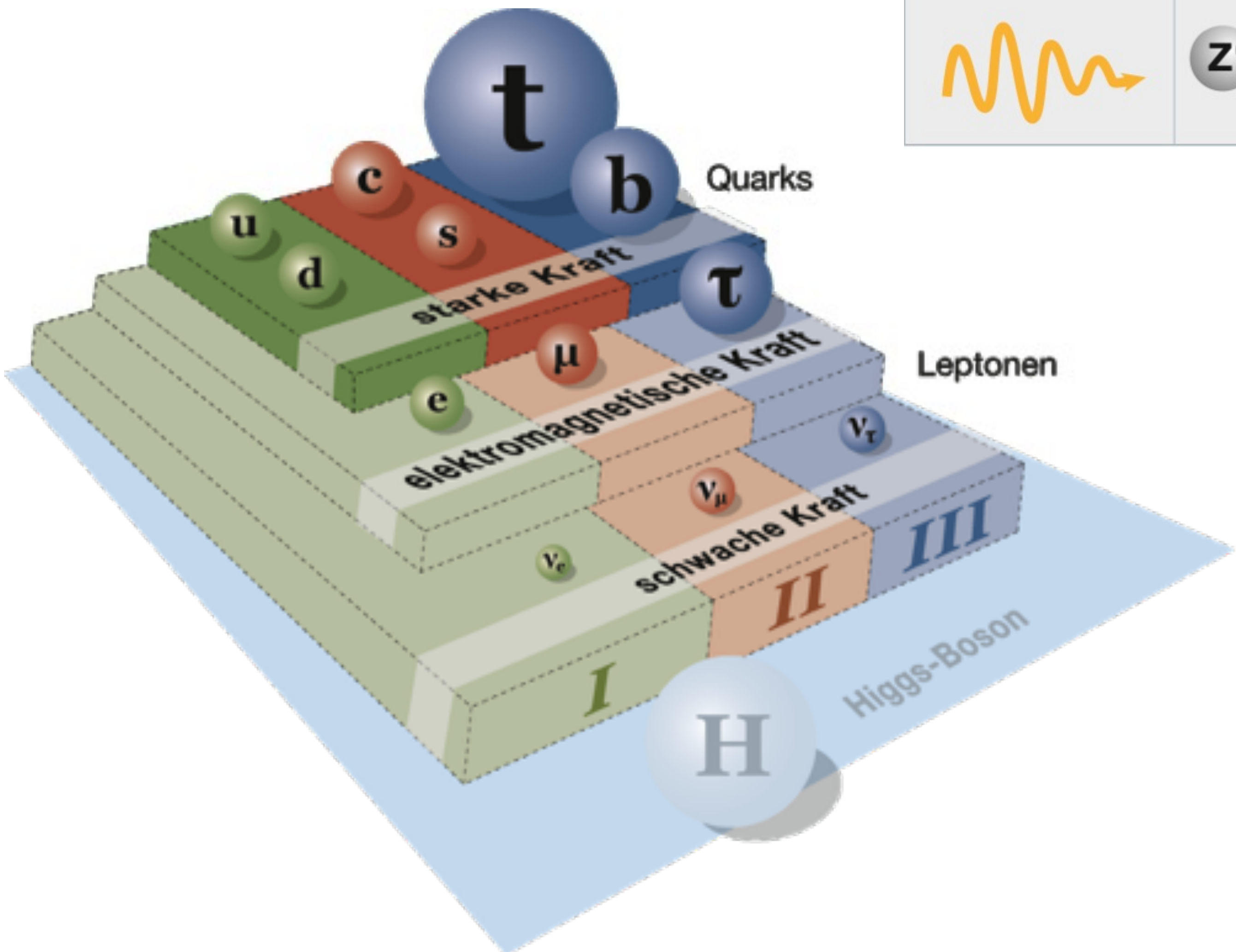
Exzellenzcluster Universe

Building Blocks of Matter

Building Blocks of Matter

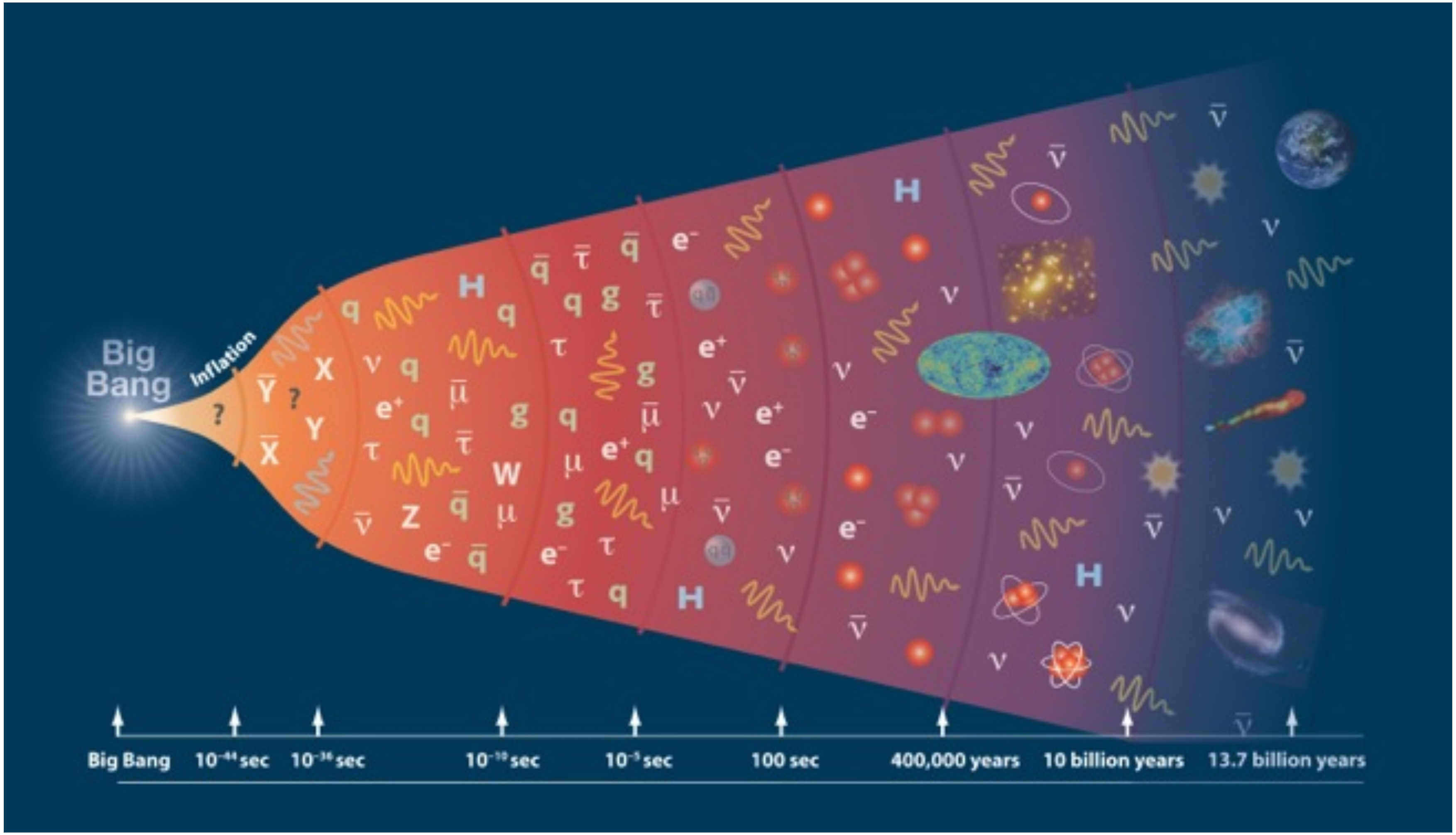
Exzellenzcluster Universe

elektromag. Kraft	schwache Kraft	starke Kraft
1 Photon	3 Bosonen	8 Gluonen
	Z^0 W^+ W^-	

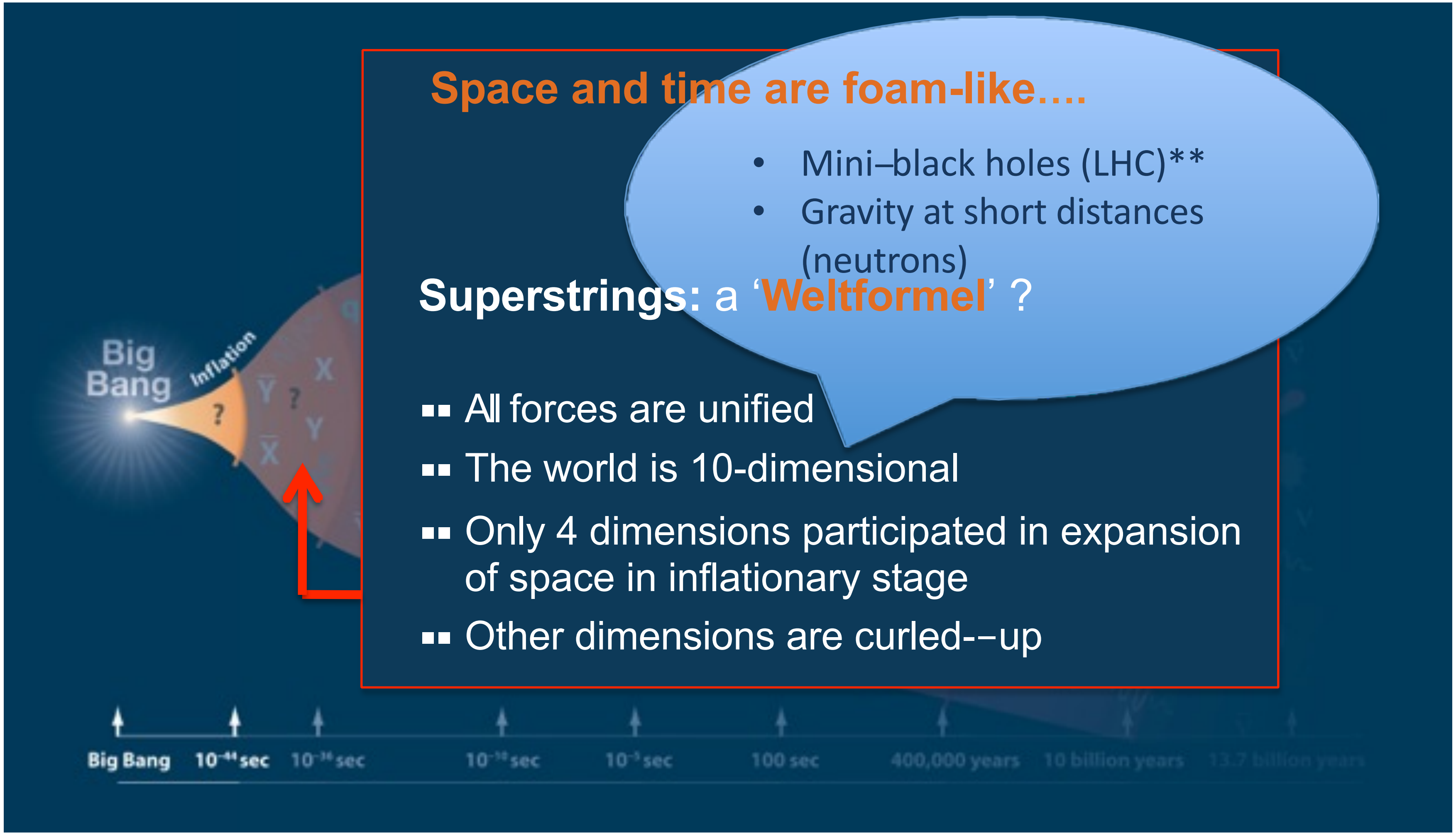


History of the Universe

Exzellenzcluster Universe



10^{-43} Seconds after Big Bang: The curtain lifts



Space and time are foam-like….

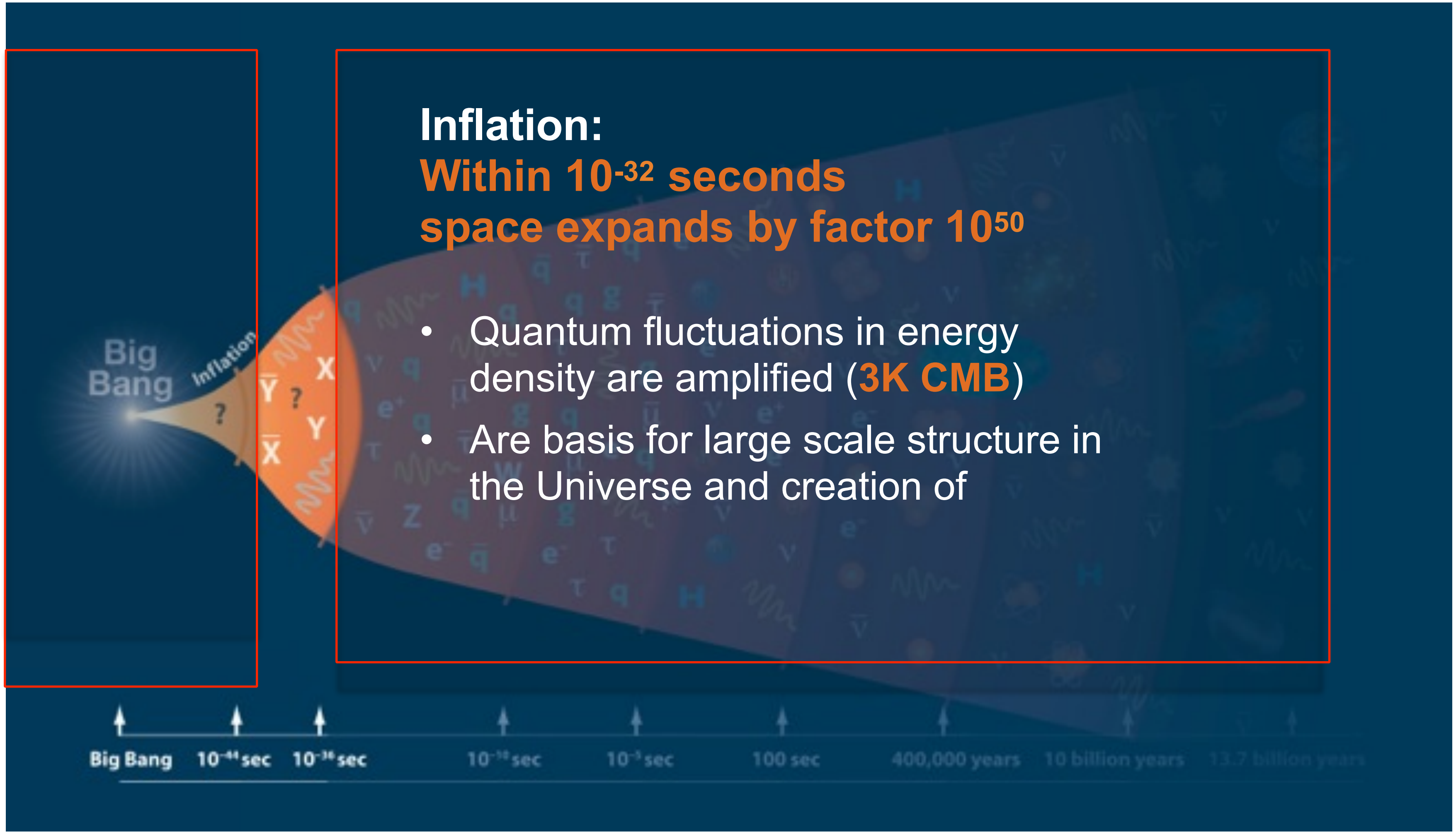
- Mini–black holes (LHC)**
- Gravity at short distances (neutrons)

Superstrings: a '**Weltformel**' ?

- All forces are unified
- The world is 10-dimensional
- Only 4 dimensions participated in expansion of space in inflationary stage
- Other dimensions are curled-–up

10^{-35} to 10^{-32} Seconds after Big Bang

Exzellenzcluster Universe

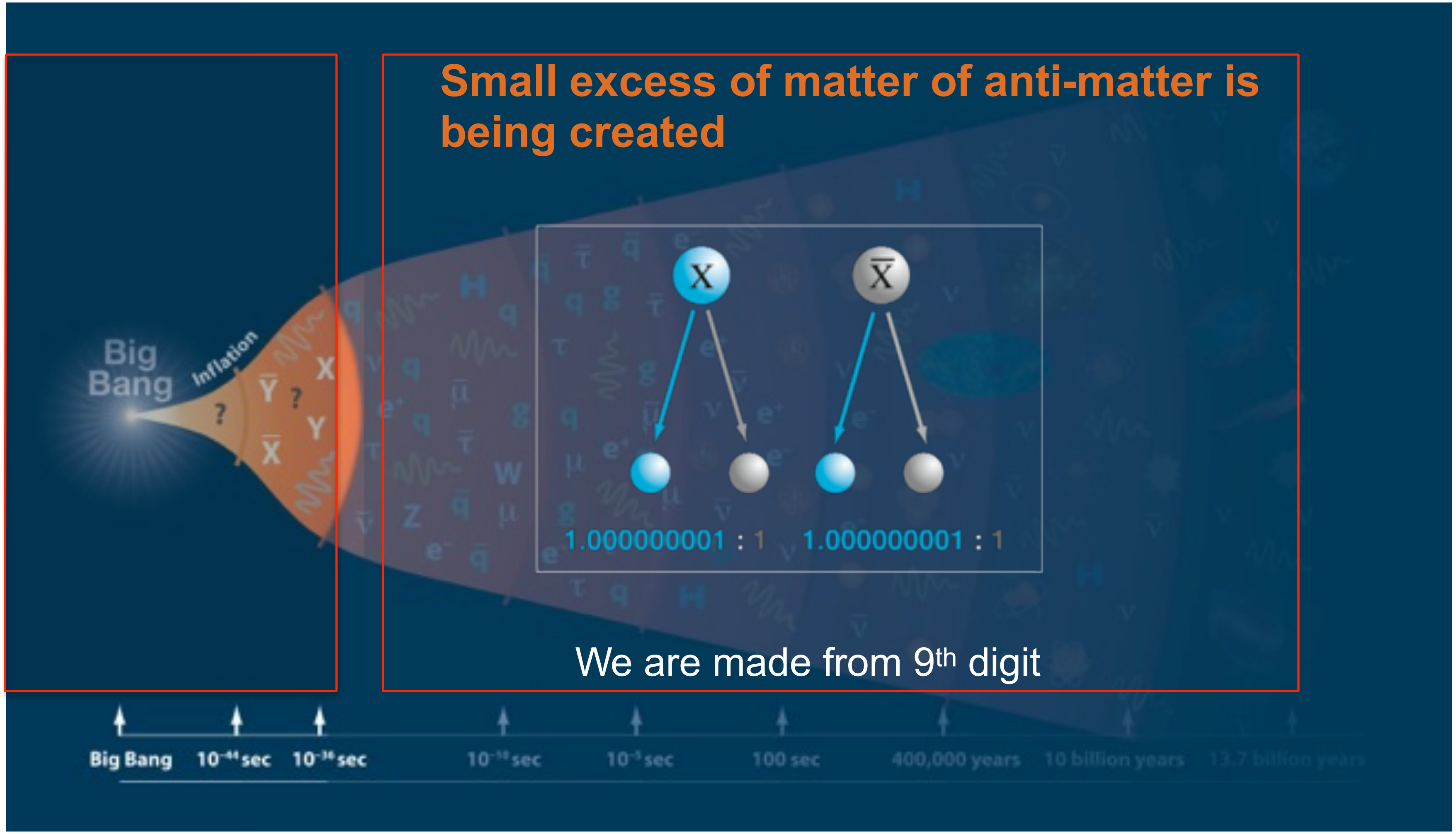


Inflation:
Within 10^{-32} seconds space expands by factor 10^{50}

- Quantum fluctuations in energy density are amplified (**3K CMB**)
- Are basis for large scale structure in the Universe and creation of

10^{-34} to 10^{-33} Seconds after Big Bang

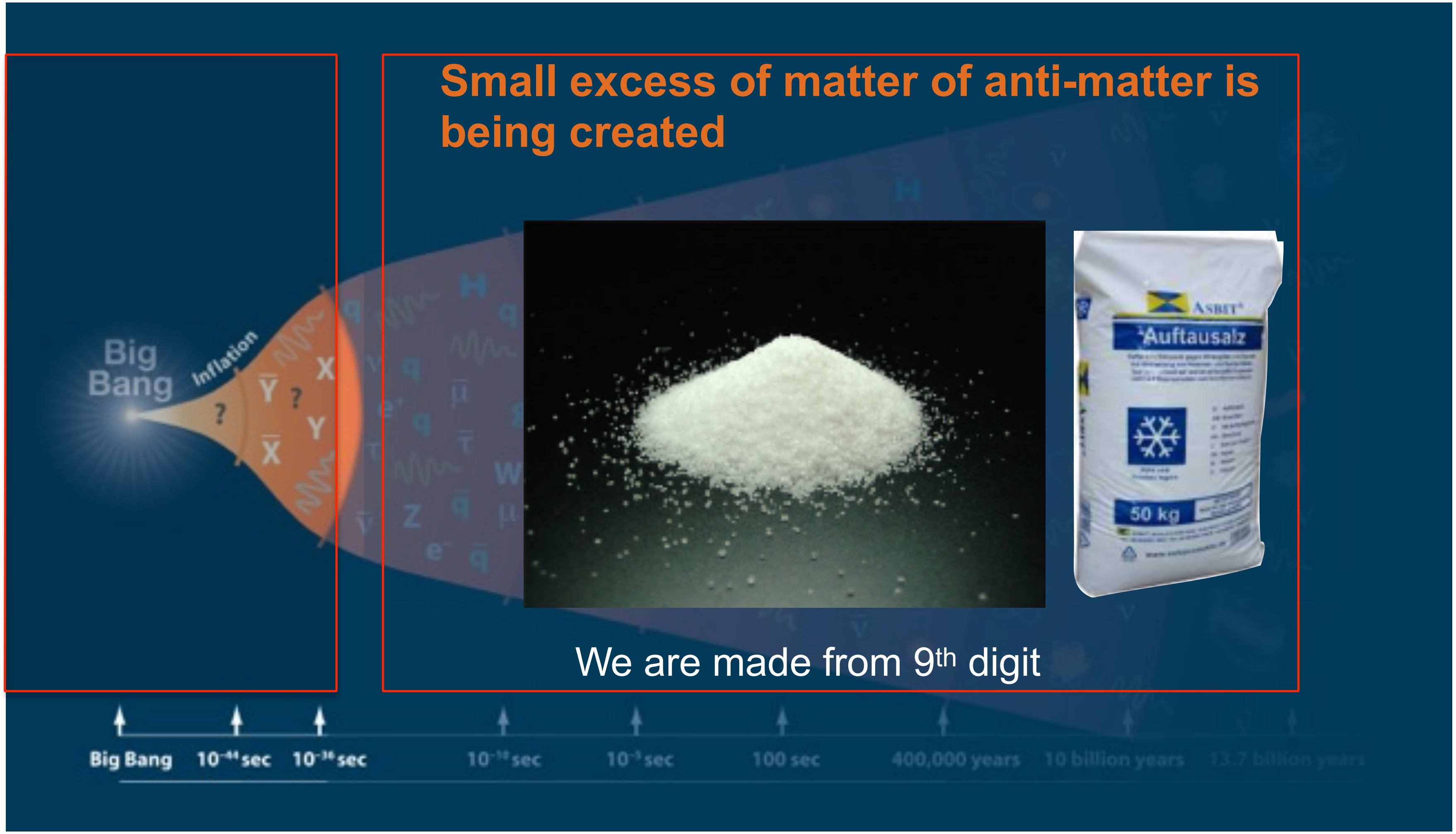
Small excess of matter of anti-matter is being created



We are made from 9th digit

10^{-34} to 10^{-33} Seconds after Big Bang

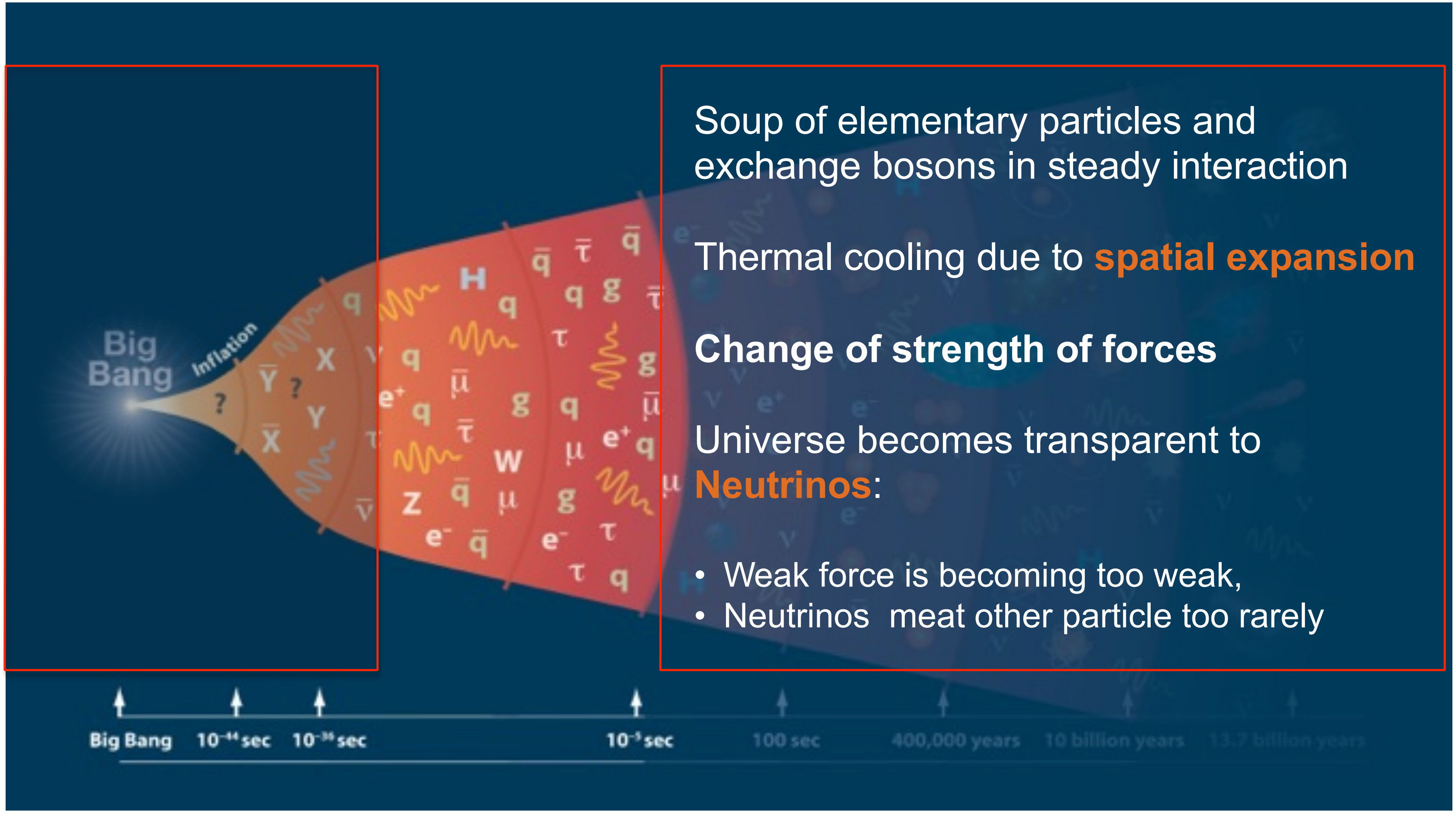
Small excess of matter of anti-matter is being created



We are made from 9th digit

Until 10^{-6} Seconds after Big Bang

Exzellenzcluster Universe



Soup of elementary particles and exchange bosons in steady interaction

Thermal cooling due to **spatial expansion**

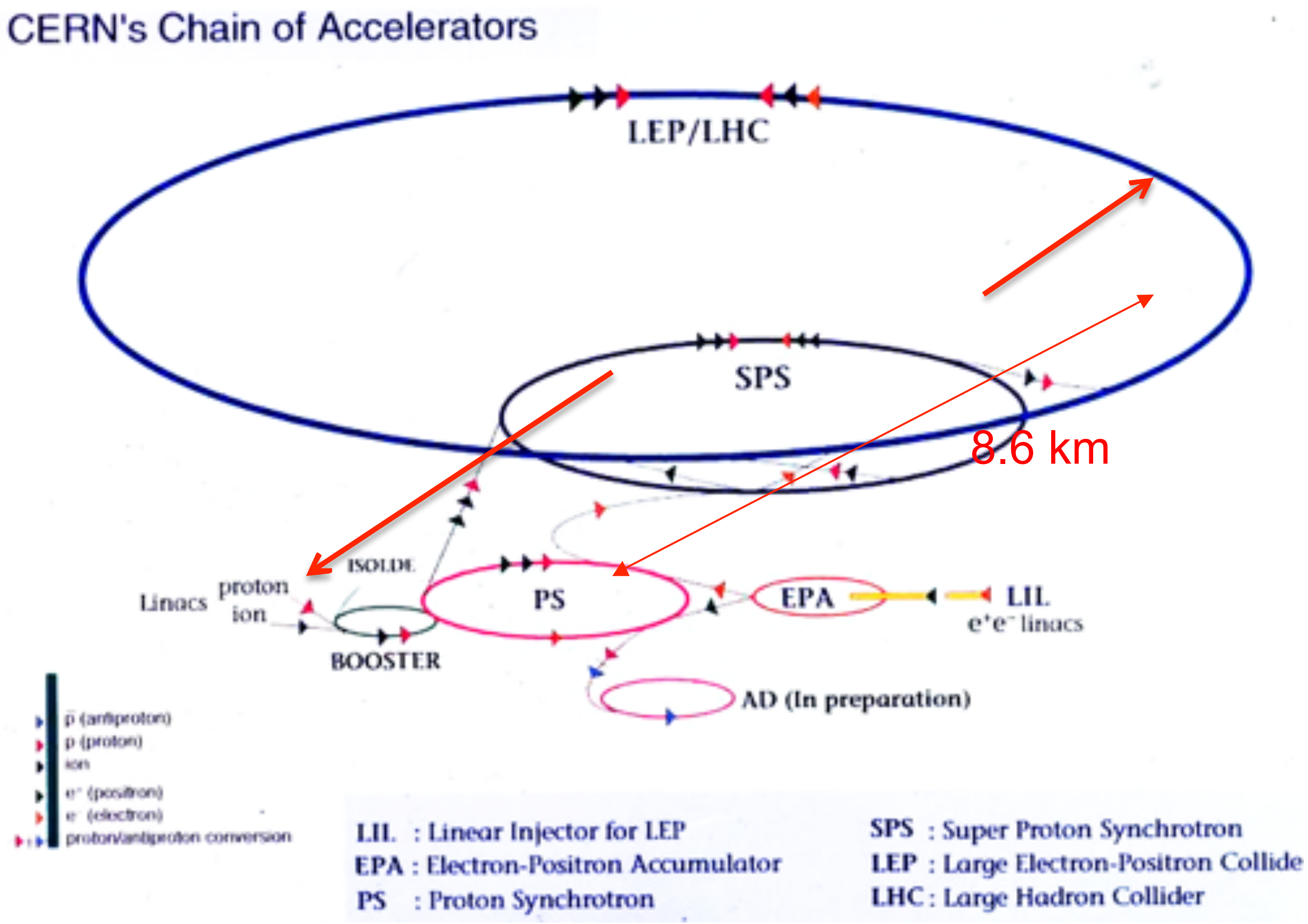
Change of strength of forces

Universe becomes transparent to **Neutrinos**:

- Weak force is becoming too weak,
- Neutrinos meat other particle too rarely

CERN – Lord of the Rings

Exzellenzcluster Universe

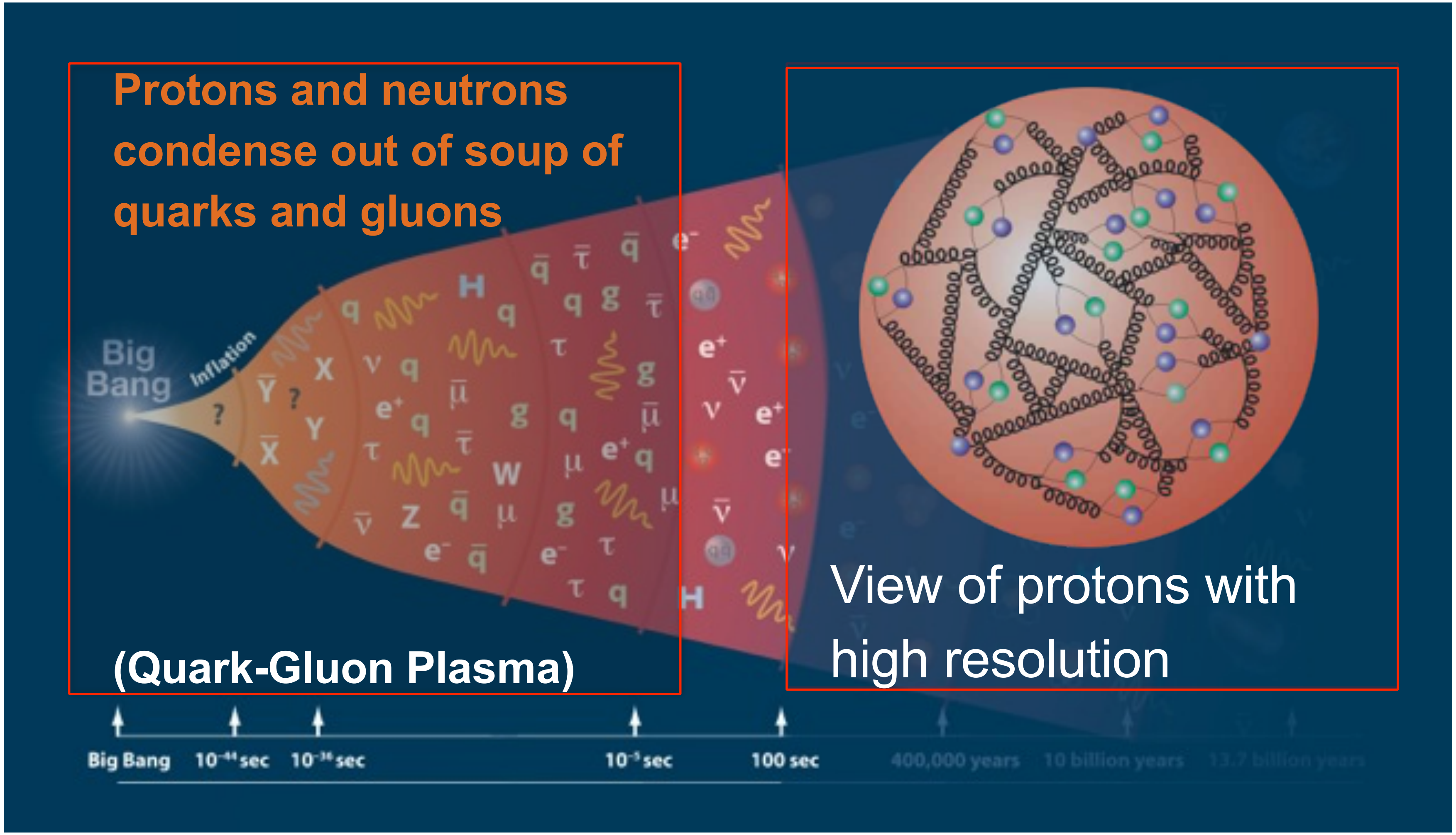


CERN Accelerator and LHC

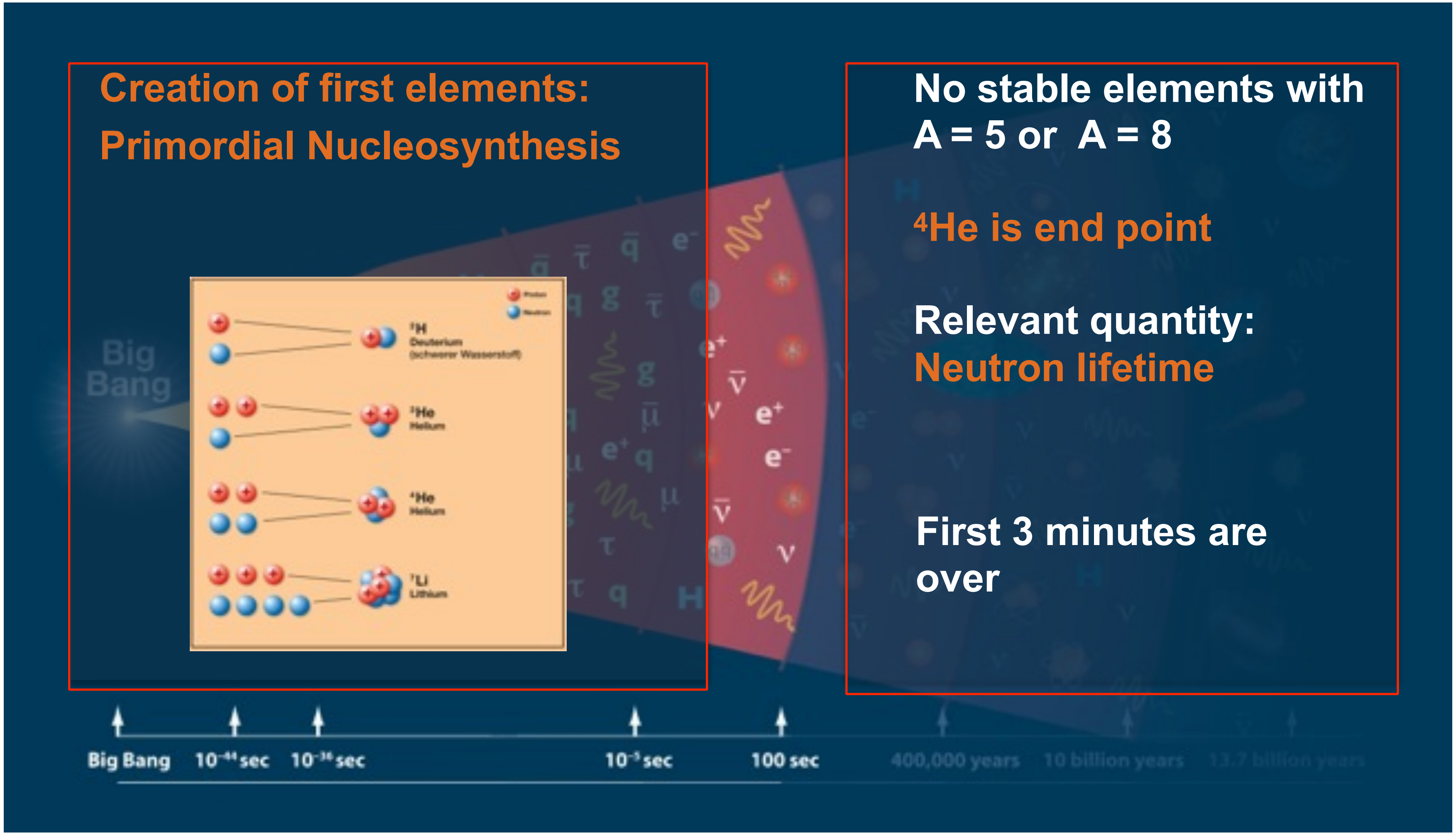
7 TeV on 7 TeV collisionen... M_p

23.11.2009

10^{-4} to 10^{-2} seconds after big bang



$10^{-2} - 10^{3}$ seconds after big bang



Creation of first elements:

Primordial Nucleosynthesis

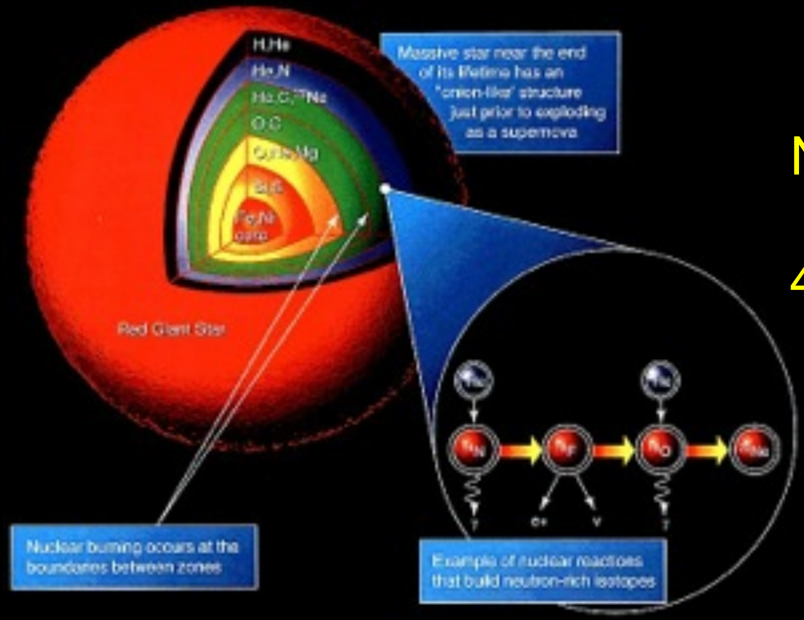
No stable elements with A = 5 or A = 8

^{4}He is end point

Relevant quantity:
Neutron lifetime

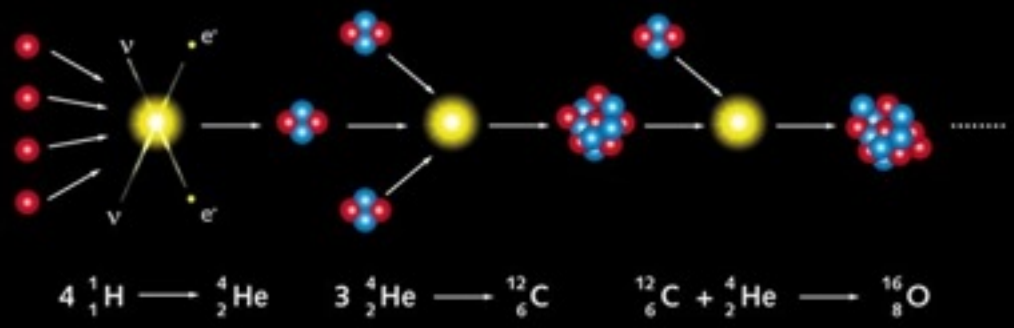
First 3 minutes are over

Synthesis of Elements – What a Star does

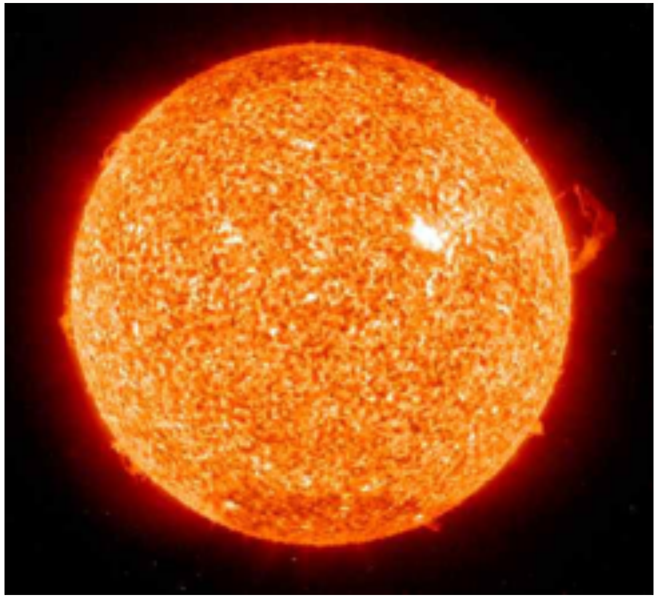


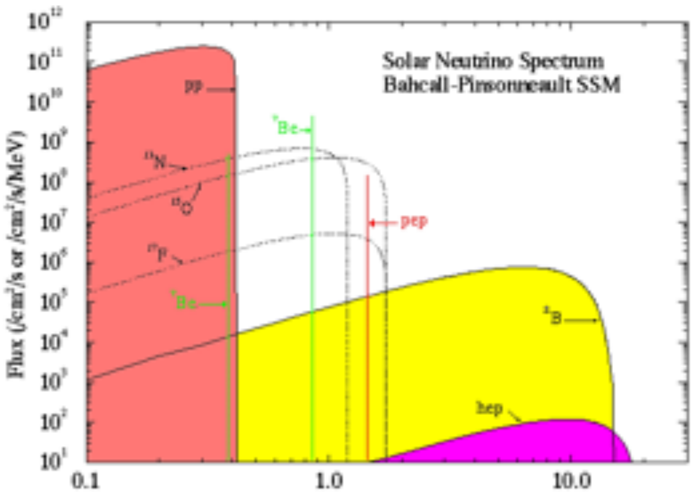
Nuclear fusion in centre of sun:

$4\ p \ \rightarrow\rightarrow {}^{4}He + 2e^{+} + 2\nu_{e}\ (+\ 26.7\ MeV)$

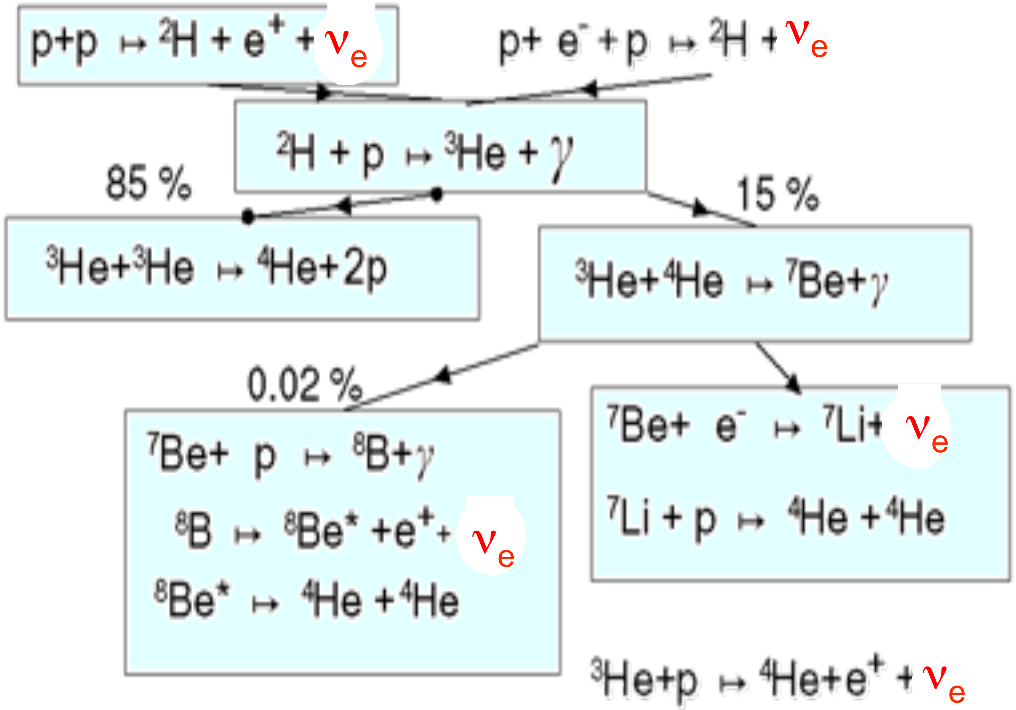


What does the sun do ?





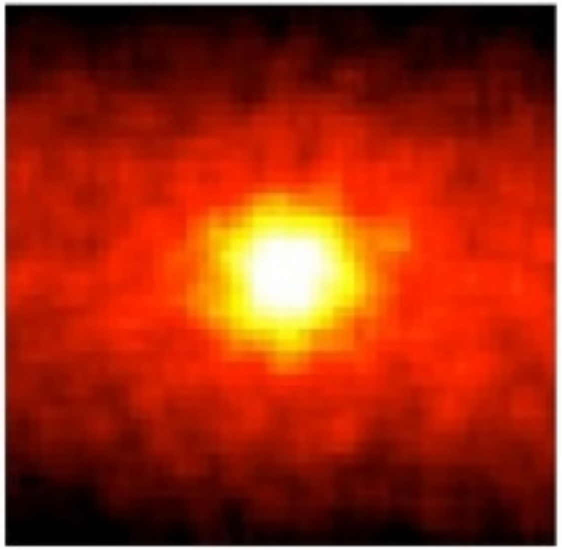
Neutrino energy [MeV]



On earth:
65 billion neutrinos per s and cm^2

Light and Neutrinos from the Sun ?

Exzellenzcluster Universe



Coulour represents number of neutrino reactions reconstructed in water tank (neutrino detector)

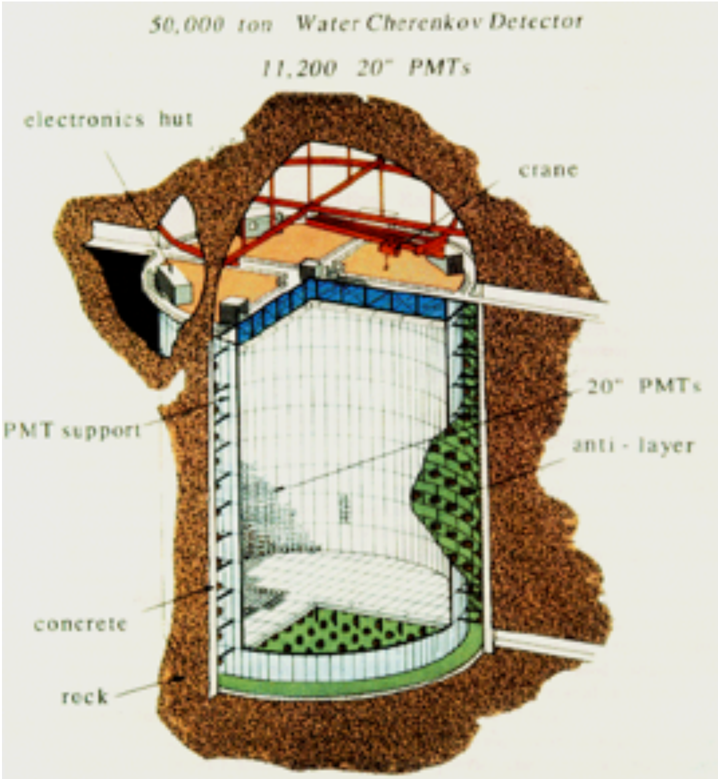
Coulour represents wavelength created by electromagnetic force

Weak force

Electromagnetic force

Light and Neutrinos from the Sun ?

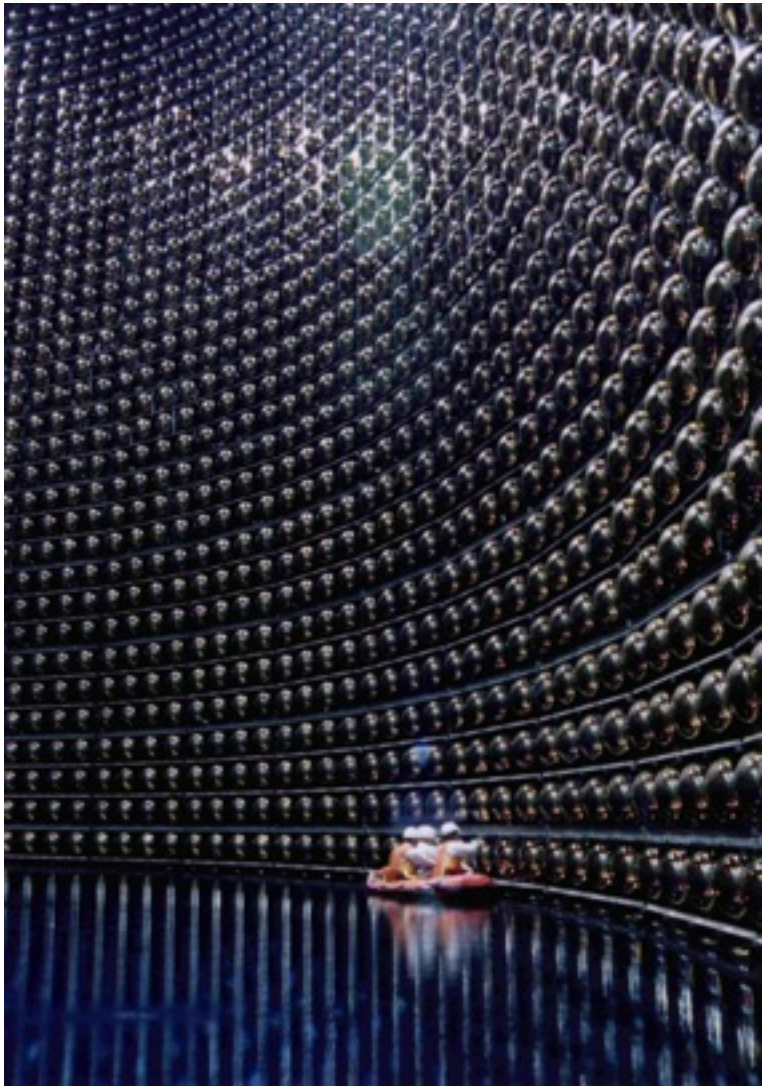
Exzellenzcluster Universe



1 km deep Kamioka Mine, Japan

40 m high
40 m Ø

11146
light- detectors
50 cm



Heavy Element formation in Supernovae

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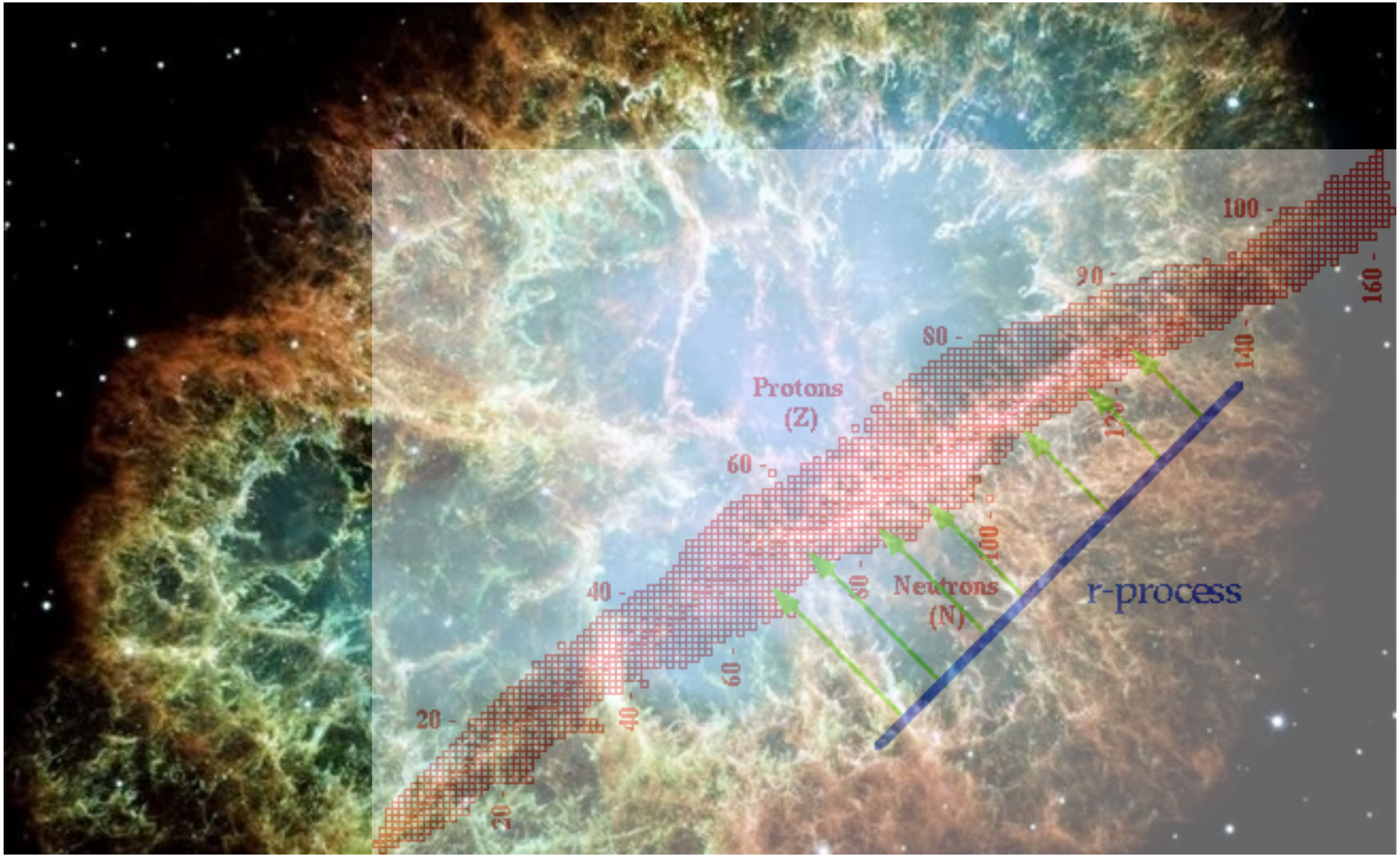
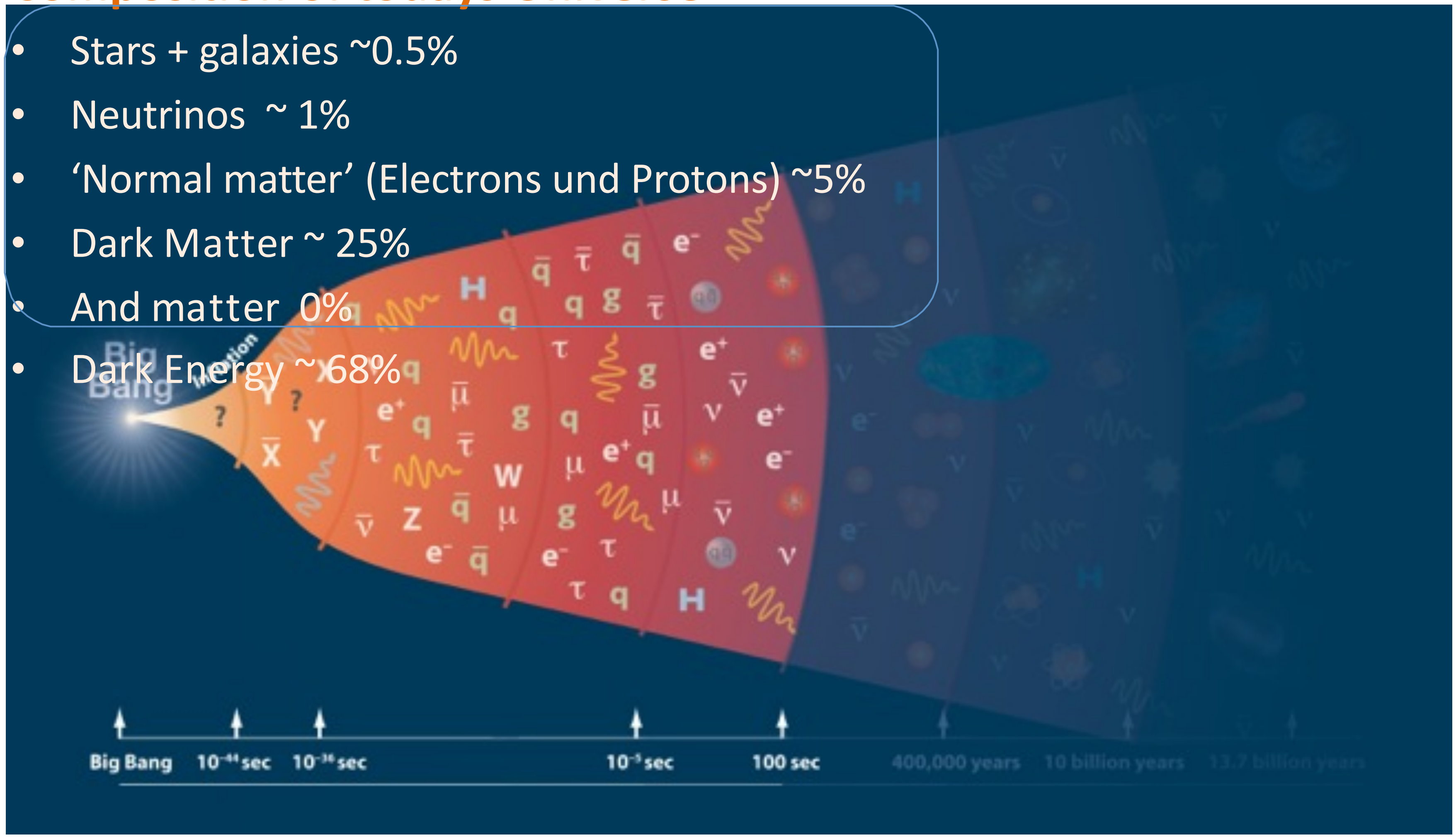


Composition of todays Universe

- Stars + galaxies ~0.5%
- Neutrinos ~ 1%
- ‘Normal matter’ (Electrons und Protons) ~5%
- Dark Matter ~ 25%
- And matter 0%
- Dark Energy ~ 68%



Dark Universe

Exzellenzcluster Universe

Composition of todays Universe

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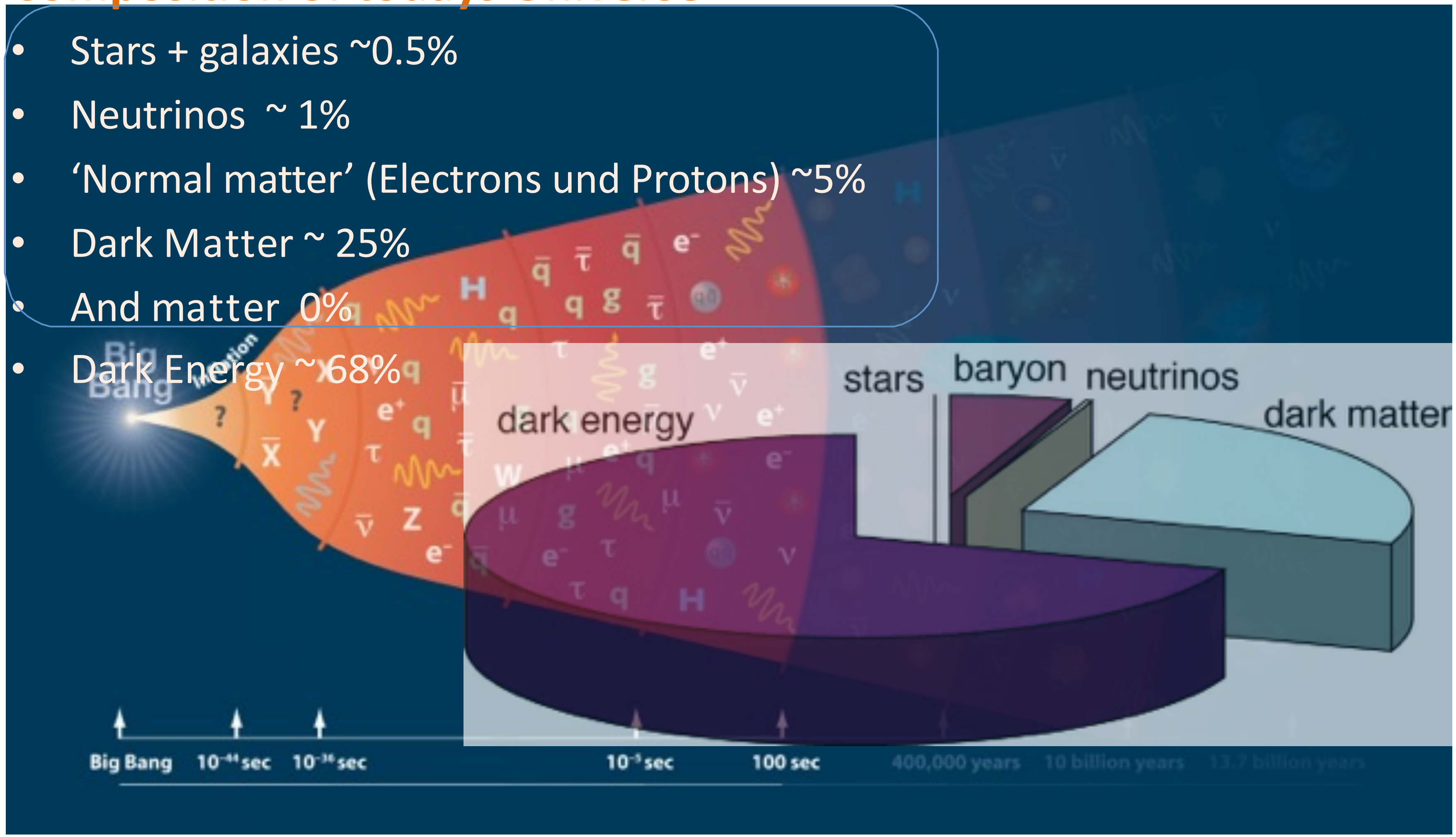


Outline of Lecture

Exzellenzcluster Universe

Introduction

Forces

Electromagnetic interaction

- Gauge principle

Strong interaction

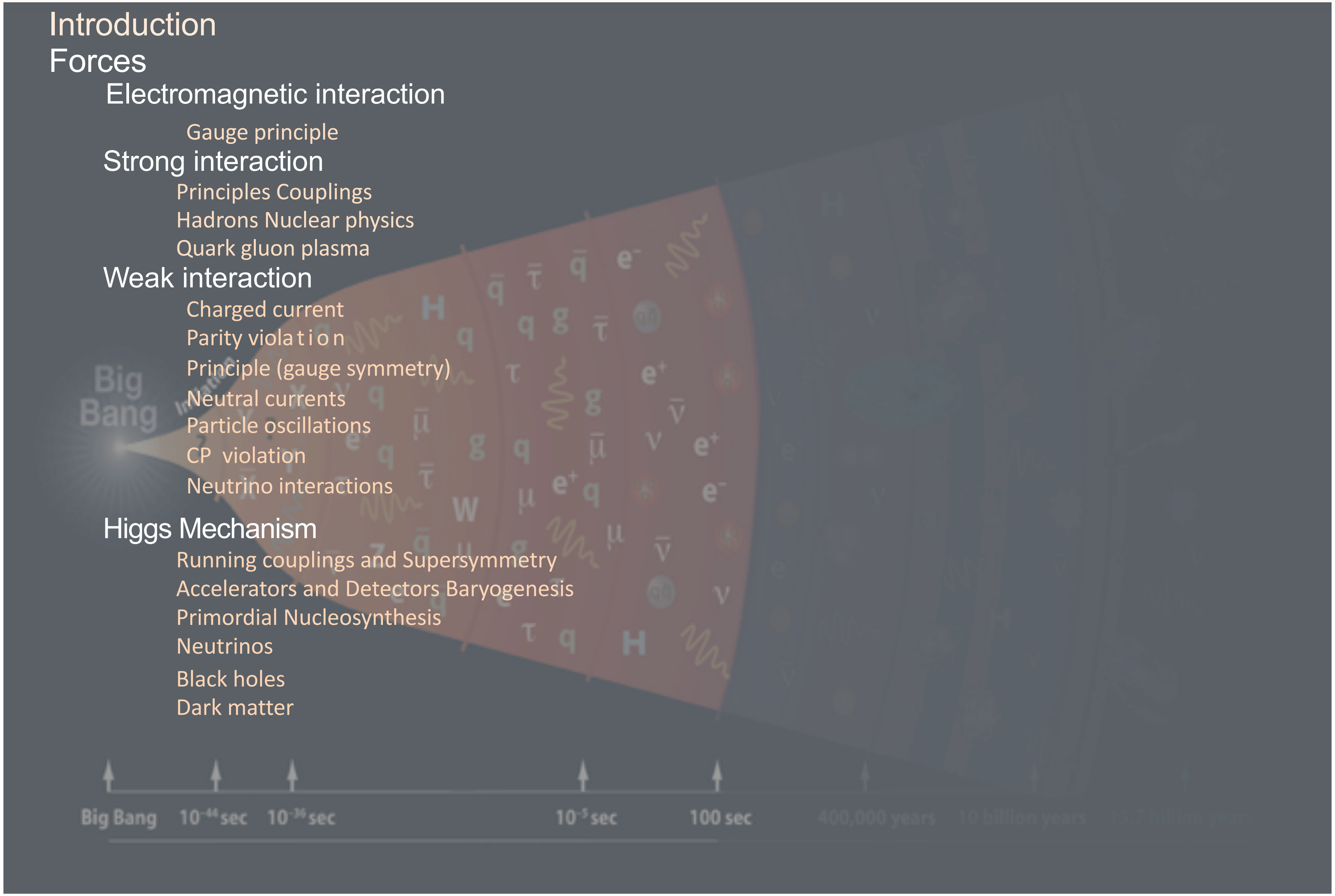
- Principles Couplings
- Hadrons Nuclear physics
- Quark gluon plasma

Weak interaction

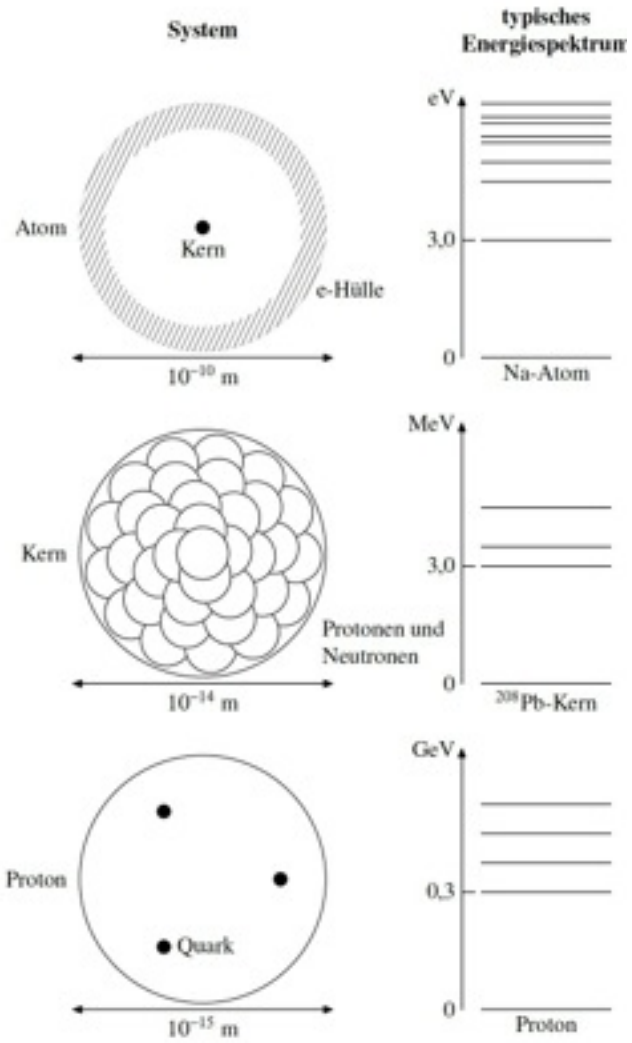
- Charged current
- Parity violation
- Principle (gauge symmetry)
- Neutral currents
- Particle oscillations
- CP violation
- Neutrino interactions

Higgs Mechanism

- Running couplings and Supersymmetry
- Accelerators and Detectors Baryogenesis
- Primordial Nucleosynthesis
- Neutrinos
- Black holes
- Dark matter



Units



Length:
Femtometer, Fermi **1 fm = 10^{-15} m**

Energy:
Electron Volt **1 eV = 1,602 $\cdot 10^{-19}$ J**
With charge: e = 1,602 $\cdot 10^{-19}$ C

Mass:
Atomic Mass Unit
1 u = 1/12 m(^{12}C) = 1,66 $\cdot 10^{-27}$ kg

Following Einstein: $E = m \cdot c^2$
[m] = MeV/c^2
1 u = 931,5 MeV/c^2

Units II

Planck - constant:

$\hbar$ = 6,582.10^{-22} MeVs = 197 MeV fm/c

$\hbar c$ = 197 MeV fm

Fine structure constant: $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$

Natural units:

Energy MeV
Mass MeV/c²
Momentum MeV/c
Length fm
Time fm/c
Charge $\sqrt{\alpha \hbar c} = e = 1{,}2\sqrt{MeV . fm}$

Frequent convention

$\hbar = c = 1$

Units III

Energy:

$$E = \sqrt{m^2c^4 + p^2c^2}$$

→ $mc^2 + p^2/2m$ For $p \ll m$

→ pc $p \gg m$ (extreme case: Photon)

Energy: electron Volt in SI units:

SI: 1 eV = $1.6022 \cdot 10^{-19}$ J

Mass:

SI: 1 eV/c² = $1.783 \cdot 10^{-36}$ kg

Momentum:

SI: 1 eV/c = $5.344 \cdot 10^{-28}$ kg m/s

Kinematics

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4 - vectors

$$P = \begin{pmatrix} E/c \\ \vec{p} \end{pmatrix} \qquad p^\mu = (p^0, p^1, p^2, p^3) = (E/c, \vec{p})$$
$$p_\mu = (p_0, p_1, p_2, p_3) = (E/c, -\vec{p})$$

Energy: $E = \sqrt{m^2c^4 + \vec{p}^{\,2}c^2}$

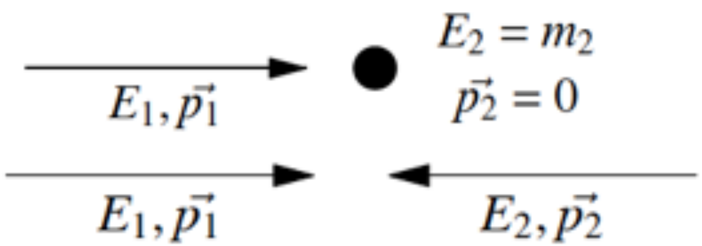
Lorentz scalars $p^2 = \sum_{\mu=0}^{3} p^\mu p_\mu \equiv p^\mu p_\mu = \frac{E^2}{c^2} - \vec{p}^{\,2} = m^2c^2$

Invariant under Lorentz transformation – same value in any system of reference

Centre of mass energy
„usable“ energy in a reaction $S = (P_1 + P_2)^2$

Lab-system

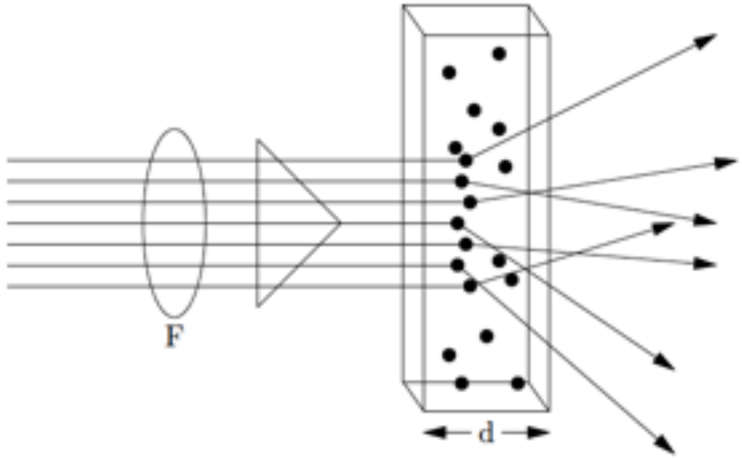
CM-system



$S = m_1^2 + m_2^2 + 2E_1 m_2$

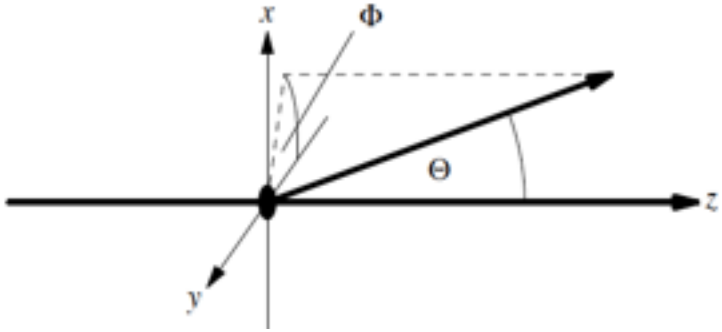
$W = \sqrt{S} = E_1 + E_2$

Reaction Rate and Cross Section



Luminosity: $L = J \cdot N_b$

Reaction rate $R = L \cdot \sigma_r$



$$N_b = n_b \cdot F \cdot d$$

target particles within beam cross section

$$J = n_a \cdot V_a = \frac{\dot{N}_a}{F} = \frac{counts}{time \cdot area}$$

Beam flux (velocity v_a) n_a

n_b : particle densities

$$\sigma_r = \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta \frac{d\sigma_r}{d\Omega}(\theta, \phi)$$

Reaction cross section

Cross section

Fermi's golden rule:

$$W_{i\to f} = \frac{2\pi}{\hbar} |\mathcal{M}_{fi}|^2 \rho(E_f)$$

Transition rate from initial state i to final state f

$$\rho(E) = \frac{dn(E)}{dE}$$

Density of final states

$$\sigma_{i\to f} = \frac{\text{Reaktionsrate } R}{\text{Luminosität } L} = \frac{N_a N_b}{L} W_{i\to f}$$

Cross section
"effective area"

$$\sigma_{i\to f} = \frac{2\pi}{\hbar V_a} |\mathcal{M}_{fi}|^2 \rho\,(E_f) V$$

Volume V will drop out via
normalization of M_{if}

Dirac Equation

$$i\hbar\frac{\partial}{\partial t}\psi = \mathcal{H}_S\psi = -\frac{\hbar^2}{2M}\vec{\nabla}^2\psi$$

Schrödinger equation

$$E = \frac{\vec{p}^{\,2}}{2M}$$

$$E \longrightarrow i\hbar\frac{\partial}{\partial t}, \quad \vec{p} \longrightarrow -i\hbar\vec{\nabla}$$

Quantization

$$x^\mu = (ct, \vec{r}) \qquad \partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

4-component nomenclature

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right) \longrightarrow i\hbar\partial^\mu$$

$$p_\mu p^\mu = m^2c^2 \qquad \left(\hbar^2\partial_\mu\partial^\mu + m^2c^2\right)\phi(x) = 0$$

Quantization of relativistic kinematics

$$\phi(x) = N e^{-ip_\mu x^\mu/\hbar}$$

Klein-Gordon equation

Dirac Equation II - Fermions

$$E = p^0 c = \pm\sqrt{m^2c^4 + \vec{p}^{\,2}c^2}$$

Solution with negative energy

Linearize in energy and momentum:

$$i\frac{\partial}{\partial t}\psi = \mathcal{H}_D\psi \qquad \text{mit} \qquad \mathcal{H}_D = \vec{\alpha}\cdot\vec{p} + \beta m = -i\vec{\alpha}\cdot\vec{\nabla} + \beta M$$

$$\underbrace{-\frac{\partial^2}{\partial t^2}}_{\widehat{=}E^2}\psi = \mathcal{H}_D\mathcal{H}_D\psi = \Big(\underbrace{-(\vec{\alpha}\cdot\vec{\nabla})^2 - iM(\vec{\alpha}\cdot\vec{\nabla}\beta + \beta\vec{\alpha}\cdot\vec{\nabla}) + \beta^2M^2}_{\stackrel{!}{=}-\vec{\nabla}^2+M^2\widehat{=}\vec{p}^{\,2}+M^2}\Big)\psi$$

relativistic energy – momentum relation

α, β: 4x4 matrices
Ψ: 4–component vector = spinor

$$\beta^2 = \mathbb{1}_{4\times4}, \quad \alpha_i^2 = \mathbb{1}_{4\times4}$$
$$\alpha_i\beta + \beta\alpha_i = 0,$$
$$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}$$

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

$$\begin{pmatrix} E-M & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -E-M \end{pmatrix}\begin{pmatrix} \chi \\ \varphi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ \text{spin-up}, \quad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ \text{spin-down}$$

$$\varphi = \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi$$

Dirac Equation III

Still we get two solutions: with positive and negative energies

$$u_s(p) = \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+m}\chi_s \end{pmatrix} \qquad v_{-s}(p) = \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E_p+m}\chi_s \\ \chi_s \end{pmatrix} \qquad \text{mit } \chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ und } \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\Psi^{>} = \mathcal{N} u_S(p)\, e^{-ip\cdot x}$	$(\gamma_\mu p^\mu - m) u_S(p) = 0$	particle
$\Psi^{<} = \mathcal{N} v_{-S}(p)\, e^{ip\cdot x}$	$(\gamma_\mu p^\mu + m) v_S(p) = 0$	anti-particle

Particles: move in +t with positive mass (energy)

Anti-particles: move in –t with positive mass (energy) or +t with negative energy

$$(i\gamma^0 \frac{\partial}{\partial t}\psi + i\beta\vec{\alpha}\cdot\vec{\nabla} - M)\psi = 0$$

Dirac equation

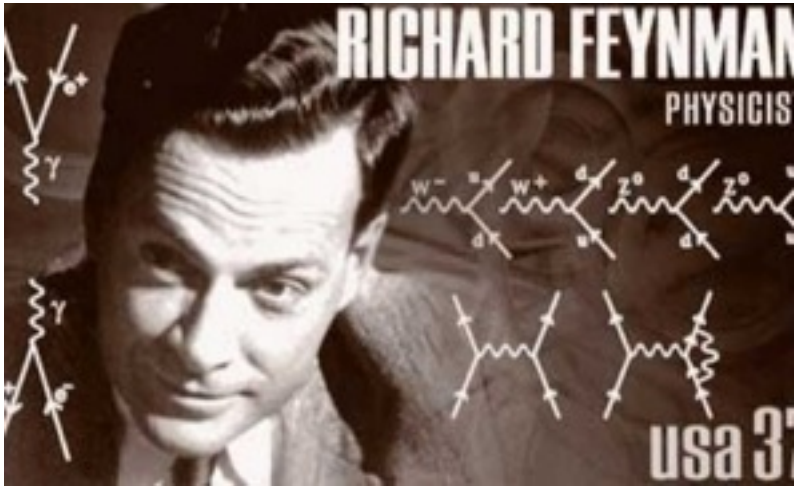
$$(i\gamma^\mu \partial_\mu - M \cdot \mathbb{1}_{4\times4})\psi = 0$$

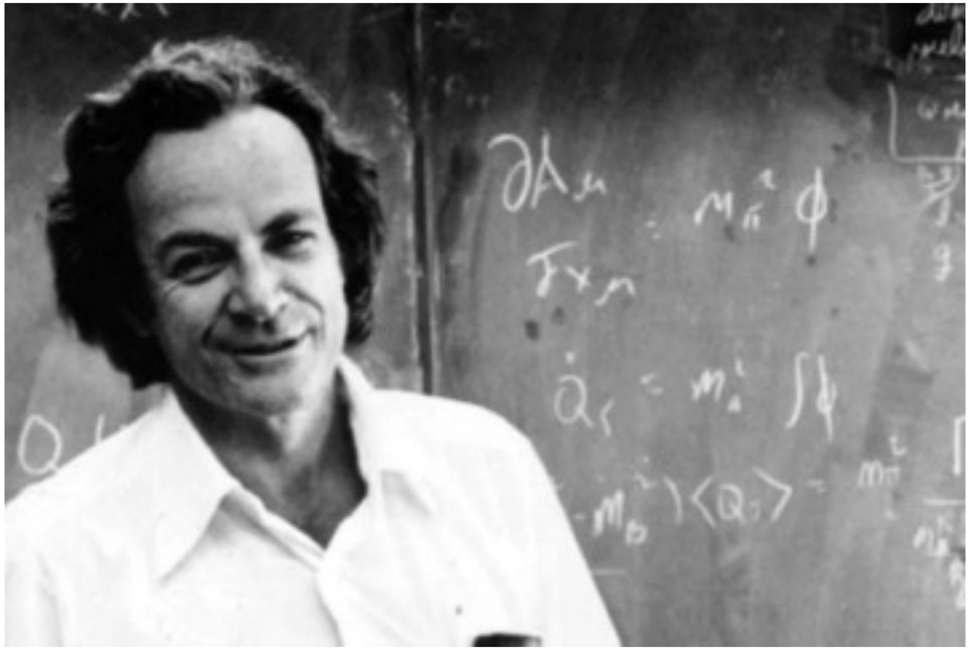
$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

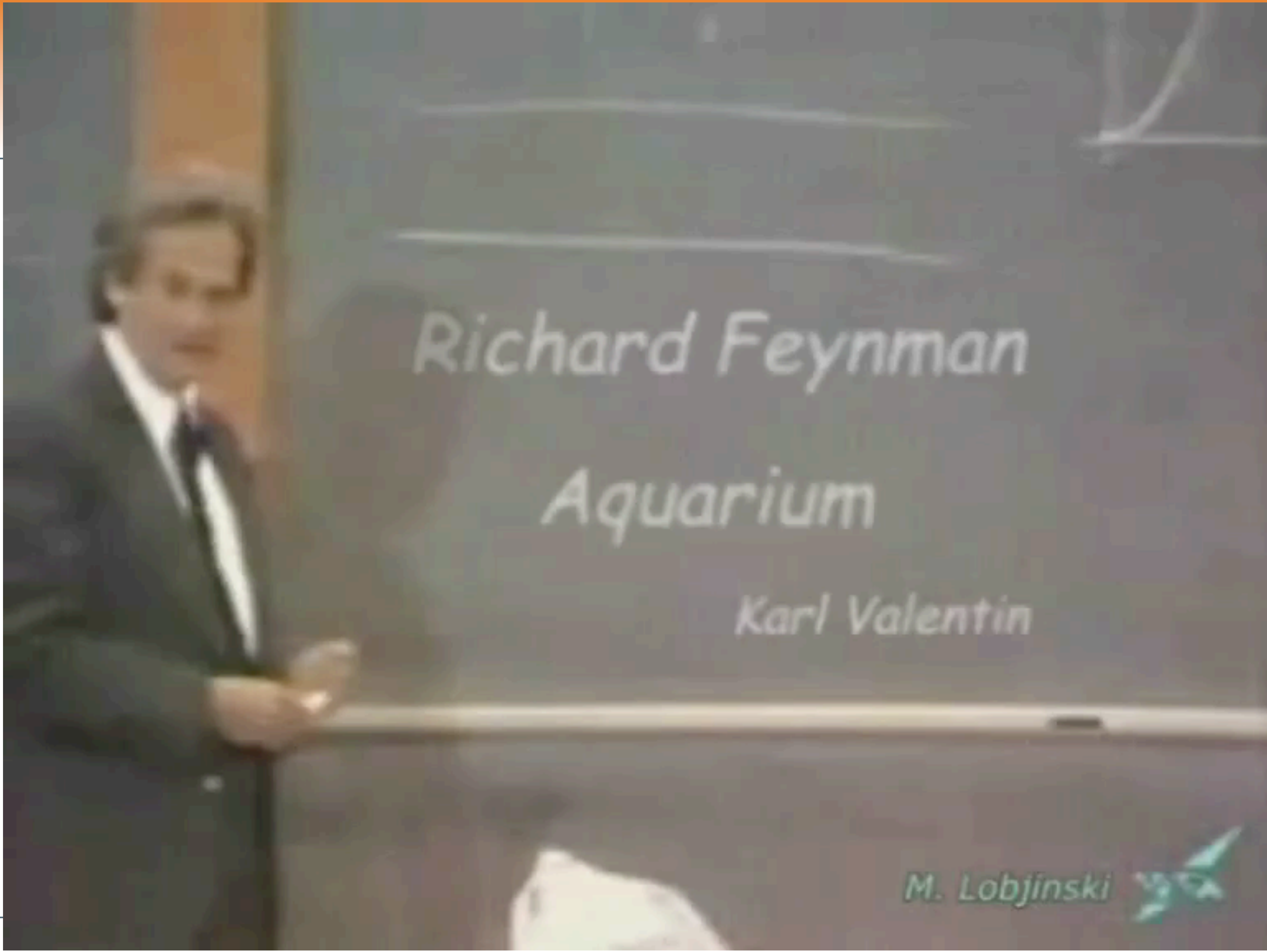
$$\vec{\gamma} = \beta\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

$$\gamma^0 = \beta = \begin{pmatrix} \mathbb{1}_{2\times2} & 0 \\ 0 & -\mathbb{1}_{2\times2} \end{pmatrix}$$

Feynman Diagram

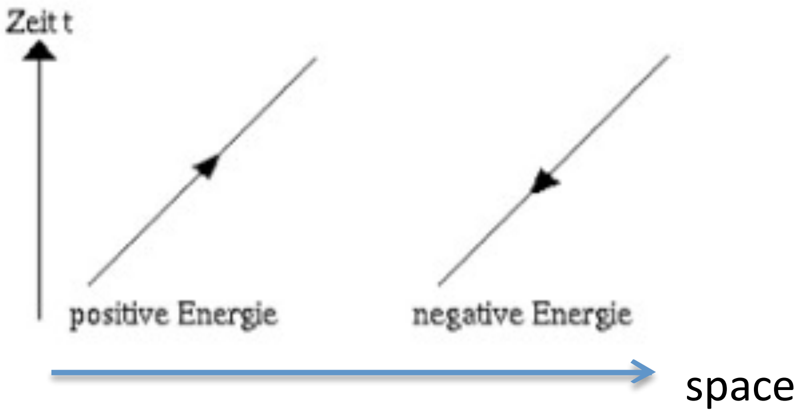


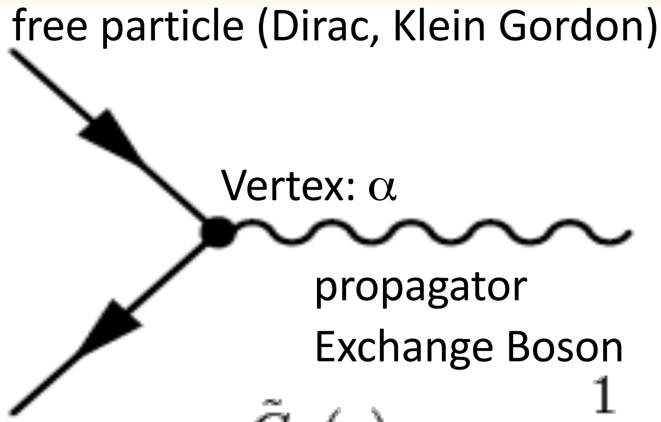



Richard Feynman
Aquarium
Karl Valentin
M. Lobjinski

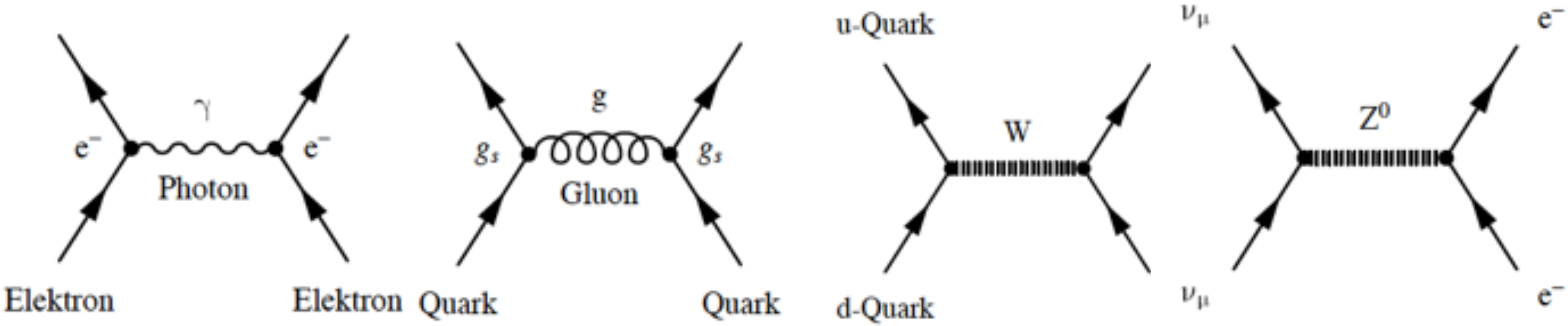
Feynman Diagrams II

Exzellenzcluster Universe





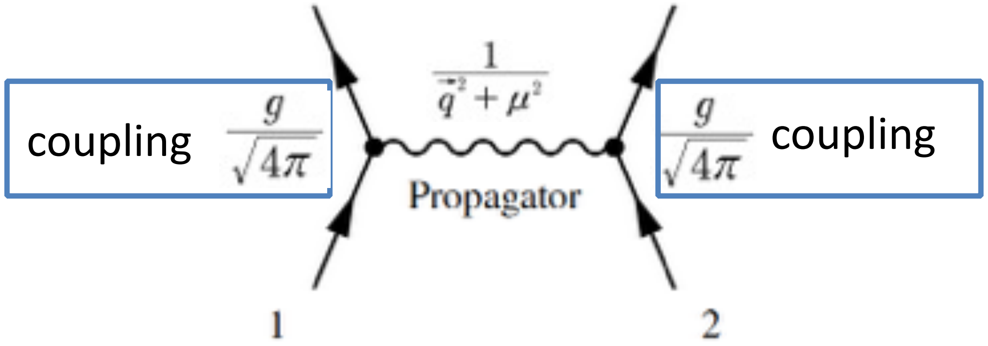
$$\tilde{G}_F(p) = \frac{1}{p^2 - m^2 + i\epsilon}.$$



$$\alpha \equiv \alpha_e = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137}$$

$$\alpha_s = \frac{g_s^2}{4\pi\hbar c} = \begin{cases} \approx \frac{1}{10} \\ \gtrsim 1 \end{cases}$$

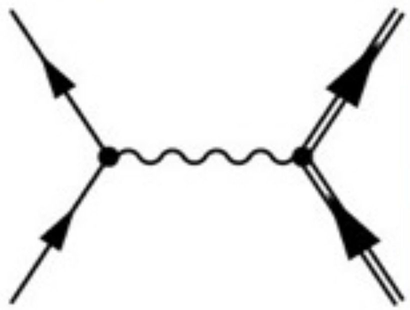
Cross section



Amplitude

$$M \sim \alpha \cdot \frac{1}{q^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \propto |\mathcal{M}_{fi}|^2 = \frac{1}{2}\sum_{s_i,s_f}\left| \quad \right|^2$$



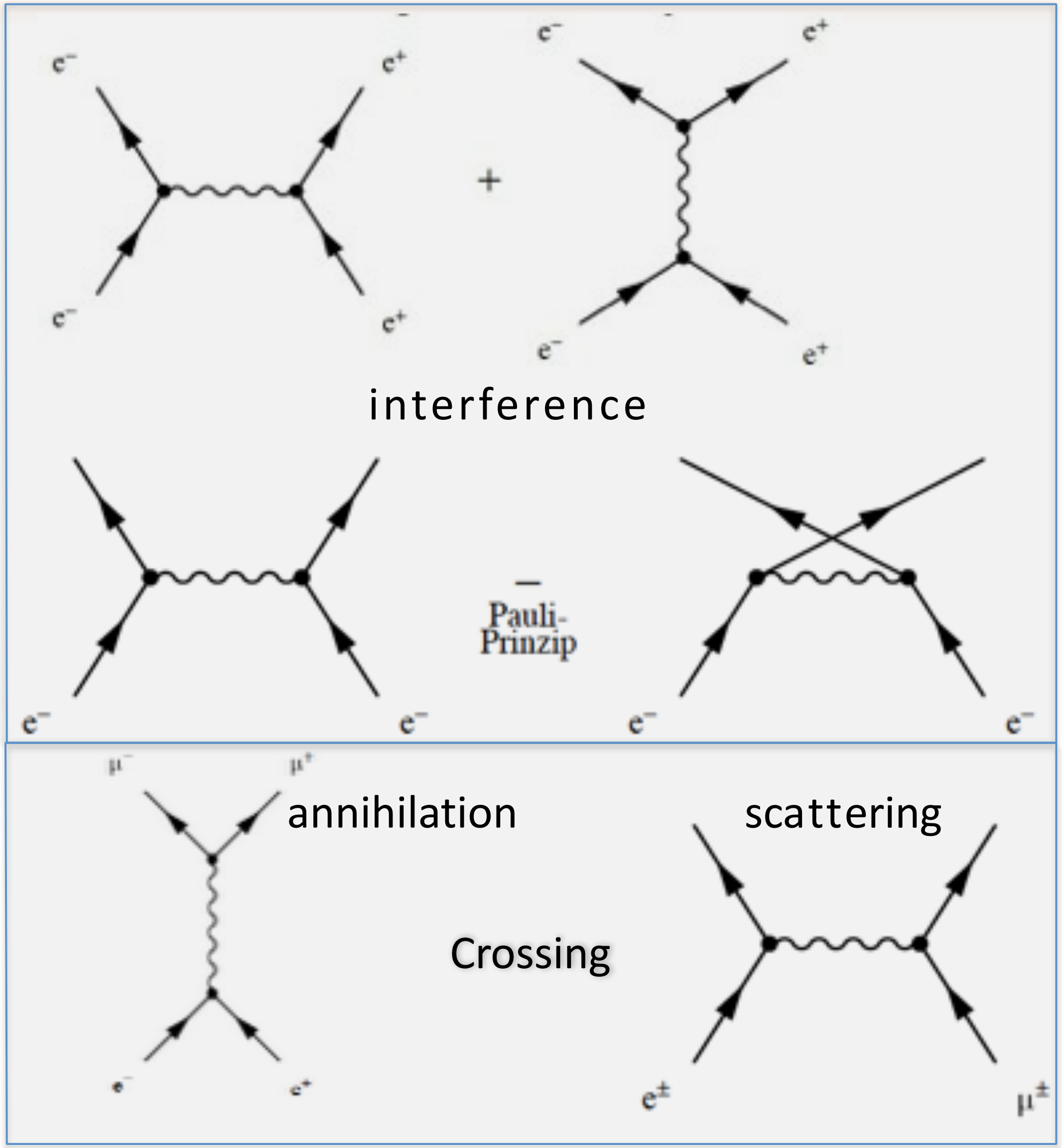
Cross section

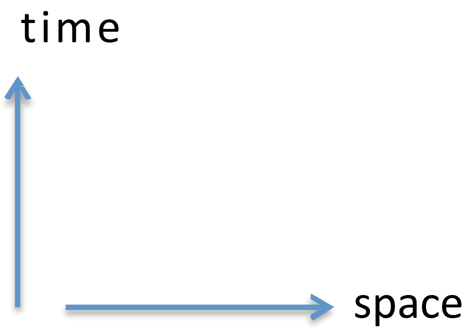
$$\sigma \sim \frac{\alpha^2}{q^4}$$

$$q^2 = (p-p')^2 = 2m_e^2 - 2\left(EE' - |\vec{p}||\vec{p}\,'|\cos\theta\right) \approx -4EE'\sin^2$$

Feynman Diagrams III

Exzellenzcluster Universe

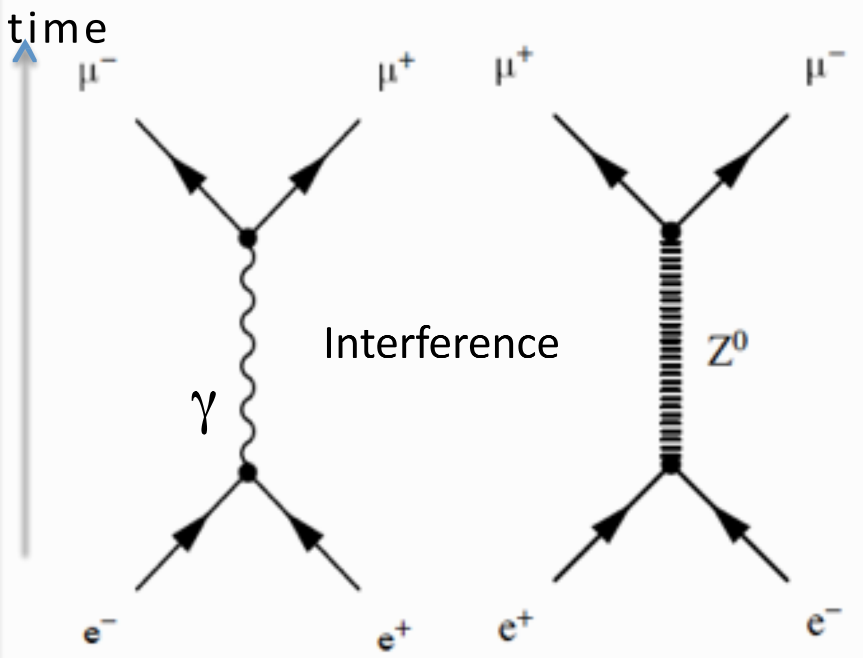




- Keep track on fundamental processes
- See similarities between processes
  - Same initial and final states: interference
- Same diagram: same amplitude
  - Except for kinematic factors
  - Rotation of diagram in t – x plane crossing

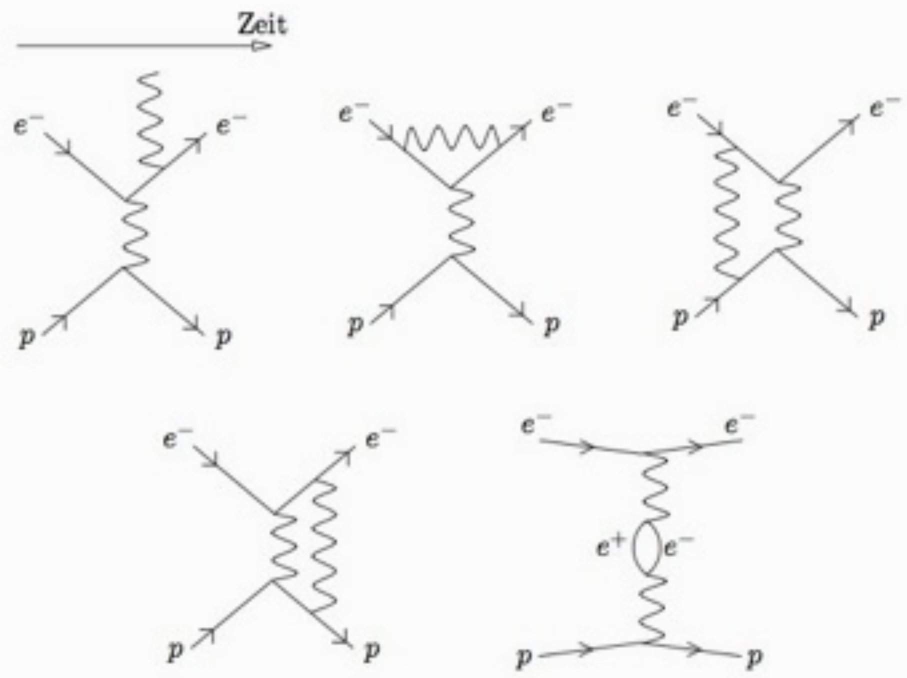
Feynman IV

Exzellenzcluster Universe



electroweak interference
„Tree diagram“

“Higher Orders – em scattering”



Some Basic Considerations

Compton wave length: $\frac{\lambda}{2\pi} = \frac{\hbar}{mc}$

De Broglie wave length: $\vec{p} = \hbar \vec{k} \quad \lambda = 2\pi/k$

Uncertainty relation: $\Delta\vec{p} \cdot \Delta\vec{x} \geqq \frac{\hbar}{2} \quad \Delta E \Delta t > \frac{\hbar}{2}$

Range of virtual particle of mass M $\Delta E \Delta t \approx mc^2 \Delta t > \frac{\hbar}{2} \rightarrow Range \approx c\Delta t > \frac{\hbar c}{2mc^2}$

Examples:

1. Electromagnetic interaction: $m_\gamma = 0$
2. Weak interaction
   - W^+, W^- exchange: $m_W = 80$ GeV/c^2 range = 0.001 fm
   - Z^0 exchange $m_Z = 90$ GeV/c^2
3. Strong interaction: $m_g = 0$
   - Gluons self interact.. range about 0.5 fm
4. Strong (nuclear) force: $m_\pi = 140$ MeV/c^2 range = 0.8 fm

Some Basic Considerations II

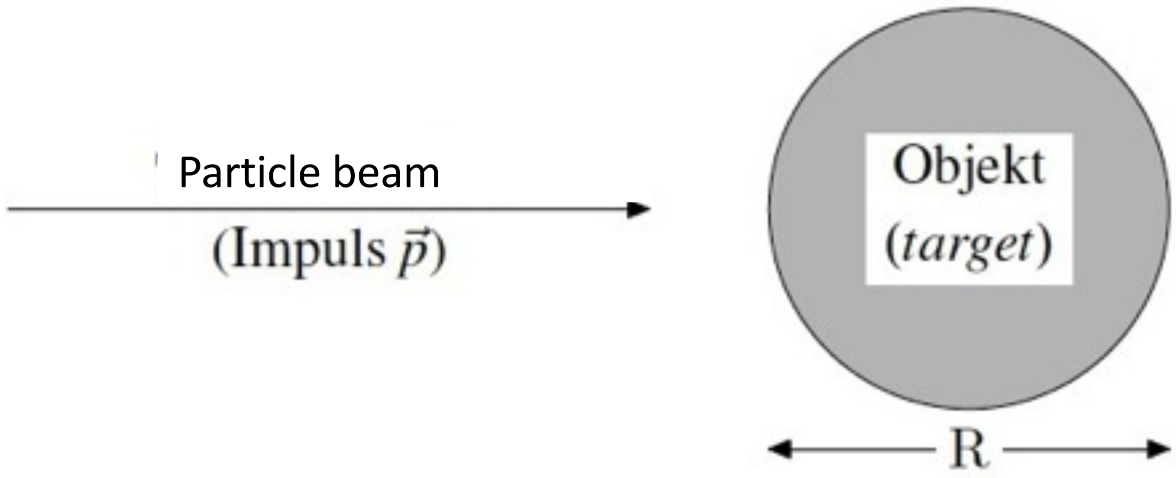
Exzellenzcluster Universe

How to resolve structure: Objects smaller than wavelength cannot be resolved:

$$\vec{p} = \hbar\vec{k} \quad \lambda = 2\pi/k,$$

In scattering: relevant quantity is momentum transfer $\Delta\vec{p}$

$$k \cdot R \gtrsim 1 \quad \text{bzw.} \quad p \cdot R \gtrsim \hbar$$



Symmetries

Lagrange function: $L\left(x_j, \dot{x}_j, t\right)$ L =T – V

Equation of motion: $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}_j} - \frac{\partial L}{\partial x_j} = 0$

Noether theorem: every invariance of the equation of motion leads to a constant of motion

Spatial translation:	$\vec{x} \longrightarrow \vec{x} + \delta\vec{x}$	Momentum conservation
Time translation:	$t \longrightarrow t + \delta t$	Energy conservation
Spatial rotation	$\vec{x} \longrightarrow D \cdot \vec{x}$	Angular momentum conservation
	$\vec{x} + \delta\vec{\omega} \times \vec{x} + \ldots$	

Continuous symmetry operation

Mirror operation:	$\vec{x} \longrightarrow -\vec{x}$	Parity conservation

Discrete symmetry operation

Symmetries and Quantum fields

$x(t) \rightarrow q(t)$ generalized coordinates — describe e.g. mass points

$q(t) \rightarrow \Phi(x)$ quantum fields — describe fermions and bosons

$\dot{q}(t) \rightarrow \frac{\partial \phi(x)}{\partial x^\mu} = \partial_\mu \phi(x)$ — x^μ is 4-vector

$L(x_j, \dot{x}_j, t) \longrightarrow \mathcal{L}(\phi(x), \partial_\mu \phi(x))$ — Lagrange density

$$\frac{d}{dt}\frac{\partial L}{d\dot{q}_i} - \frac{\partial L}{dq_i} = 0 \rightarrow \frac{d}{dt}\frac{2m\vec{v}_i}{2} - \frac{\partial V}{\partial \vec{r}_i} = 0 \rightarrow \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi(x))} - \frac{\partial \mathcal{L}}{\partial \phi(x)} = 0$$

Euler–Lagrange equation

Classical mechanics: L = T – V

$m\dot{\vec{v}} - \vec{\nabla} V(\vec{r}, t)$ Equation of motion

Field theory:
assume spin 0 particle

Klein Gordon equation

$$\mathcal{L}_{\text{Klein-Gordon}} = \frac{1}{2}\partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{m^2}{2}\phi^2(x) \longrightarrow \partial_\mu \partial^\mu \phi(x) - m^2 \phi(x) = 0$$

Symmetries and Quantum fields II

Field theory:
assume spin 1/2 particle (spinor ψ)

Dirac equation

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(x)\left[i\gamma_\mu \partial^\mu - M\right]\psi(x) \quad \longrightarrow \quad \left(i\gamma_\mu \partial^\mu - M\right)\psi(x) = 0$$

Examples for symmetries in quantum mechanics

$$\mathcal{H}\,\psi = E\psi \qquad \mathcal{H} = -\frac{\vec{\nabla}^2}{2m} + V(r) \qquad \psi(\vec{r}) = R_{n\ell}(r)Y_{\ell m}(\vartheta,\varphi)$$

Central rotationally symmetric potential

$$\left[\mathcal{H}, \vec{L}\right] = 0$$

Leads to angular momentum conservation and (2m+1) degeneracy

$$\mathcal{P}\,\psi(\vec{r}) = \psi(-\vec{r}) \quad \text{if} \quad [\mathcal{H}, \mathcal{P}] = 0 \quad \longrightarrow \quad \mathcal{P}\,\psi(\vec{r}) = \pm\psi(-\vec{r})$$

Eigenstates have defined parity

Discrete Symmetries and Quantum fields

Consider parity: $\mathcal{P}: \vec{r} \rightarrow \vec{r}\,' = -\vec{r}$ polar coordinates: $\theta \rightarrow \pi - \theta$

$$\mathcal{P}\mathcal{P}\,\psi(\vec{r},t) := \mathcal{P}\,\psi(-\vec{r},t) = \psi(\vec{r},t) \Leftrightarrow \mathcal{P}^2 = 1 \qquad \mathcal{P}\,\psi = P\psi, \quad P = \pm 1$$

polare Vektoren	axiale Vektoren
Ort $\vec{r}$ Impuls $\vec{p} = M\frac{d\vec{r}}{dt}$ Elektrisches Feld $\vec{E} = -\vec{\nabla}\phi$ (erzeugt von der Ladungsdichte ρ) Vektorpotential $\vec{A}$ (erzeugt von der Stromdichte $\vec{j} = \rho\vec{V}$)	Drehimpuls $\vec{L} = \vec{r} \times \vec{p}$ Spin $\frac{1}{2}\sigma$ Magnetfeld $\vec{B} = \vec{\nabla} \times \vec{A}$
P = -1	P = +1

$$Y_{\ell m}(\pi - \theta, \pi + \varphi) = (-1)^{\ell}\, Y_{\ell m}(\theta, \varphi) \qquad P_{\text{Bahn}} = (-1)^{\ell}$$

Consider time reversal: $\mathcal{T}: t \rightarrow -t.$

momentum: $\vec{p} \rightarrow -\vec{p}$

angular momentum: $\vec{L} \rightarrow -\vec{L}, \vec{\sigma} \rightarrow -\vec{\sigma}$

wave function: $\Psi(\vec{x},t) = \exp(i\vec{p}\cdot\vec{x} - iEt) \overset{\mathcal{T}}{\Rightarrow} \exp(-i\vec{p}\cdot\vec{x} - iEt)$

$$\mathcal{T}\,\Psi(\vec{x},t) = \Psi^*(\vec{x},-t)$$

Time reversal of Langrangian: process has same amplitudes in both time directions

"detailed balance"

Consider charge conjugation: $c\,|q\rangle = |\bar{q}\rangle \qquad c\,|\bar{q}\rangle = |q\rangle$

For particles being their own antiparticles (γ, π^0): $c\,|\alpha\rangle = C_\alpha |\alpha\rangle \qquad C_\alpha = \pm 1$

Consider electromagnetic fields:

$$c\,(\phi, \vec{A}) = -(\phi, \vec{A}) \quad \text{vector potential}$$
$$c\,(\vec{E}, \vec{B}) = -(\vec{E}, \vec{B}) \quad \text{fields}$$

$$c\,|\gamma\rangle = -|\gamma\rangle \qquad C(\gamma) = -1$$

Many more symmetries exist ⟶ conserved quantum numbers

CPT-Theorem:

All locally Lorentz-invariant field theories are immune towards a combined application of C, P and T transformations:

CPT is an exact symmetry

Consequences:

- Lifetimes and masses of particle and antiparticle are equal
- If a Langrangian is not invariant under C,P or T, it must also violate the combination of PT, CT, CP, respectively

Discrete Symmetries

start

identical to start

Discrete Symmetries

start

identical to start

antiparticle
e^+

particle
e^-

using Escher ©
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Discrete Symmetries

start

identical to start

antiparticle **particle**

e^+ **e^-**

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Discrete Symmetries

Matter

start

mirror image

P

identical to start

antiparticle **particle**

e^+ e^-

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Discrete Symmetries

Matter

Antimatter

start

identical to start

mirror image

P

C

antiparticle

e^+

particle

e^-

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Discrete Symmetries

start

Matter

Antimatter

identical to start

mirror image

time →

P

C

antiparticle

e^+

particle

e^-

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Discrete Symmetries

start | Matter | Antimatter | identical to start

mirror image | time → | ← time

P | C | T

antiparticle

e^{+}

particle

e^{-}

World is invariant under CPT

using Escher ©

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Gauge theories:

What has a symmetry to do with an interaction ?

Sketch for electromagnetic interaction:

Dirac equation (describes equation of motion for fermions):

$$(i\gamma_\mu \partial^\mu - m)\psi(x) = 0 \qquad \partial^\mu = \frac{\partial}{\partial x^\mu}$$

Sum of derivates of all 4 coordinates

$$x = (t, \vec{x})$$

Corresponding Lagrange density for a free particle:

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(x)(i\gamma_\mu \partial^\mu - m)\psi(x), \qquad \bar{\psi} = \psi^\dagger \gamma^0$$

$$\int \mathrm{d}t\, L_{\text{Dirac}} = \int \mathrm{d}^4x\, \mathcal{L}_{\text{Dirac}}$$ Action will be extreme (path integral)

At extremum, action should be insensitive w.r.t. small changes of ψ

Interactions and Symmetries II

Consider simple symmetry operation (gauge transformation):

$$\psi(x) \longrightarrow e^{i\theta}\psi(x) \equiv U\psi(x)\,, \quad \bar{\psi}(x) \longrightarrow \bar{\psi}(x)e^{-i\theta}$$

$$e^{i\theta} \in \mathrm{U}(1) \text{ mit } U^{\dagger} = U^{-1}$$ U: unitary

$$\theta = \begin{cases} \text{const} & \text{global gauge transformation} \\ \theta(x) & \text{local gauge transformation} \end{cases}$$

Global gauge transformation: trivial

Consider local gauge transformation

$$\partial_\mu \left(U\psi(x)\right) = e^{i\theta(x)}\partial_\mu \psi(x) + ie^{i\theta(x)}\psi(x)\partial_\mu \theta(x)$$

$$\mathcal{L}_{\text{Dirac}} \rightarrow \mathcal{L}_{\text{Dirac}} - \bar{\psi}(x)\gamma_\mu \psi(x)\partial^\mu \theta(x) \neq \mathcal{L}_{\text{Dirac}}$$

Although absolute phases not measurable: no free Dirac particles possible

$\longrightarrow$ We must change expression for Lagrange density to obtain invariance !

Interactions and Symmetries III

Exzellenzcluster Universe

Introduce Gauge Field $\mathcal{A}_\mu(x) \xrightarrow[\text{transf.}]{\text{Eich-}} \mathcal{A}_\mu(x) + \partial_\mu \theta(x)$

and add $\underset{\text{normale Ableitung}}{\partial_\mu} \longrightarrow \partial_\mu - \mathrm{i}e\underset{\substack{\uparrow \\ \text{Photonfeld}}}{A_\mu(x)} =: \underset{\text{eichkovariante Ableitung}}{D_\mu(x)}$ minimal substitution

invariance $U^\dagger \left[\mathrm{i}\gamma_\mu D^\mu - m\right] U\psi(x) = \left[\mathrm{i}\gamma_\mu D^\mu - m\right] \psi(x) = 0$

$$\mathcal{L}_{\text{Koppl}} = \bar{\psi}\left(\mathrm{i}\gamma_\mu D^\mu - m\right)\psi = \bar{\psi}(x)\left(\mathrm{i}\gamma_\mu \partial^\mu + \gamma_\mu \mathcal{A}^\mu(x) - m\right)\psi(x)$$

contains coupling of gauge field to fermion introducing interaction
Gauge field has properties of photon field

Requirement of invariance under local U(1) symmetry
introduces electromagnetic interaction

We may say: local change of θ at point x_0 requires θ to change everywhere to restore invariance
Communication of phase change to all x is equivalent to interaction

Example: QED

For QED: gauge field is well known: $\mathcal{A}_\mu = -eA_\mu \qquad D_\mu(x) = \partial_\mu + ieA_\mu(x)$

Field strengths are summarized in the field strength tensor:

$$F_{\mu\nu}(x) = \frac{i}{e}\left[D_\mu(x), D_\nu(x)\right] = \frac{i}{e}\left[\partial_\mu - ieA_\mu(x), \partial_\nu - ieA_\nu(x)\right] = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) = -F_{\nu\mu}(x)$$

Derivatives can be interchanged: gauge invariance guaranteed

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

Gauge field boson (photon) contributes to total energy: Add kinetic term in Lagrangian

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(x)\left(i\gamma_\mu D^\mu - m\right)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

Additional (explicit) mass term would not be invariant to U(1) $\mathcal{L}^{(\gamma)}_{\text{Masse}} = \frac{m_\gamma^2}{2}A^\mu(x)A_\mu(x)$

Photon must be massless : range infinite

Symmetries and Quantum fields III

Exzellenzcluster Universe

Symmetries are all transformation fields leaving Langrange density invariant
$\longrightarrow$ conserved currents

$$L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^\mu A_\mu \quad \text{with} \quad F^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\nu \quad \text{field strength tensor}$$

$$\frac{\partial L}{\partial(\partial_\mu A_\nu)} = -\frac{1}{4} \frac{\partial}{\partial(\partial_\mu A_\nu)} \left[\left(\partial_\alpha A_\beta - \partial_\beta A_\alpha \right) \left(\partial^\alpha A^\beta - \partial^\beta A^\alpha \right) \right]$$

$$= -\frac{1}{2} g^{\alpha\alpha} g^{\beta\beta} \frac{\partial}{\partial(\partial_\mu A_\nu)} \left[\left(\partial_\alpha A_\beta \right)^2 - \left(\partial_\beta A_\alpha \right) \left(\partial_\alpha A_\beta \right) \right]$$

$$= \partial^\mu A^\nu + \partial^\nu A^\mu = -F^{\mu\nu}$$

$$\frac{\partial L}{\partial A_\nu} = -j^\nu$$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi(x))} - \frac{\partial \mathcal{L}}{\partial \phi(x)} = 0$$

kovariant Maxwell equ. — conserved current

$$\Rightarrow \partial_\mu F^{\mu\nu} = j^\nu \quad , \quad \partial_\nu j^\nu = 0$$

Other Gauge Theories

$SU(3)_{colour}$

Describes strong interaction

- symmetry : rotation in colour space
  (colour not distinguishable)

$SU(2)_L$

Describes weak interaction

- symmetry : rotation in weak isospin (left handed fermions) $U(1)_Y$

Describes hypercharge interaction

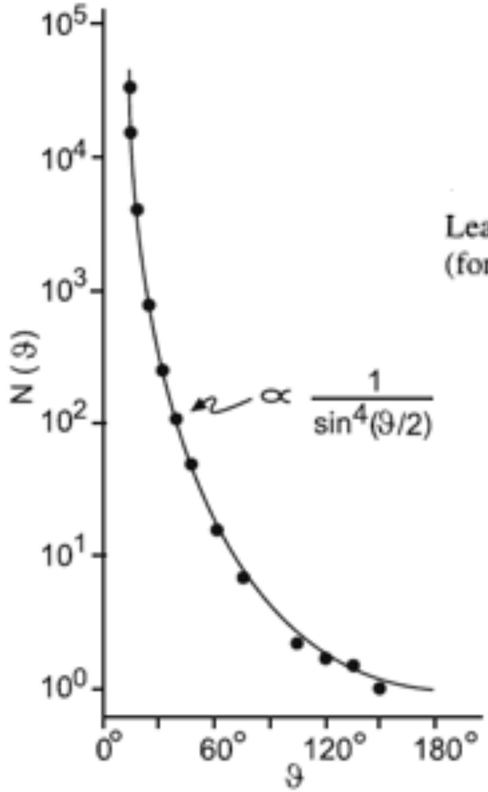
- symmetry : local phase transformation
  (right handed fermions) $SU(2)_L \times U(1)_Y$

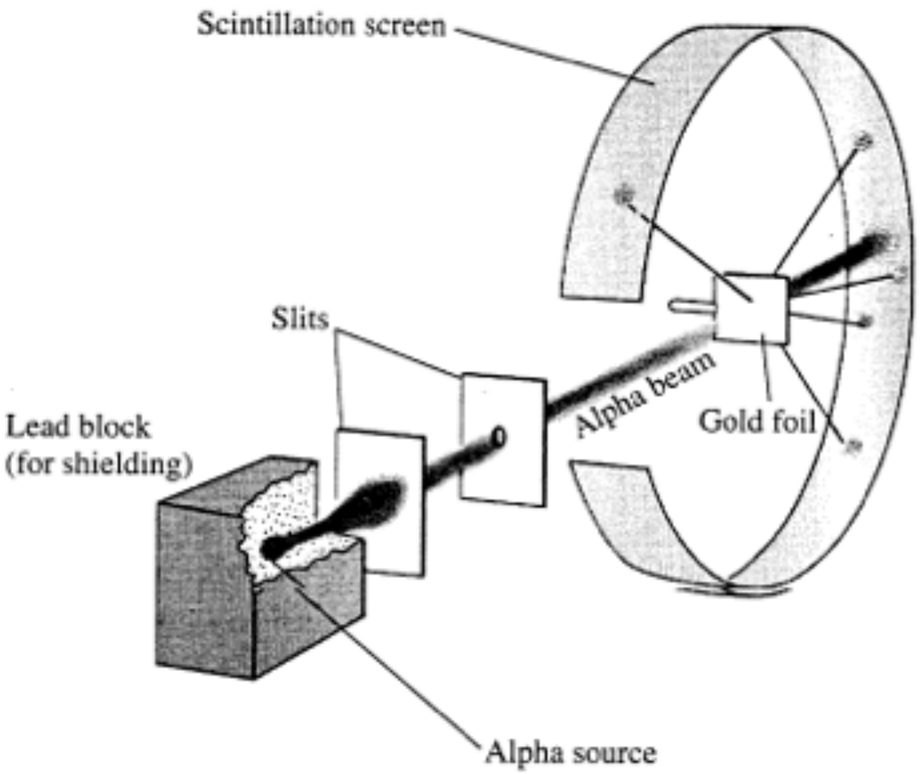
Describes electroweak interaction

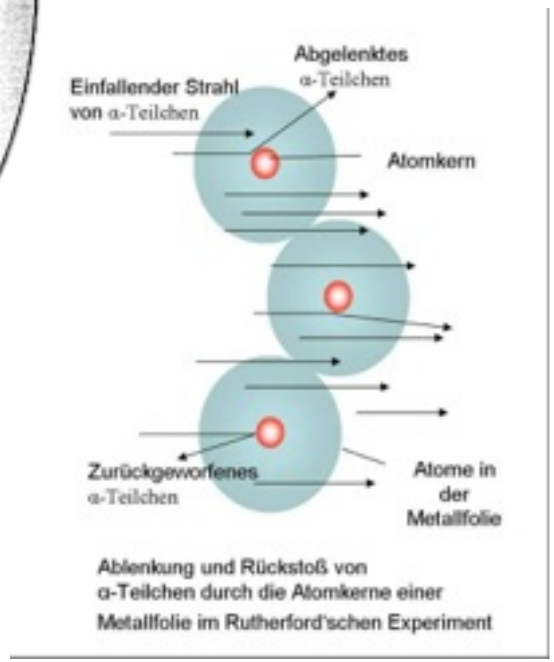
Example: Electromagnetic process

Rutherford's scattering experiment

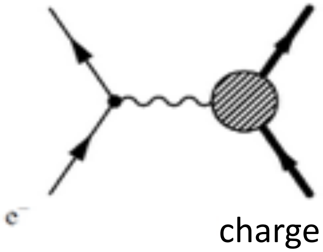
Geiger et al. (1909)





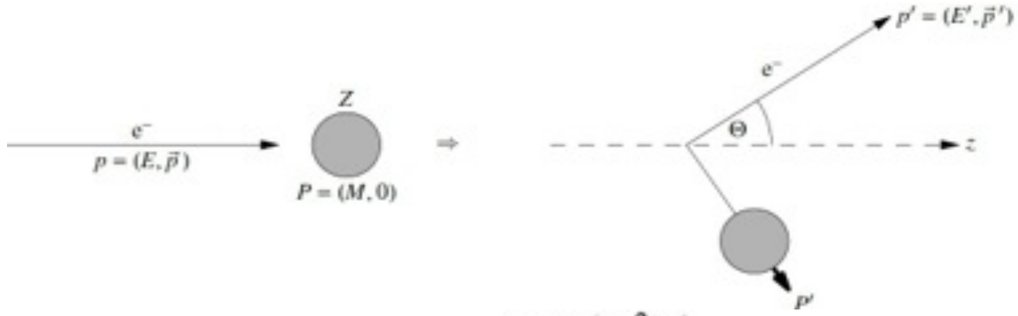


e^- - Scattering to Study Particle Structures



$$\int d^3r\rho(\vec{r}) = Z$$

charge density

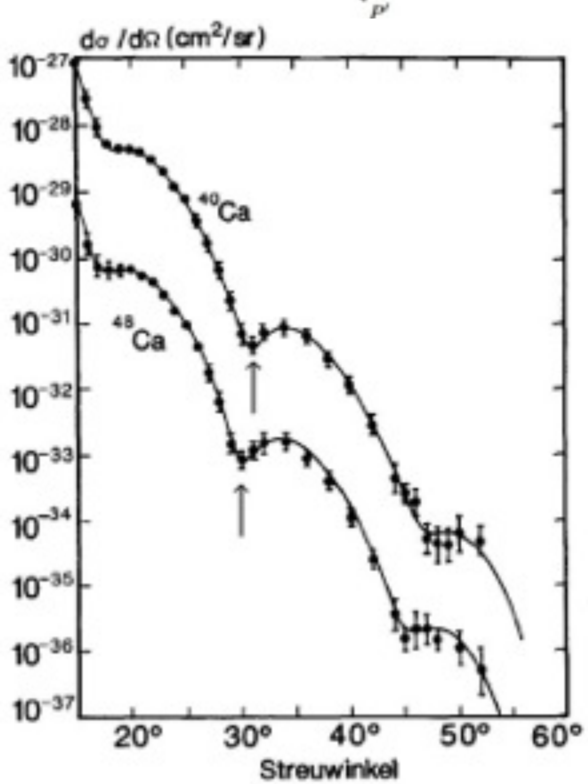


$$\frac{c\hbar\tilde{q}}{c} = \frac{200\text{MeV}\cdot\text{fm}}{c}\cdot\frac{1}{r} \quad\Rightarrow\quad \hbar\tilde{q} = q \geq 50\text{MeV}$$

$$r \approx 1-10\,\text{fm}\;(r \approx r_0 A^{\frac{1}{3}})$$

$$q = |\vec{p}-\vec{p}'| = \sqrt{p^2 + (p')^2 - 2pp'\cos\theta}$$

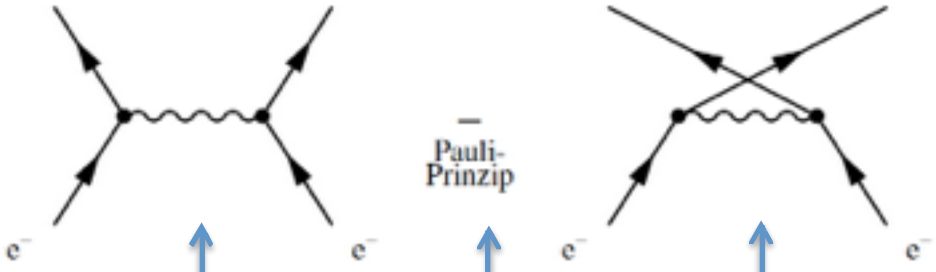
p_{in} = 500 – 700 MeV/c assuming θ = 180^0

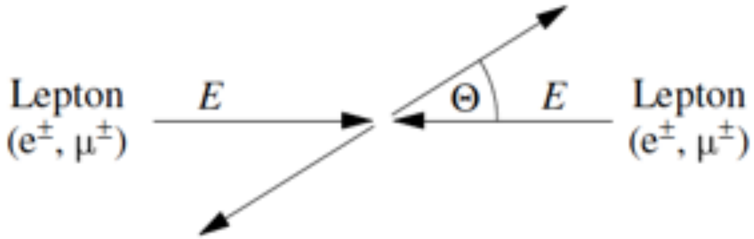


Observe “diffractive” pattern

- “location of minimal reveals size (radius) of nuclei
- Drop with angle reveals
  shape of charge distribution

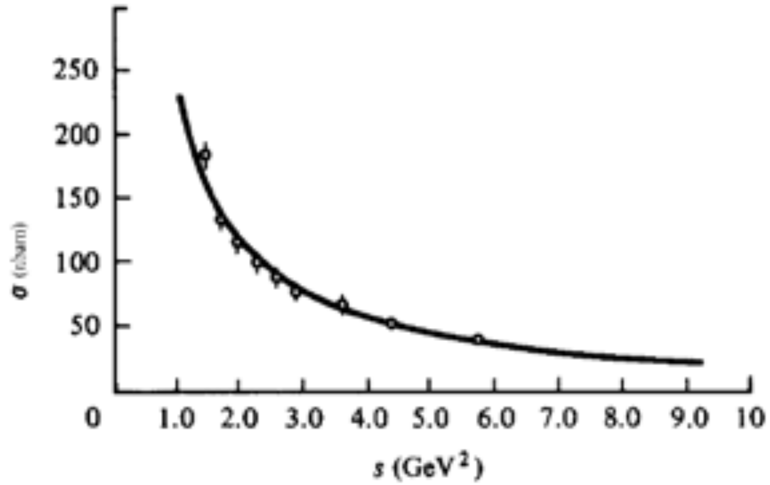
High Energy e-e scattering





$$F(\theta) = \frac{1+\cos^4\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} + \underbrace{\frac{2}{\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}}_{\text{Interferenz-term}} + \frac{1+\sin^4\frac{\theta}{2}}{\cos^4\frac{\theta}{2}}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}F(\theta), \qquad s = W^2 = 4E^2$$



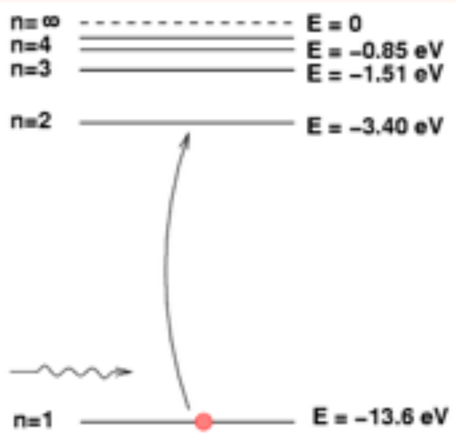
Bhabha-scattering:

- cross section exactly calculable
- experimental data agree with calculation
- "finite-size" effects of electron is small

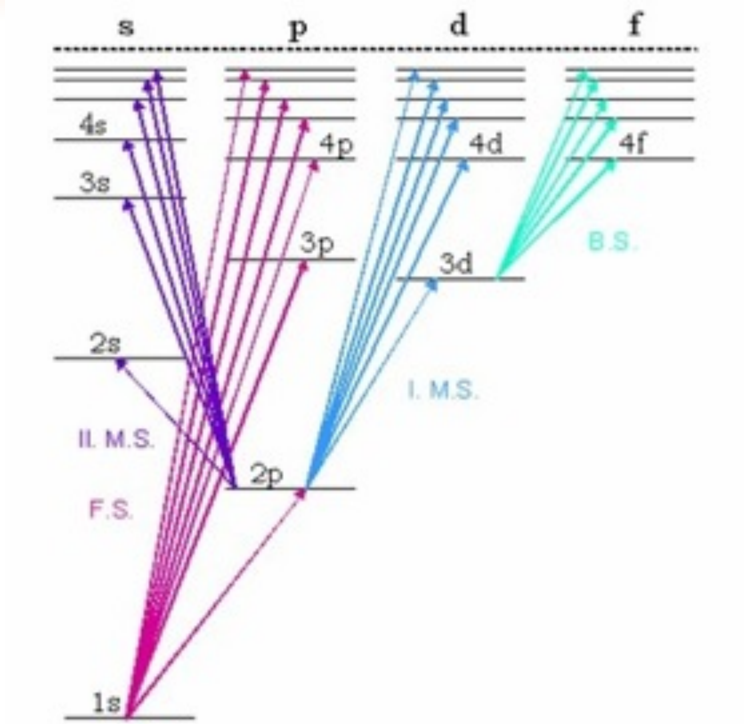
$$r_e < 10^{-3}\ \text{fm} = 10^{-18}\ \text{m}$$

Hydrogen Atom

Exzellenzcluster Universe

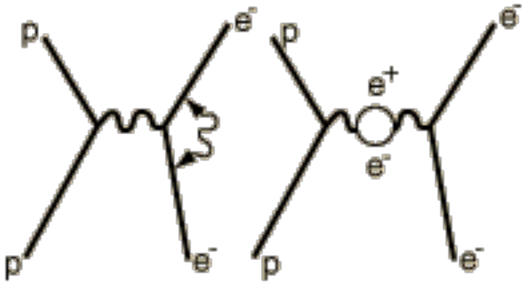


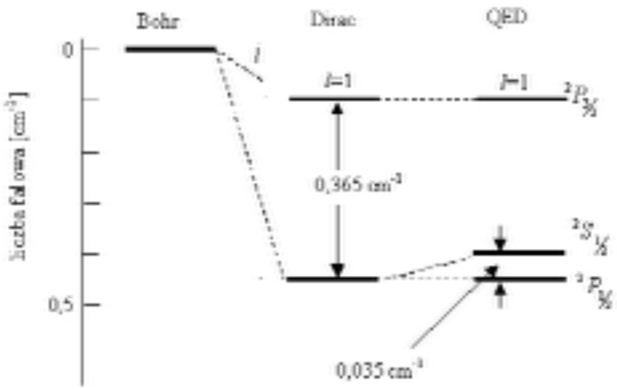
central potential
spin-orbit interaction (FS)
spin-orbit interaction (HFS)



What do we learn about QED or particle physics ?

vacuum polarization
(Lamb-shift)





Hydrogen Atom II

Exzellenzcluster Universe

Radius of proton:

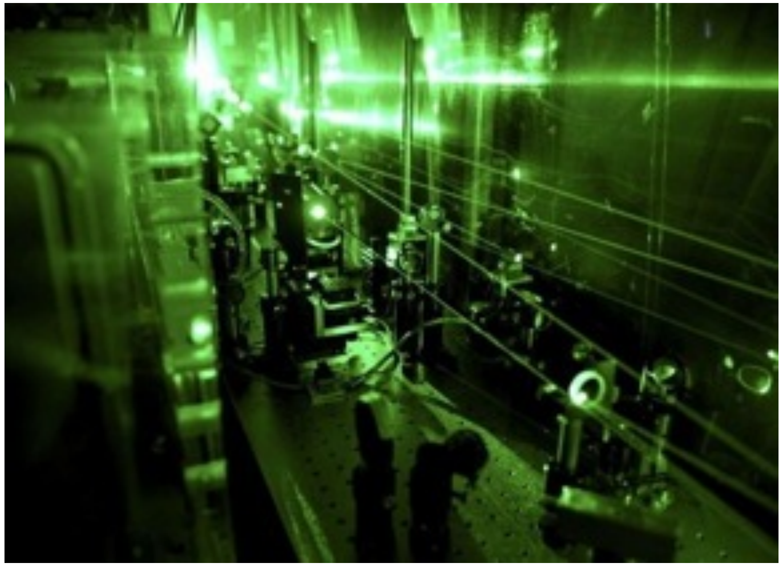
Test Coulomb potential close to proton
Deviation of spectrum from calculations reveal finite charge distribution

Average distance of 1S electron: (Bohr radius) a_0 = 0.5 Angstrom

$$a_0 = \frac{\hbar^2}{m_e e^2}$$

Exchange electron by muon ($m_\mu = 207\ m_e$) Bohr radius of muonic hydrogen: $2.5\ 10^{-13}$ m

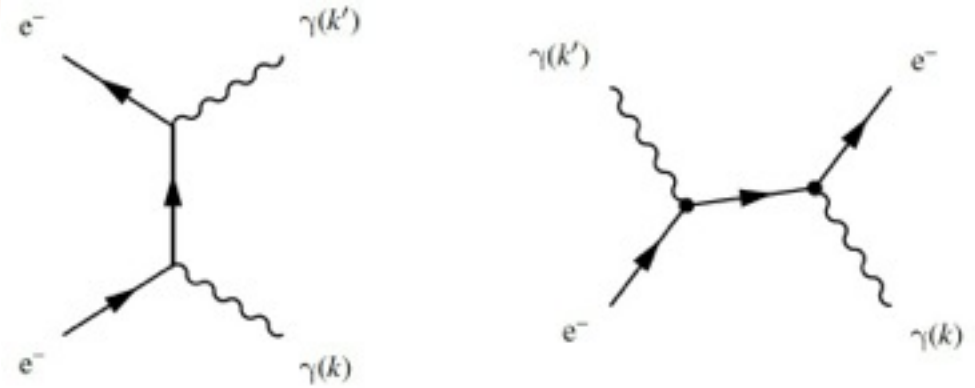
Experiment combines :
particle physics (production muonic hydrogen)
atomic physics (Laser spectroscopy)

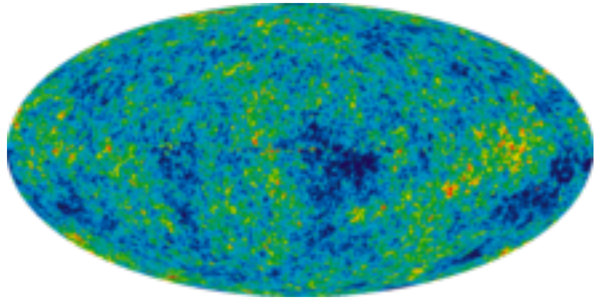


$\langle r_p^2 \rangle^{1/2} = 0.84184 \pm 0.00067$ fm $= 0.8\ 10^{-15}$ m

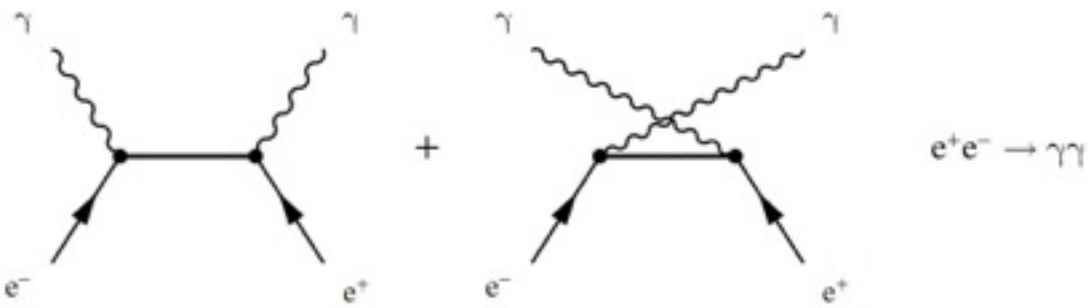
Lifetime of muon: 2.2 μs !!

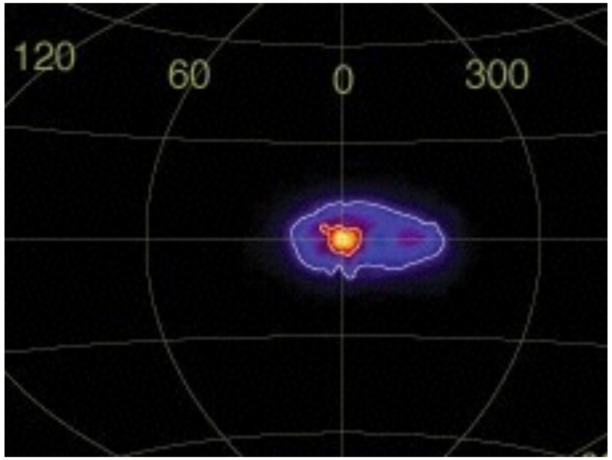
Relevance for Astrophysics





Early Universe: Hydrogen formation in competition with ionization by hot CMB
Reionization after first star formation





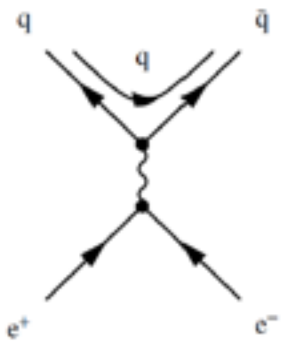
INTEGRAL satellite: Antimatter cloud surrounding our galaxy   511keV line

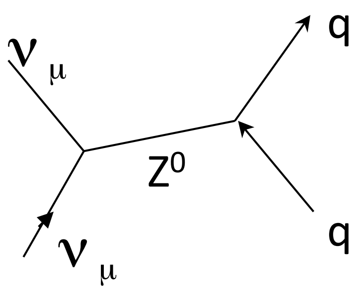
Origin: X-ray binaries

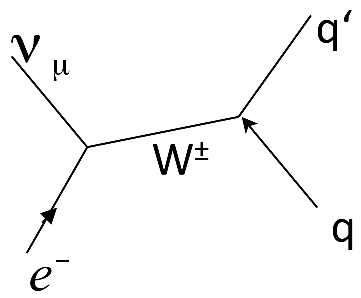
Strong Interaction

Players:

Quarks and Gluons

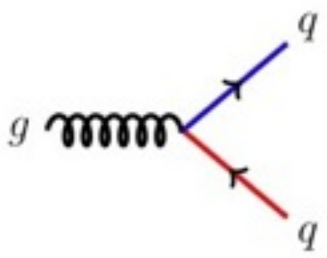


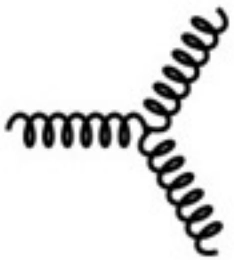




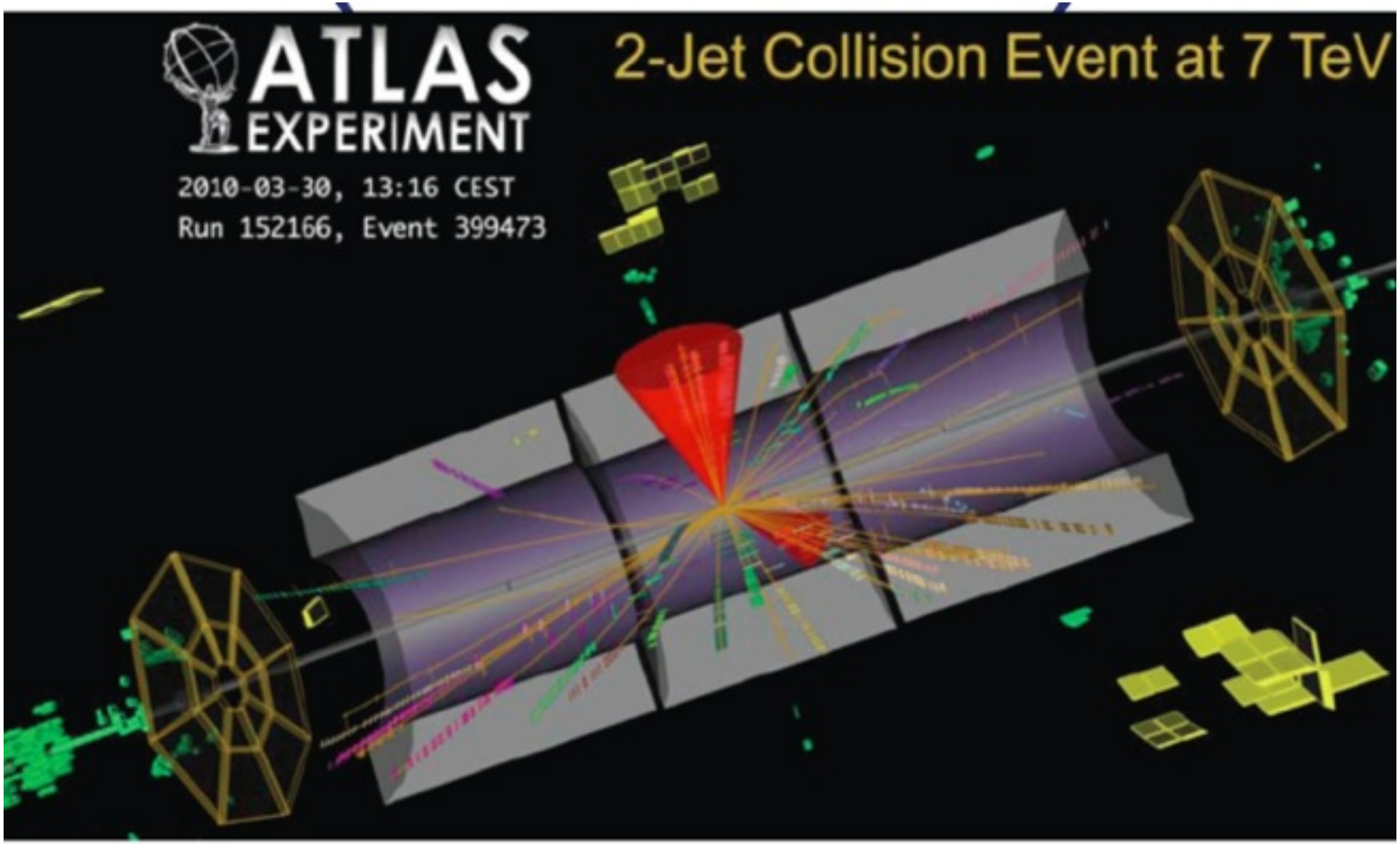
Quarks carry:
electric charge (–1/3, +2/3) ⟶ coupling to photon
colour charge (r,g,b) ⟶ coupling to gluons
weak charge ⟶ coupling to W, Z

Gluons carry:
Colour charge (e.g. $r\bar{b}$…) ⟶ coupling to quarks and gluons





Strong Interaction



How do we know about Quarks ?

Free quarks not observed

searching for fractionally charged particles

- at accelerators:
- Cosmic rays

Where Quarks are Found

Exzellenzcluster Universe

Quarks:

postulated to explain large number of particles found, inteaction strongly

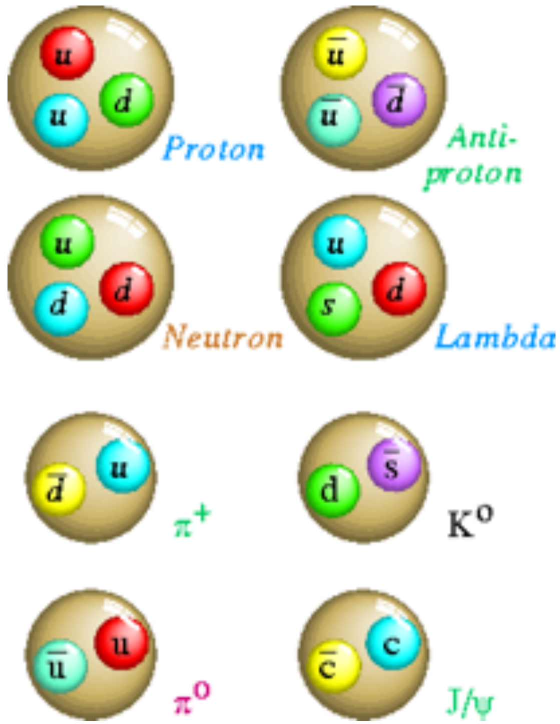
- particles (lifetimes below 10^{-16} s)
  - p, n, Λ, Σ, Ξ, Ω baryons
  - $\pi^{\pm}$, π^0, K, D, B mesons…
- Resonances (“states” decaying strongly with lifetimes above 10^{-20} s)

Originally “model particles”
carriers of quantum numbers

Idea:

electric charge: +⅔ and –⅓
baryon number: ⅓
spin: ½ (fermion) colour: r,g,b

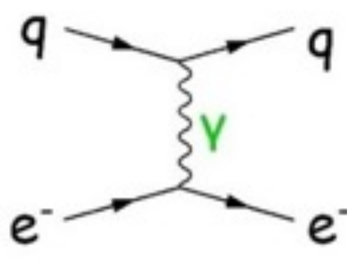
Baryons: qqq
Mesons: $q\bar{q}$



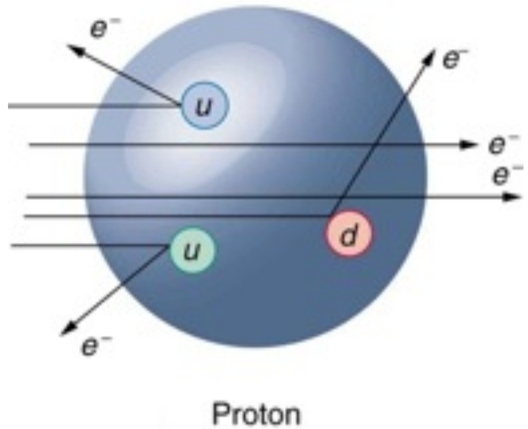
How Quarks are Found II

Scatter electrons (muons)

- Point like particles
- Carry electric charge
- Energy: wavelength of exchanged photon: $\lambda < 0.2$fm (resolve proton)



elastic scattering



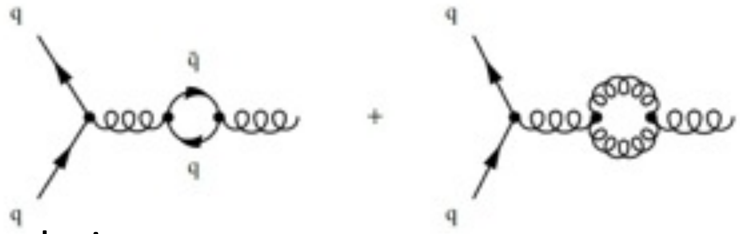
Findings:

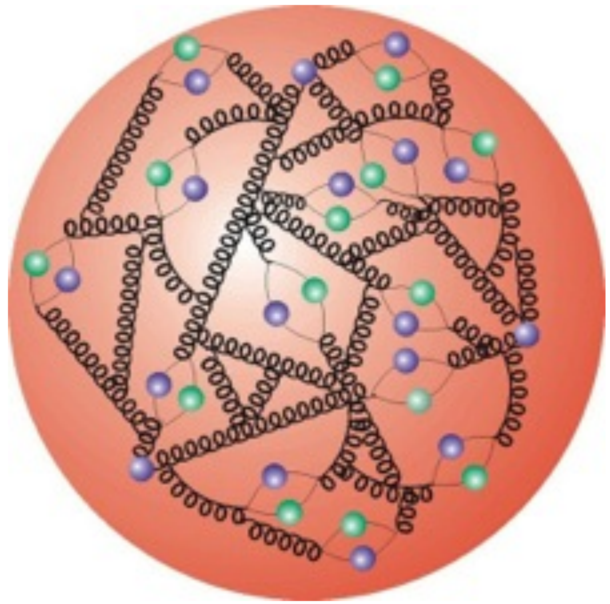
- Scattering distribution follows “elastic” 2-body scattering
- no dependence on “wavelength (momentum transfer Q^2)
- Cross section: $\approx \sum z_i^2$
- Quarks move inside nucleon with distribution function q(x)

How Quarks are Found III

Nucleon inside is very complicated

- Quarks are bound by gluons (“elastically tied together)
- Gluons self–interact
- Gluons form $q\bar{q}$–pairs
    - also antiquarks in nucleon
    - quantum effects depend on “size scale”
        → picture of nucleon changes slightly with resolution
- Gluons carry 50% of momentum





Spin of the nucleon

Quarks and gluons carry spin:

- nucleon has spin S=1/2
- how does spin of quarks and gluons add up to S=1/2 ?
- active field of research

$e^+ e^-$ Collider (DESY)

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