Singlet extensions of the MSSM with $\mathbb{Z}_4^R$-symmetry

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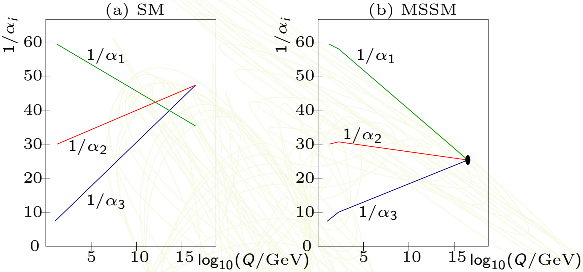
MIAPP - Anticipating 14 TeV

Based on:

- M. Ratz, P. V.: arXiv:1502.07207, to be published in AHEP

Motivation for Supersymmetry

- Unification of gauge coupling constants α_i, $i = 1, 2, 3$



- Hierarchy stabilisation
- Dark matter candidate
- ...

$\Rightarrow$ Minimal Supersymmetric Extension of the Standard Model (MSSM)

Motivation against Supersymmetry

- Little hierarchy problem:
  - Higgs mass a bit large
  - SUSY not yet found

$\Rightarrow$ Singlet extension of the MSSM

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M. Dine, N. Seiberg, S. Thomas 2007

- Add SM singlet N coupled to $H_u\,H_d$

$$\mathcal{W} \supset \underbrace{\mu H_u\,H_d}_{\mu\text{–term}} + \lambda_N\,N\,H_u\,H_d + \frac{1}{2}\mu_N\,N^2$$

- Integrate out N

$$\Rightarrow \mathcal{W}_{\text{eff}} \supset \mu H_u\,H_d - \frac{\lambda_N}{2\mu_N}\,(H_u\,H_d)^2$$

- Raise Higgs mass, ameliorate the fine–tuning
- But: μ–problem & proton decay $\Rightarrow$ symmetry? $\Rightarrow \mathbb{Z}_4^R$

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The $\mathbb{Z}_4^R$ symmetry

If one demands:

1. anomaly freedom (discrete Green-Schwarz mechanism)
2. fermion masses (Yukawa couplings & neutrino mass operator)
3. consistency with SU(5)
4. gauge coupling unification (anomaly universality)

$\Rightarrow$ only R symmetries can forbid the μ–term

If additionally

5. consistency with SO(10)

$\Rightarrow$ only $\mathbb{Z}_4^R$ symmetry can forbid the μ–term, where

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$$R(\text{matter}) = 1\,, \qquad R(\text{Higgs}) = 0\,, \qquad R(\mathcal{W}) = 2\,.$$

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- $\mathbb{Z}_4^R$ charges:

$$R(\text{matter}) = 1\,, \qquad R(\text{Higgs}) = 0\,, \qquad R(\mathcal{W}) = 2\,.$$

- Most general superpotential:

$$\begin{aligned}\mathcal{W} &= \mu H_u\,H_d + Y^u\,Q\,\overline{U}\,H_u + Y^d\,Q\,\overline{D}\,H_d + Y^e\,L\,\overline{E}\,H_d \\ &\quad + \kappa\,L\,H_u + \lambda\,L\,L\,\overline{E} + \lambda'\,L\,Q\,\overline{D} + \lambda''\,\overline{U}\,\overline{D}\,\overline{D} \\ &\quad + \frac{1}{M_\mathrm{P}}\left(L\,H_u\,L\,H_u + Q\,Q\,Q\,L + \overline{U}\,\overline{U}\,\overline{D}\,\overline{E} + \ldots\right)\end{aligned}$$

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$\mathbb{Z}_4^R$ broken to matter parity by anomaly $\Rightarrow \mu \sim \langle\mathcal{W}\rangle \sim m_{3/2}$

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- Add SM singlet N coupled to $H_u\,H_d$

$$\mathcal{W} \supset \lambda_N\, N\, H_u\, H_d \quad \Rightarrow \quad R(N) = 2$$

- Hence, linear term unconstrained

$$\mathcal{W} \supset \Lambda^2\, N + \lambda_N\, N\, H_u\, H_d \quad \text{with} \quad \Lambda \sim M_\mathrm{P}$$

- Forbid linear term $\Lambda^2\, N$ by symmetry?

Such symmetry would forbid μ–term

$\Leftrightarrow$

Only $\mathbb{Z}_4^R$ can forbid μ–term

- No symmetry! Use holomorphic zeros!

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NMSSM with $\mathbb{Z}_4^R$ symmetry

- NMSSM with $\mathbb{Z}_4^R \times \mathrm{U}(1)_\mathsf{anom}$
- $\mathrm{U}(1)_\mathsf{anom}$ is anomalous with FI–term $\xi > 0$
- $\xi > 0$ cancelled by SM singlet ϕ with charges

$$R(\phi) = 0 \quad \text{and} \quad Q_\mathsf{anom}(\phi) = -1$$

- Hence, $\mathrm{U}(1)_\mathsf{anom}$ D–term:

$$D_\mathsf{anom} = \xi - |\phi|^2 \stackrel{!}{=} 0 \quad \text{with} \quad \varepsilon = \frac{\langle\phi\rangle}{M_\mathsf{P}} \sim \sin\vartheta_\mathsf{Cabbibo} \sim 0.2$$

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NMSSM with $\mathbb{Z}_4^R$ symmetry

- $\mathrm{U}(1)_{\mathrm{anom}}$ anomaly cancelled by Green-Schwarz with axion S
- Axion S shifts under $\mathrm{U}(1)_{\mathrm{anom}}$ gauge transformation

$$S \mapsto S + \frac{\mathrm{i}}{2}\delta_{\mathrm{GS}}\Lambda(x) \quad \textit{with} \quad \delta_{\mathrm{GS}} \sim \xi > 0$$

- Non–perturbative term $e^{-b\,S}$ with charges

$$R(e^{-b\,S}) = 2 \quad \text{and} \quad Q_{\mathrm{anom}}(e^{-b\,S}) = s = \frac{b}{2}\delta_{\mathrm{GS}} > 0$$

- Non–perturbative superpotential contains

$$\mathcal{W}_{\text{non–pert.}} \supset M_P^3 \left(\frac{\phi}{M_P}\right)^s e^{-b\,S}$$

e.g. Arkani-Hamed, Dine & Martin 1998

- Gravitino mass $m_{3/2}$

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NMSSM with $\mathbb{Z}_4^R$ symmetry

- Charges

	$\mathcal{W}$	ϕ	$H_u H_d$	N	$e^{-b\,S}$
$\mathbb{Z}_4^R$	2	0	0	2	2
$\mathrm{U}(1)_{\mathsf{anom}}$	0	-1	$h>0$	$n<0$	$s>0$

- At perturbative level

$$\mathcal{W} \;\supset\; \lambda\, N\, H_u\, H_d \quad \text{with} \quad \lambda \;\sim\; \varepsilon^{n+h}$$

- Including non-perturbative terms: generalised NMSSM

M. Drees 1989

$$\mathcal{W}_{\text{eff}} \;\supset\; f^2\, N + \mu\, H_u\, H_d + \lambda\, N\, H_u\, H_d + \mu_N\, N^2 + \kappa\, N^3$$

where

$$f \;\sim\; \varepsilon^{n/2} m_{3/2} \quad , \quad \mu \;\sim\; \varepsilon^{h} m_{3/2}$$

$$\mu_N \;\sim\; \varepsilon^{2n} m_{3/2} \quad , \quad \kappa \;\sim\; \varepsilon^{3n} \frac{m_{3/2}^2}{M_{\mathrm{P}}^2}$$

M. Ratz, P.V. 2015

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M. Ratz, P.V. 2015

NMSSM with $\mathbb{Z}_4^R$ symmetry

- Charges

	$\mathcal{W}$	ϕ	$H_u H_d$	N	$e^{-b\,S}$
$\mathbb{Z}_4^R$	2	0	0	2	2
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M. Drees 1989

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Example: NMSSM with $\mathbb{Z}_4^R$ symmetry

- Charges $(h, n, s) = (1, -1, 2)$

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NMSSM with $\mathbb{Z}_4^R$ symmetry: SUSY vacua

- Effective superpotential

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- Supersymmetric vacua $F = D = 0$
  - SUSY unbroken, EW broken

$$\langle N\rangle \ = \ -\frac{\mu}{\lambda} \ \sim \ -\,\varepsilon^{|n|}\,m_{3/2}$$

$$\langle H_u\rangle \ = \ \langle H_d\rangle \ = \ \frac{\sqrt{2\mu\,\mu_N - \lambda\,f^2}}{\lambda} \ \sim \ \varepsilon^{-h/2}\,m_{3/2}$$

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Conclusion

- The $\mathbb{Z}_4^R$ NMSSM
- Holomorphic zeros forbid linear term
- Effective superpotential

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Backup

Green-Schwarz mechanism

- Symmetry generator α is anomalous if action invariant but path integral measure not, i.e.

$$D\Psi D\bar{\Psi} \mapsto e^{i\int \mathrm{d}^4 x A(\alpha)} D\Psi D\bar{\Psi}$$

where

$$A(\alpha) = \frac{1}{32\pi^2}\mathrm{Tr}\left(\alpha F\tilde{F}\right) - \frac{1}{384\pi^2} R\tilde{R}\,\mathrm{Tr}\left(\alpha\right)$$

and $A(\alpha) \neq 0$. Álvarez-Gaumé, Witten/ Álvarez-Gaumé, Ginsparg

- Note: $\int \mathrm{d}^4 x A(\alpha) = 2\pi \frac{\text{integer}}{N}$ for $\mathbb{Z}_N$,
  i.e. phase can be non-trivial for discrete transformation α

Green-Schwarz mechanism

- For example, under U(1) gauge transformation Λ

$$V \mapsto V + \frac{i}{2}\left(\Lambda - \Lambda^\dagger\right) \qquad \text{vector superfield}$$

$$\Phi^{(f)} \mapsto e^{-iq^{(f)}\Lambda}\Phi^{(f)} \qquad \text{chiral superfields}$$

- From $A(\alpha) \supset \text{Tr}\left(\alpha F\tilde{F}\right) \neq 0 \quad \Rightarrow$

$$\sum_f {q^{(f)}}^3 \neq 0 \quad \text{and} \quad \sum_{r^{(f)}} \ell\left(r^{(f)}\right) q^{(f)} \neq 0$$

$\Rightarrow$ Anomaly!

Green-Schwarz mechanism

- Add Dilaton superfield S

$$L \supset -\int \mathrm{d}^4\theta \ln\left(S + S^\dagger - \delta_{\mathrm{GS}} V\right) + \int \mathrm{d}^2\theta \frac{S}{4} W_\alpha W^\alpha + \mathrm{h.c.}$$

with transformation property

$$S \mapsto S + \frac{i}{2}\delta_{\mathrm{GS}}\Lambda \quad \text{with} \quad \delta_{\mathrm{GS}} = \frac{1}{6\pi^2}\sum_f {q^{(f)}}^3$$

- Second term compensates non-invariance of path integral measure

$\Rightarrow$ Anomaly cancelled!

Discrete Green-Schwarz mechanism

- α is $\mathbb{Z}_N$ transformation
- Chiral superfields transform

$$\Phi^{(f)} \mapsto e^{-2\pi i q^{(f)} \frac{1}{N}} \Phi^{(f)}$$

- Assume $\int \mathrm{d}^4 x A(\alpha) = 2\pi \frac{\text{integer}}{N}$
- Again, add Dilaton superfield S with discrete shift trafo

$$S \mapsto S + \frac{i}{2}\Delta_{\mathrm{GS}} \quad \text{with} \quad \Delta_{\mathrm{GS}} = \frac{1}{\pi N} A_{G-G-\mathbb{Z}_N}$$

$\Rightarrow$ Anomaly cancelled!

$\mathbb{Z}_N$ (R) symmetry

- Matter superfields with charges $q^{(f)}$
  $\Rightarrow$ fermion components with charges $q^{(f)} - R$,
  with $R = 0$ or 1 for non-R or R symmetry
- Superpotential $\mathcal{W}$ has charge $2R$
- Anomaly coefficients for gauge group G

$$A_{G-G-\mathbb{Z}_N} = \sum_{r^{(f)}} \ell\left(r^{(f)}\right)\left(q^{(f)} - R\right) + \ell\,(\text{adj})\,R$$

- Anomaly cancellation

$$A_{G-G-\mathbb{Z}_N} = \rho \bmod \eta \quad \text{where} \quad \eta = \begin{cases} N & \text{for } N \text{ odd,} \\ N/2 & \text{for } N \text{ even.} \end{cases}$$

$\rho \neq 0$ Green-Schwarz anomaly cancellation

SU(5) universal charges

Consider simple situation:

- SU(5) universal charges for matter $q_{\mathbf{10}_i}$ and $q_{\overline{\mathbf{5}}_i}$
- MSSM $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ anomaly coefficients

$$
\begin{aligned}
A_{\mathrm{SU}(3)_C-\mathrm{SU}(3)_C-\mathbb{Z}_N} &= \frac{1}{2}\sum_i \left[3q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 3R \\
A_{\mathrm{SU}(2)_L-\mathrm{SU}(2)_L-\mathbb{Z}_N} &= \frac{1}{2}\sum_i \left[3q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 2R \\
&\quad + \frac{1}{2}\left(q_H + q_{\bar{H}} - 2R\right)
\end{aligned}
$$

SU(5) universal charges

- Anomaly cancellation
  (Allowing for the Green-Schwarz mechanism)

$$A_{\mathrm{SU}(2)_L-\mathrm{SU}(2)_L-\mathbb{Z}_N} - A_{\mathrm{SU}(3)_C-\mathrm{SU}(3)_C-\mathbb{Z}_N} = 0 \bmod \eta$$
$$\Rightarrow \quad q_H + q_{\bar{H}} = 4R \bmod 2\eta$$

- In contrast, μ-term allowed if

$$q_H + q_{\bar{H}} = 2R \bmod N \,.$$

- For non-R symmetry ($R = 0$): In the SU(5) case, anomaly cancellation generically implies the $\mu-$problem

SU(5) universal charges for $\mathbb{Z}_N^R$

Demand: Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg and P. V. 2010

- $\mathbb{Z}_N^R$-charges for matter are SU(5) universal
- $G - G - \mathbb{Z}_N^R$ anomalies are universal for $G \in$ SM
- Yukawa couplings and Weinberg operator are allowed

Result: N divides 24 and:

N	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_H	$q_{\bar{H}}$	ρ
4	1	1	0	0	1
6	5	3	4	0	0
8	1	5	0	4	1
12	5	9	4	0	3
24	5	9	16	12	9
3	2	0	1	0	0
6	2	0	4	0	0

(last two cases allow $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}} \Rightarrow$ Proton decay)

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