Updated NNLO QCD predictions for the weak radiative B-meson decays

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in collaboration with

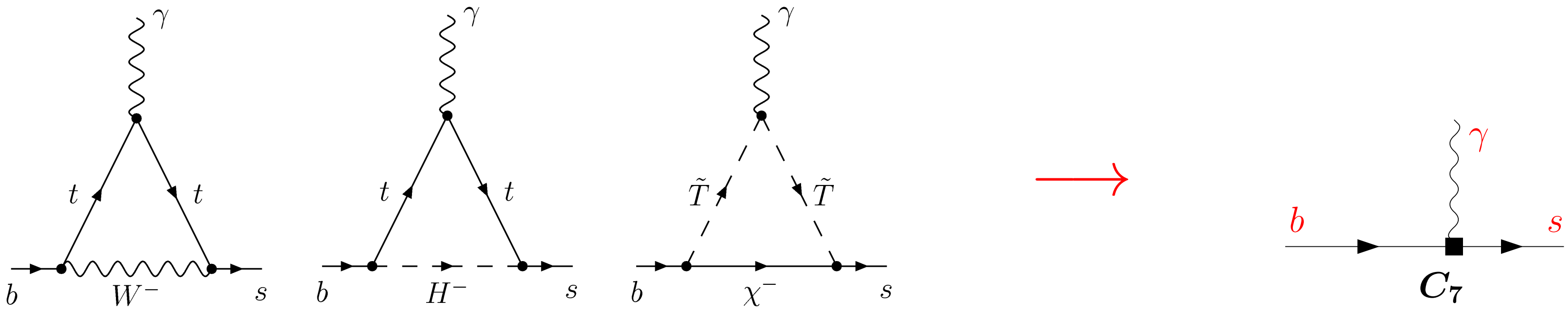
H. M. Asatrian, R. Boughezal, M. Czakon, T. Ewerth, A. Ferroglia, P. Fiedler, P. Gambino, C. Greub, U. Haisch, T. Huber, M. Kamiński, G. Ossola, M. Poradziński, A. Rehman, T. Schutzmeier, M. Steinhauser and J. Virto,

based on arXiv:1503.01789, to appear in the PRL

and arXiv:1503.01789, JHEP 1504 (2015) 168.

1. Introduction
2. $\bar{B} \to X_s\gamma$ at $\mathcal{O}(\alpha_s^2)$ and bounds on $M_{H^\pm}$
3. $\bar{B} \to X_d\gamma$ and the ratio R_γ
4. Outlook: calculations without interpolation in m_c
5. Summary

Information on electroweak-scale physics in the $b \to s\gamma$ transition is encoded in an effective low-energy local interaction:



$$b \in \bar{B} \equiv (\bar{B}^0 \text{ or } B^-)$$

The inclusive $\bar{B} \to X_s\gamma$ decay rate is well approximated by the corresponding perturbative decay rate of the b-quark:

$$\Gamma(\bar{B} \to X_s\,\gamma)_{E_\gamma > E_0} \;=\; \Gamma(b \to X_s^p\,\gamma)_{E_\gamma > E_0} \;+\; \left(\begin{array}{c}\text{non-perturbative effects}\\ (2 \pm 5)\% \\ \text{Benzke et al., arXiv:1003.5012}\end{array}\right)$$

provided E_0 is large $(E_0 \sim m_b/2)$
but not too close to the endpoint $(m_b - 2E_0 \gg \Lambda_{\rm QCD})$.

Conventionally, $E_0 = 1.6\,\text{GeV} \simeq m_b/3$ is chosen.

CP-averaged decay rates

$$\Gamma_0 = \frac{\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^0 \to X_{\bar{s}}\gamma)}{2}, \qquad \Gamma_\pm = \frac{\Gamma(B^- \to X_s\gamma) + \Gamma(B^+ \to X_{\bar{s}}\gamma)}{2}.$$

CP- and isospin-averaged branching ratio in an untagged measurement at $\Upsilon(4S)$

$$\mathcal{B}_{s\gamma} \;=\; \tau_{B^0}\,\Gamma\,\left(\frac{1+r_f r_\tau}{1+r_f} \;+\; \Delta_{0\pm}\frac{1-r_f r_\tau}{1+r_f}\right).$$

where

$\Gamma = (\Gamma_0 + \Gamma_\pm)/2$ (isospin average)

$\Delta_{0\pm} = (\Gamma_0 - \Gamma_\pm)/(\Gamma_0 + \Gamma_\pm)$ (isospin asymmetry)

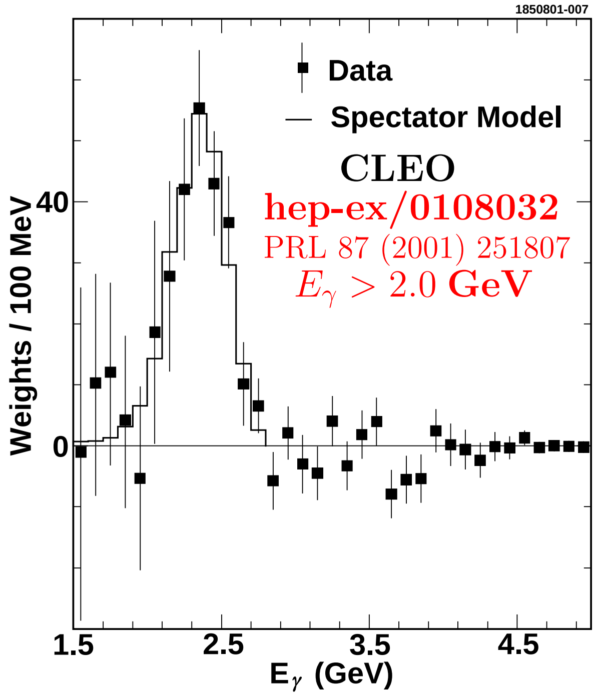
$r_\tau = \tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.004$ (measured lifetime rate)

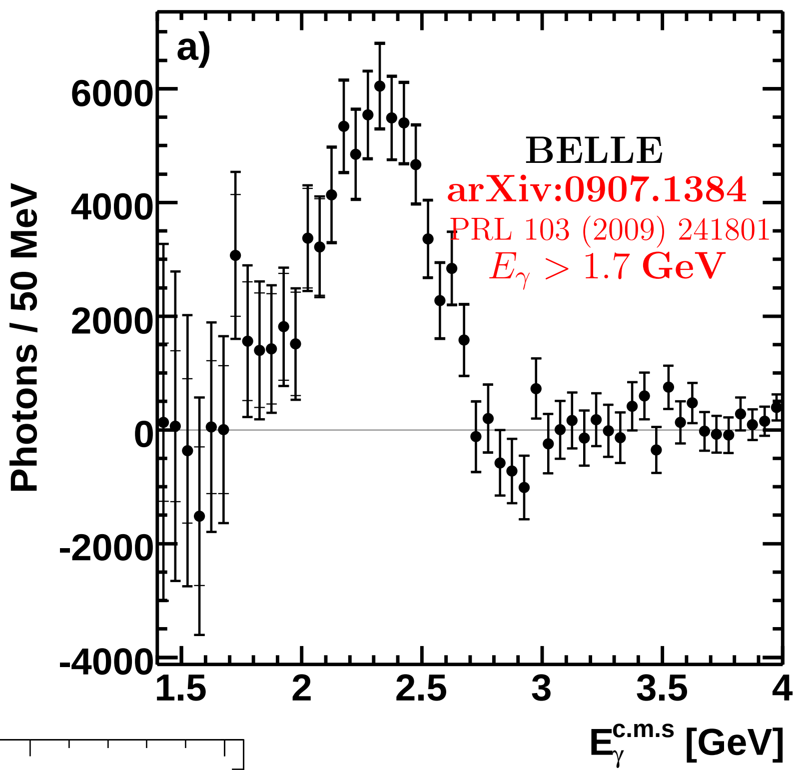
$r_f = f^{+-}/f^{00} = 1.059 \pm 0.027$ (measured production rate at $\Upsilon(4S)$)

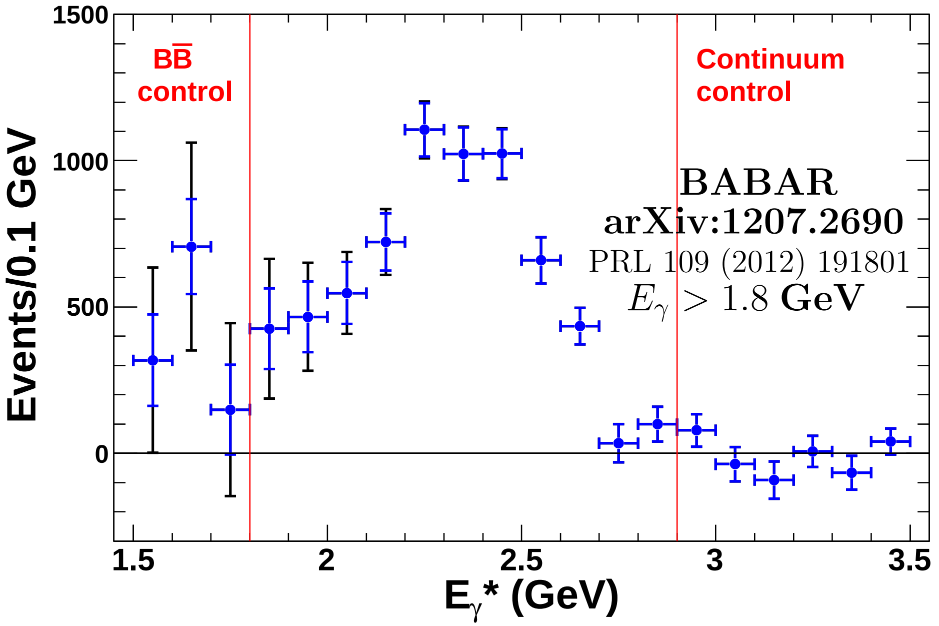
The term proportional to $\Delta_{0\pm}$ contributes only at a permille level, which follows from the measured value of $\Delta_{0\pm} = -0.01 \pm 0.06$ (for $E_\gamma > 1.9\,\text{GeV}$).

The final state strangeness (-1 for X_s and $+1$ for $X_{\bar{s}}$) and neutral B-meson flavours have been specified upon ignoring effects of the $B^0\bar{B}^0$ and $K^0\bar{K}^0$ mixing. Taking the $K^0\bar{K}^0$ mixing into account amounts to replacing X_s and $X_{\bar{s}}$ by $X_{|s|}$ with an unspecified strangeness sign, which leaves Γ_0 and $\Gamma_\pm$ invariant. Next, taking the $B^0\bar{B}^0$ mixing into account amounts to using in Γ_0 the time-integrated decay rates of mesons whose flavour is fixed at the production time. Such a change leaves Γ_0 practically unaffected because mass eigenstates in the $B^0\bar{B}^0$ system are very close to being orthogonal ($|p/q| = 1$) and having the same decay width.

The "raw" photon energy spectra in the inclusive measurements of $\mathcal{B}_{s\gamma}$







The peaks are centred around

$$\tfrac{1}{2} m_b \simeq 2.35 \text{ GeV}$$

which corresponds to a two-body $b \to s\gamma$ decay.

Broadening is due to (mainly):

- perturbative gluon bremsstrahlung,
- motion of the b quark inside the $\bar{B}$ meson,
- motion of the $\bar{B}$ meson in the $\Upsilon(4S)$ frame.

Experimental world average for $\mathcal{B}_{s\gamma}$

$$\mathcal{B}_{s\gamma}^{\rm exp} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$ [HFAG, arXiv:1412.7515]

for the photon energy $E_\gamma > E_0 = 1.6$ GeV. The averaging involves an extrapolation from the measurements performed at $E_0 \in [1.7, 2.0]$ GeV.

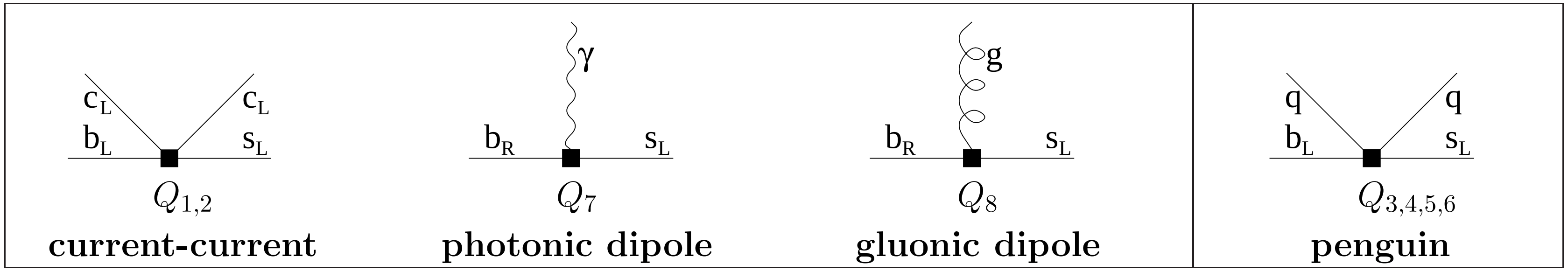
Applying the same extrapolation method to the available $\bar{B} \to X_d\gamma$ measurement [BABAR, arXiv:1005.4087]**, one finds** [Crivellin, Mercolli, arXiv:1106.5499]**:**

$$\mathcal{B}_{d\gamma}^{\rm exp} = (1.41 \pm 0.57) \times 10^{-5}.$$

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

$$L_{\text{weak}} \sim \Sigma\, C_i(\mu_b)\, Q_i$$

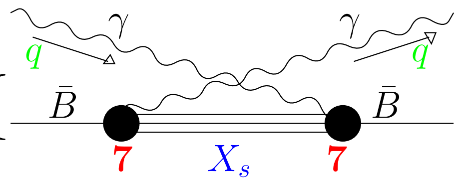
8 operators matter for $\mathcal{B}^{\text{SM}}_{s\gamma}$ when the higher-order EW and/or CKM-suppressed effects are neglected:



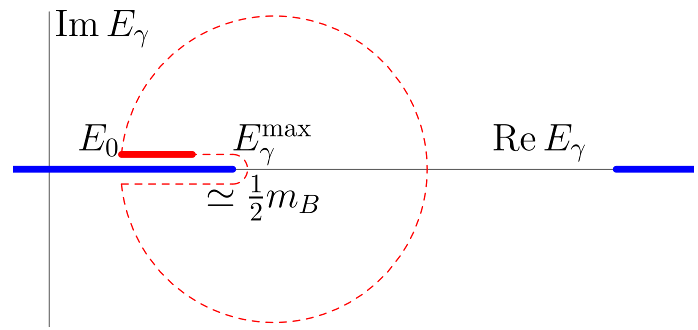
$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = |C_7|^2\, \Gamma_{77}(E_0) + (\text{other})$$

Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \text{Im}\{\ \ldots\ \} \equiv \text{Im}\, A$$



Integrating the amplitude A over E_γ:



OPE on the ring $\Rightarrow$ Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\text{QCD}}}{m_b}$ and α_s that begins with

$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3}, \ldots;\ \frac{\alpha_s \mu_\pi^2}{(m_b - 2E_0)^2},\ \frac{\alpha_s \mu_G^2}{m_b(m_b - 2E_0)}; \ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\text{QCD}})$ are extracted from the semileptonic $\bar{B} \to X_c e \bar{\nu}$ spectra and the B–$B^\star$ mass difference.

NNLO QCD corrections to $\bar{B} \to X_s \gamma$

The relevant perturbative quantity:

$$\frac{\Gamma[b \to X_s \gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2 \, \Gamma[b \to X_u e \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\rm em}}{\pi} \underbrace{\sum_{i,j} C_i \, C_j \, K_{ij}}_{P(E_0)}$$

Expansions of the Wilson coefficients and K_{ij}:

$$C_i(\mu_b) = C_i^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} C_i^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 C_i^{(2)}(\mu_b) + \ldots$$

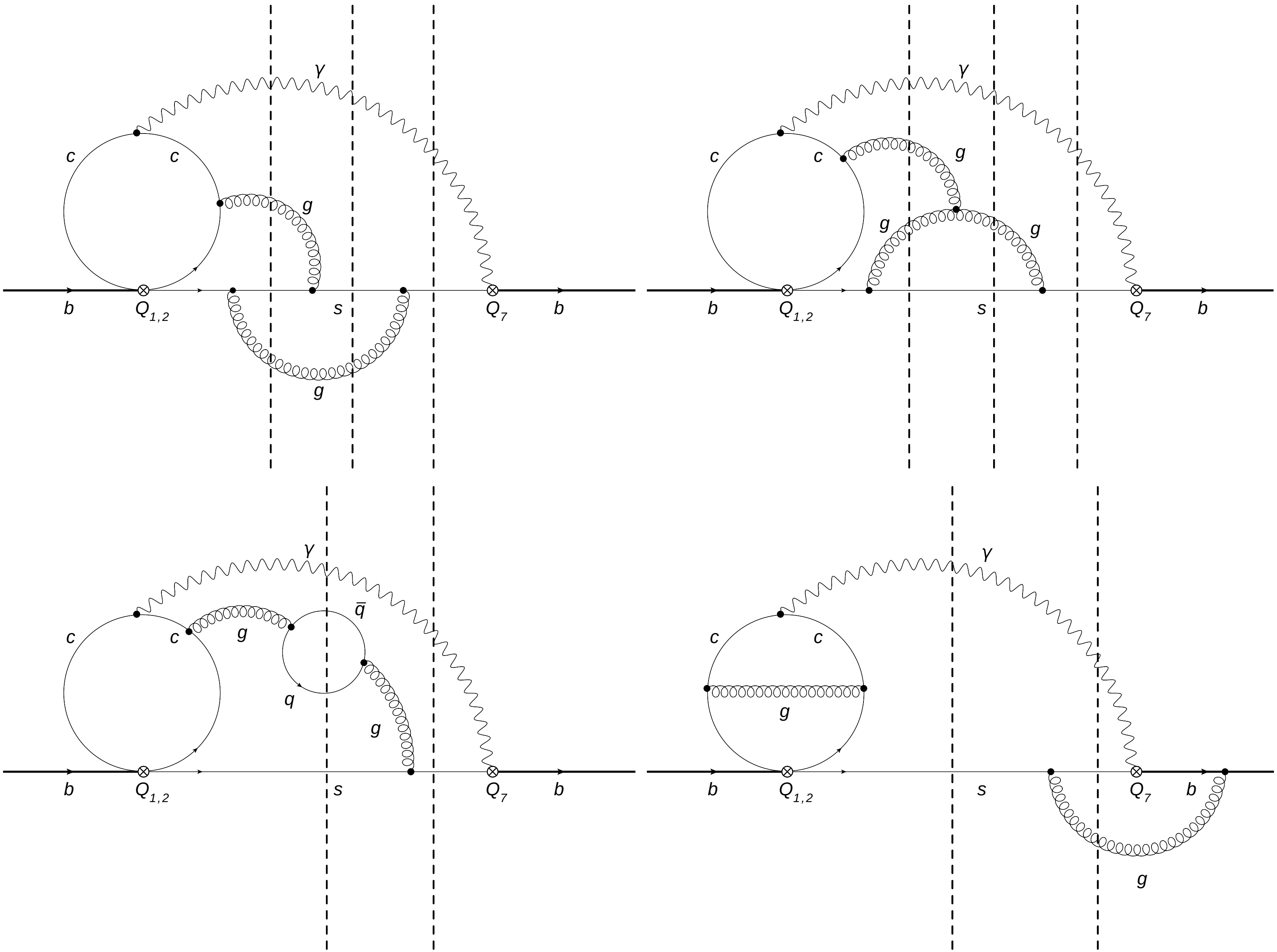
$$K_{ij} = K_{ij}^{(0)} + \frac{\alpha_s(\mu_b)}{4\pi} K_{ij}^{(1)} + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 K_{ij}^{(2)} + \ldots \qquad \mu_b \sim \frac{m_b}{2}$$

Most important at the NNLO: $K_{77}^{(2)}$**,** $K_{27}^{(2)}$ **and** $K_{17}^{(2)}$**.**

They depend on $\frac{\mu_b}{m_b}$, $\delta = 1 - \frac{2E_0}{m_b}$ **and** $z = \frac{m_c^2}{m_b^2}$**.**

Evaluation of $K_{27}^{(2)}$ and $K_{17}^{(2)}$ for $m_c = 0$:

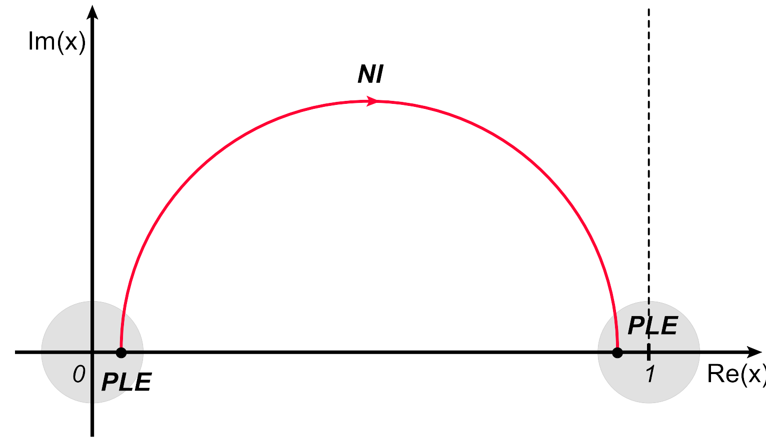
[M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier, M. Steinhauser, JHEP 1504 (2015) 168]



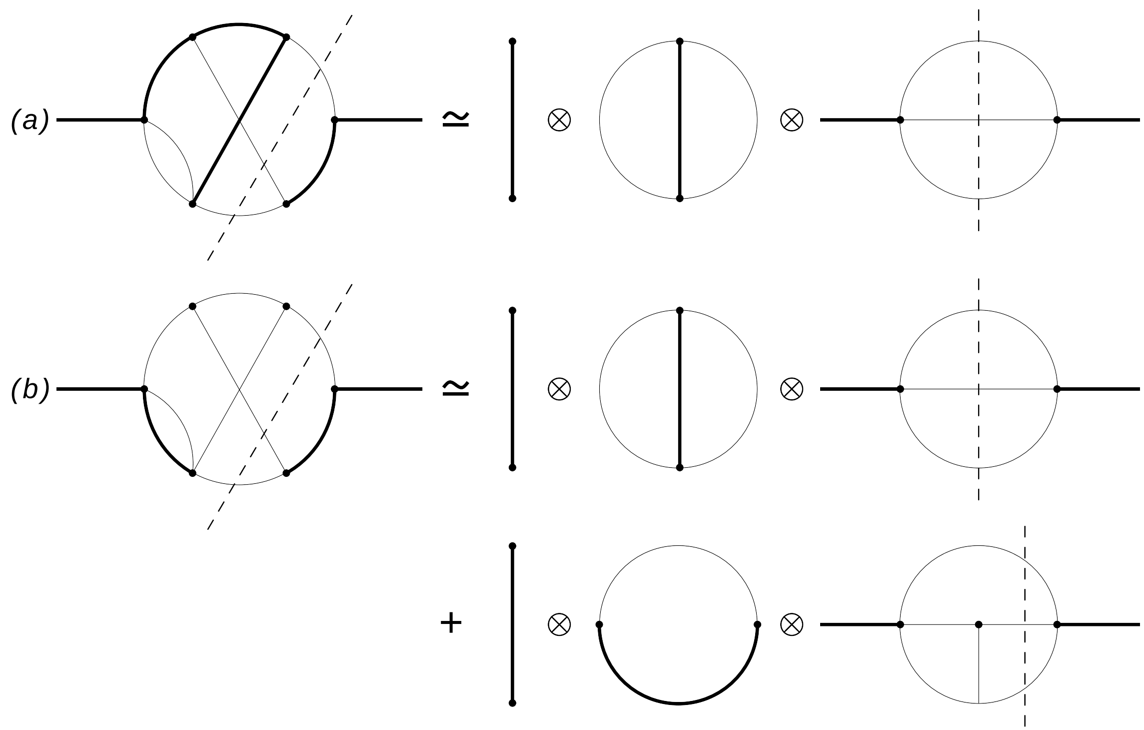
Master integrals and differential equations:

	n_D	n_{OS}	n_{eff}	$n_{massless}$
2-particle cuts	**292**	**92**	**143**	**9**
3-particle cuts	**267**	**54**	**110**	**11**
4-particle cuts	**292**	**17**	**37**	**7**

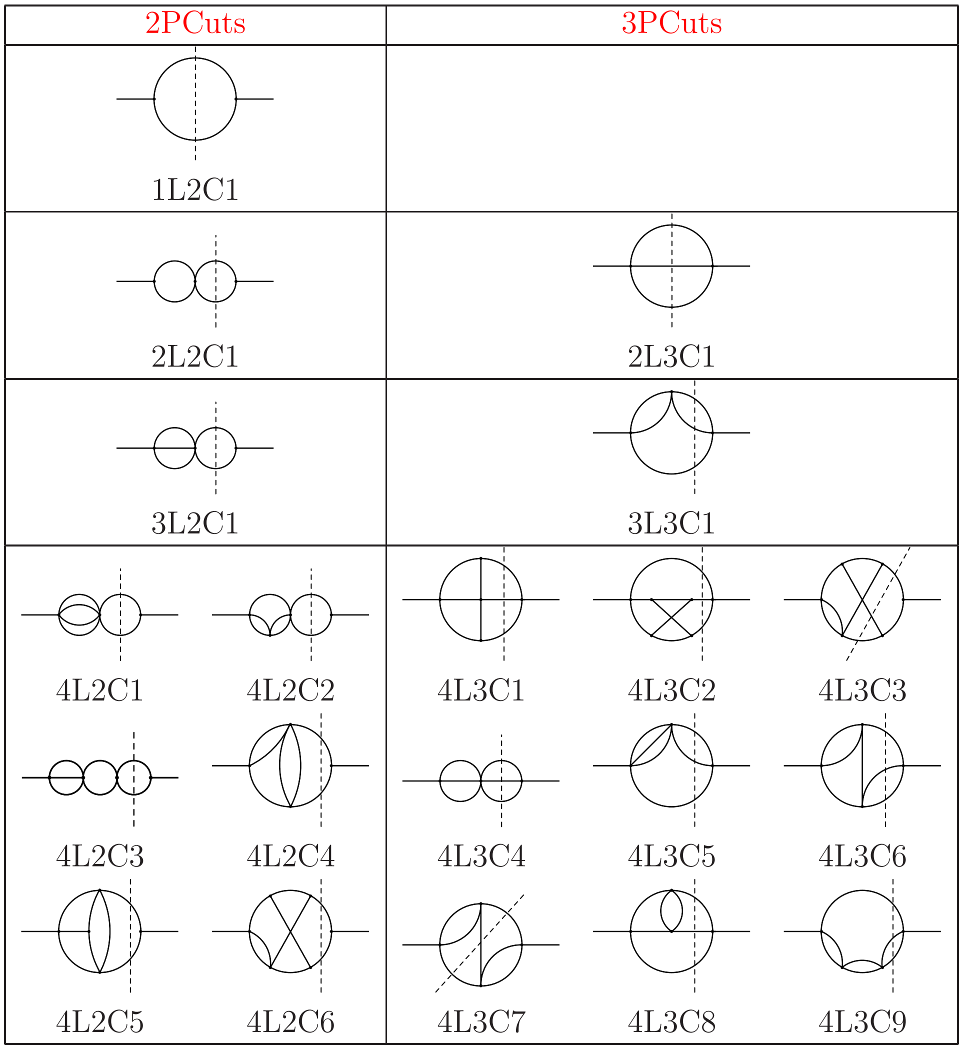
$$\frac{d}{dx} I_i(x) = \sum_j R_{ij}(x) I_j(x), \qquad x = \frac{p^2}{m_b^2}.$$

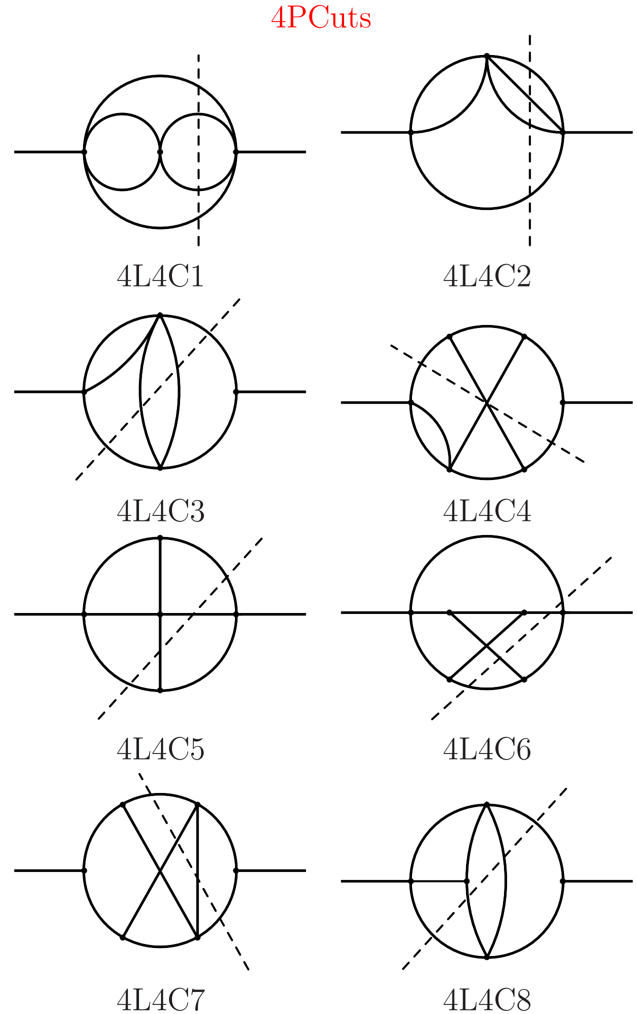


Boundary conditions in the vicinity of $x = 0$:



Massless integrals for the boundary conditions:





Results for the NNLO corrections:

$$\begin{aligned} K_{27}^{(2)}(r,\delta) &= A_2 + F_2(z,\delta) - \frac{3}{2}\beta_0^{n_f=3} f_q(z,\delta) + f_b(z) + f_c(z) + \frac{4}{3}\phi_{27}^{(1)}(z,\delta)\ln z \\ &+ \left[(8L_c - 2x_m)\, z\frac{d}{dz} + (1-\delta)x_m \frac{d}{d\delta}\right] f_{NLO}(z,\delta) + \frac{416}{81}x_m \\ &+ \left(\frac{10}{3}K_{27}^{(1)} - \frac{2}{3}K_{47}^{(1)} - \frac{208}{81}K_{77}^{(1)} - \frac{35}{27}K_{78}^{(1)} - \frac{254}{81}\right) L_b - \frac{5948}{729}L_b^2, \end{aligned}$$

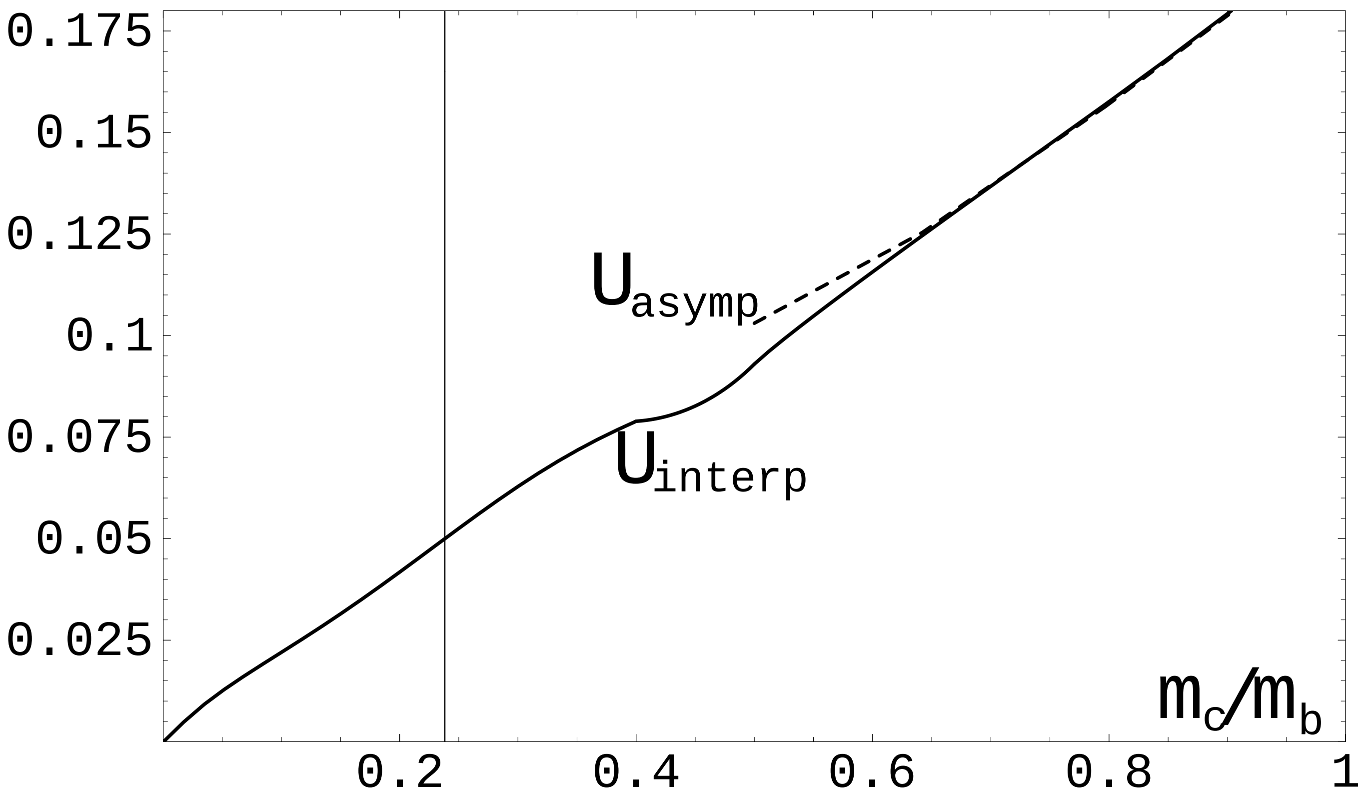
$$K_{17}^{(2)}(r,\delta) = -\frac{1}{6}K_{27}^{(2)}(z,\delta) + A_1 + F_1(z,\delta) + \left(\frac{94}{81} - \frac{3}{2}K_{27}^{(1)} - \frac{3}{4}K_{78}^{(1)}\right) L_b - \frac{34}{27}L_b^2,$$

where $F_i(0,0) \equiv 0$**,** $A_1 \simeq 22.605$**,** $A_2 \simeq 75.603$ **from the present calculation.**

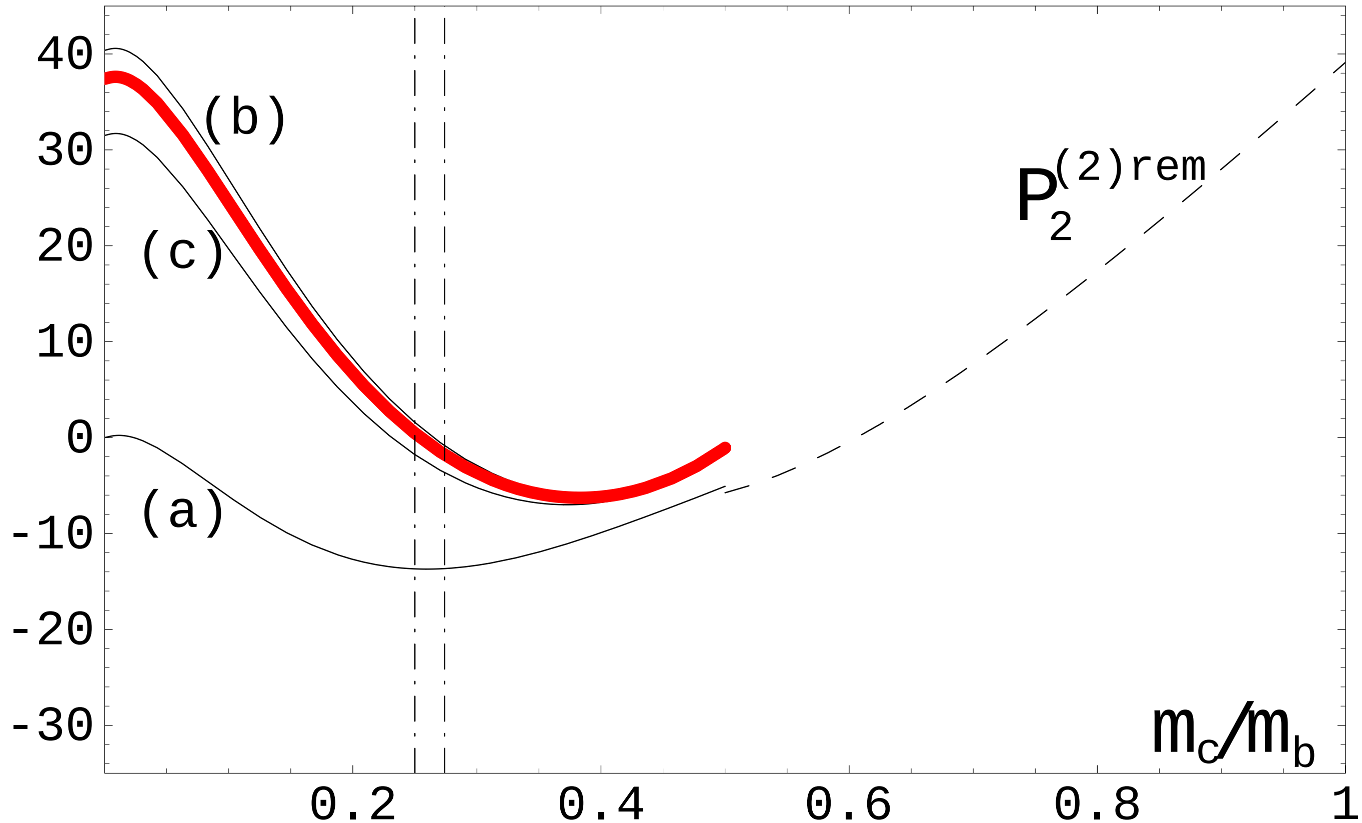
Next, we interpolate in m_c **by assuming that** $F_i(r,0)$ **are linear combinations of** $f_q(r,0)$**,** $f_{NLO}(r,0)$**,** $r\frac{d}{dr}f_{NLO}(r,0)$ **and a constant term. The known large-**r **behaviour of** F_i **[hep-ph/0609241] and the condition** $F_i(0,0) \equiv 0$ **fix these linear combinations in a unique manner.**

Effect of the interpolated contribution on the branching ratio

$$\frac{\Delta\mathcal{B}_{s\gamma}}{\mathcal{B}_{s\gamma}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2} \frac{C_1^{(0)}(\mu_b)F_1(z,\delta)+\left(C_2^{(0)}(\mu_b)-\frac{1}{6}C_1^{(0)}(\mu_b)\right)F_2(z,\delta)}{C_7^{(0)\mathrm{eff}}(\mu_b)}$$



Comparison to the interpolation in hep-ph/0609241

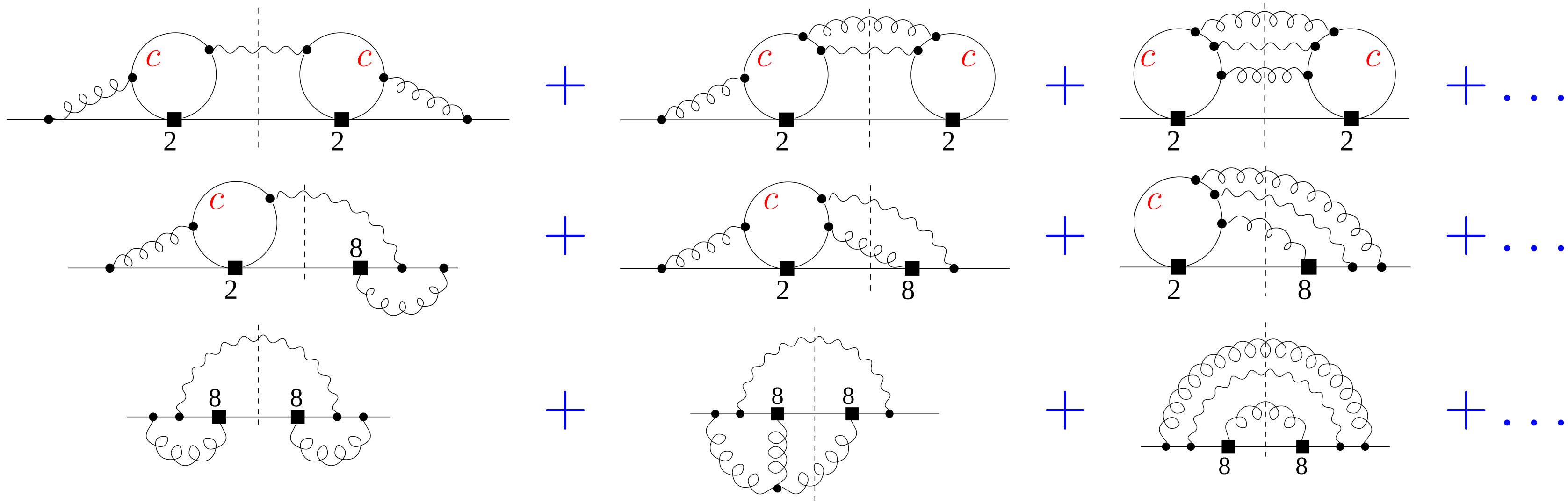


Interferences not involving the photonic dipole operator are treated as follows:

K_{22}**:**
(and analogous K_{11} & K_{12})

K_{28}**:**
(and analogous K_{18})

K_{88}**:**



Two-particle cuts are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only. The NLO+(NNLO BLM) corrections are not big $(+3.8\%)$.

Incorporating other perturbative contributions evaluated after the previous phenomenological analysis in hep-ph/0609232:

1. Four-loop mixing (current-current) $\to$ (gluonic dipole)
   M. Czakon, U. Haisch, MM, JHEP 0703 (2007) 008 [hep-ph/0612329]
2. Diagrams with massive quark loops on the gluon lines
   R. Boughezal, M. Czakon and T. Schutzmeier, JHEP 0709 (2007) 072 [arXiv:0707.3090]
   H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, Phys. Lett. B 647 (2007) 173 [hep-ph/0611123]
   T. Ewerth, Phys. Lett. B 669 (2008) 167 [arXiv:0805.3911]
3. Complete interference (photonic dipole)–(gluonic dipole)
   H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola,
   Phys. Rev. D 82 (2010) 074006 [arXiv:1005.5587]
4. New BLM corrections to contributions from 3-body and 4-body final states for interferences not involving the photonic dipole
   A. Ferroglia and U. Haisch, Phys. Rev. D 82 (2010) 094012 [arXiv:1009.2144]
   MM and M. Poradziński, Phys. Rev. D 83 (2011) 014024 [arXiv:1009.5685]
5. LO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from 4-quark operators ("penguin" or CKM-suppressed)
   M. Kamiński, MM and M. Poradziński, Phys. Rev. D 86 (2012) 094004 [arXiv:1209.0965]
6. NLO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from interferences of the above operators with $Q_{1,2,7,8}$
   T. Huber, M. Poradziński, J. Virto, JHEP 1501 (2015) 115 [arXiv:1411.7677]

Taking into account new non-perturbative analyses:

M. Benzke, S. J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099 [arXiv:1003.5012]
T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 830 (2010) 278 [arXiv:0911.2175]

Updating the parameters (Parametric uncertainties go down to 2.0%)

P. Gambino, C. Schwanda, Phys. Rev. D 89 (2014) 014022
A. Alberti, P. Gambino, K. J. Healey, S. Nandi, Phys. Rev. Lett. 114 (2015) 061802

Updated SM estimate [arXiv:1503.01789, arXiv:1503.01791]:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.36 \pm 0.23) \times 10^{-4}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, **3%** from the interpolation in m_c

3% higher order $\mathcal{O}(\alpha_s^3)$, **2%** parametric

It is very close the the experimental world average [arXiv:1412.7515]:

$$\mathcal{B}_{s\gamma}^{\rm exp} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

Experiment agrees with the SM to much better than $\sim 1\sigma$ **level.**

Uncertainties: TH $\sim 7\%$, **EXP** $\sim 6.5\%$.

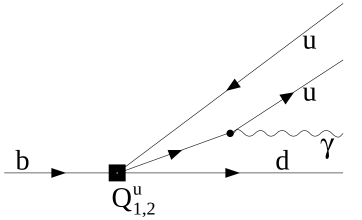
$\Rightarrow$

Stronger bounds on the $H^{\pm}$ mass in the Two-Higgs-Doublet-Model II:

$$M_{H^{\pm}} > 480\,\text{GeV at } 95\%\text{C.L.}$$

$$M_{H^{\pm}} > 358\,\text{GeV at } 99\%\text{C.L.}$$

$\bar{B} \to X_d\gamma$



$$\mathcal{L}_{\rm eff} \sim V_{td}^* V_{tb} \left[\sum_{i=1}^{8} C_i Q_i + \kappa_d \sum_{i=1}^{2} C_i (Q_i - Q_i^u) \right]$$

$$\kappa_d = (V_{ud}^* V_{ub})/(V_{td}^* V_{tb}) = \left(0.007^{+0.015}_{-0.011}\right) + i \left(-0.404^{+0.012}_{-0.014}\right)$$

$$\mathcal{B}_{d\gamma}^{\rm SM} = \left(1.73^{+0.12}_{-0.22}\right) \times 10^{-5} \quad \text{for } E_0 = 1.6\,\text{GeV}$$

- Rough: m_b/m_q varied between $10 \sim m_B/m_K$ and $50 \sim m_B/m_\pi$ $\Rightarrow$ 2% to 11% of $\mathcal{B}_{d\gamma}$.
- Fragmentation functions give a similar range [H. M. Asatrian and C. Greub, arXiv:1305.6464].
- Collinear logarithms and isolated photons

The ratio R_γ

$$R_\gamma^{\rm SM} \equiv \left(\mathcal{B}_{s\gamma}^{\rm SM} + \mathcal{B}_{d\gamma}^{\rm SM} \right) / \mathcal{B}_{c\ell\nu} = (3.31 \pm 0.22) \times 10^{-3}$$

Generic (but CP-conserving) beyond-SM effects

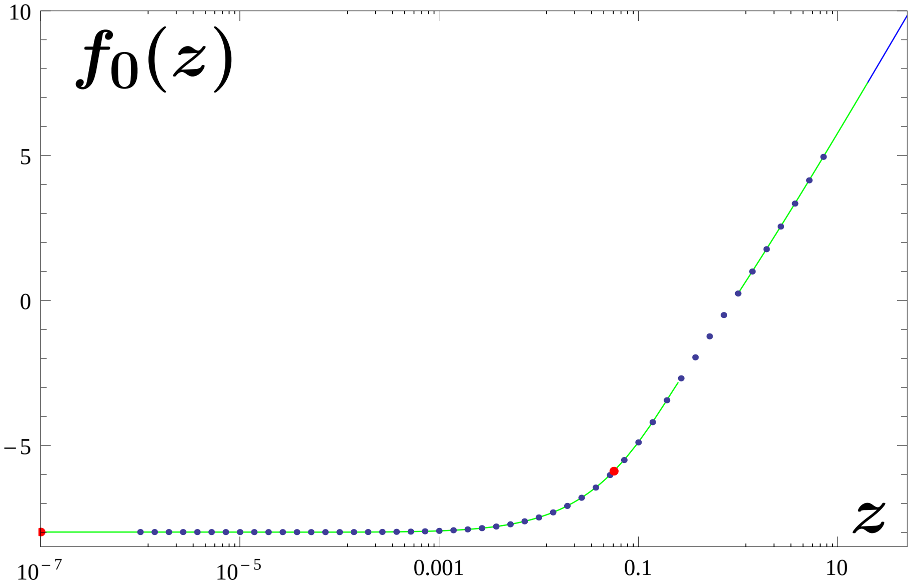
$$\mathcal{B}_{s\gamma} \times 10^4 = (3.36 \pm 0.23) - 8.22\,\Delta C_7 - 1.99\,\Delta C_8$$
$$R_\gamma \times 10^3 = (3.31 \pm 0.22) - 8.05\,\Delta C_7 - 1.94\,\Delta C_8.$$

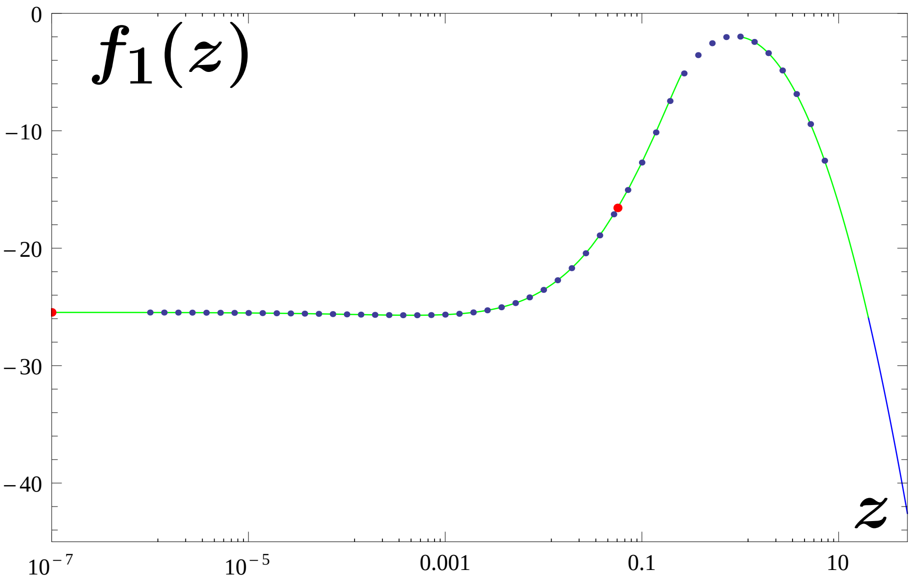
Outlook: generalizing the K_{27} NNLO calculation to arbitrary $z = m_c^2/m_b^2$.

Method: differential equations in z for the master integrals.

Results for the bare NLO contributions up to $\mathcal{O}(\epsilon)$:

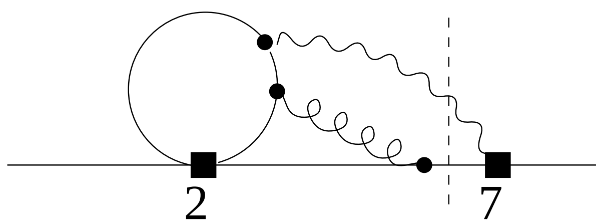
$$\tilde{G}_{27}^{(1)2P} = -\frac{92}{81\epsilon} + f_0(z) + \epsilon f_1(z) \xrightarrow{z\to 0} -\frac{92}{81\epsilon} - \frac{1942}{243} + \epsilon\left(-\frac{26231}{729} + \frac{259}{243}\pi^2\right)$$





Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.
Lines: large- and small-z asymptotic expansions

Large-z expansions of the 11 master integrals are from M. Steinhauser.



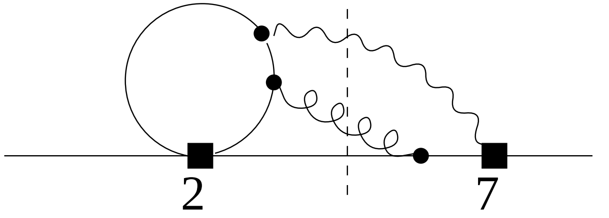
Small-z expansions of $\tilde{G}_{27}^{(1)2P}$:

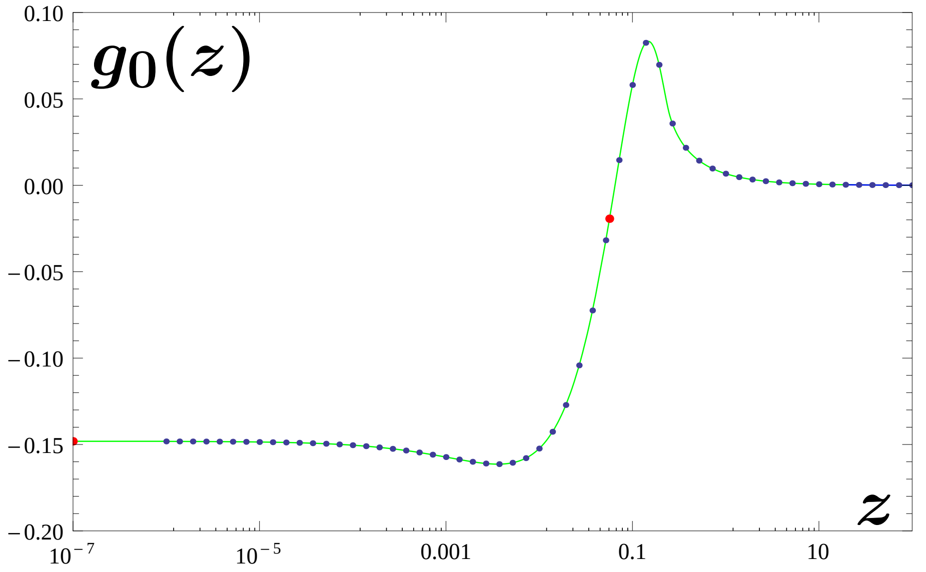
f_0 from C. Greub, T. Hurth, D. Wyler, hep-ph/9602281, hep-ph/9603404,
A. J. Buras, A. Czarnecki, MM, J. Urban, hep-ph/0105160,

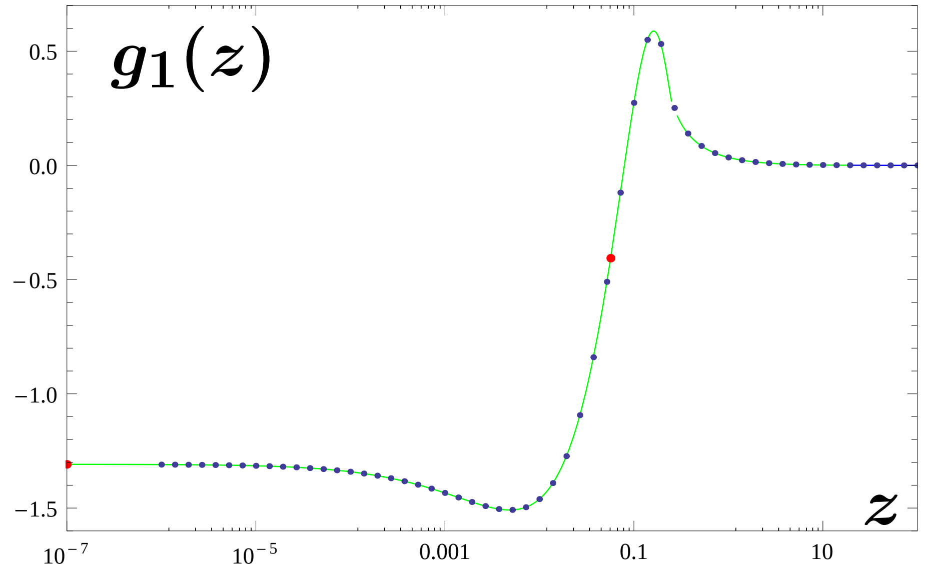
f_1 from H.M. Asatrian, C. Greub, A. Hovhannisyan, T. Hurth and V. Poghosyan, hep-ph/0505068.

Analogous results for the 3-body final state contributions ($\delta = 1$):

$$\tilde{G}^{(1)3P}_{27} = g_0(z) + \epsilon g_1(z) \xrightarrow{z\to 0} -\frac{4}{27} - \frac{106}{81}\epsilon$$







Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.

Lines: exact result for g_0, as well as large- and small-z asymptotic expansions for g_1 from A. Rehman.

$$g_0(z) = \begin{cases} -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, s\, L + \frac{16}{9}z(6z^2-4z+1)\left(\frac{\pi^2}{4} - L^2\right), & \text{for } z \le \frac{1}{4}, \\ -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, t\, A + \frac{16}{9}z(6z^2-4z+1)\, A^2, & \text{for } z > \frac{1}{4}, \end{cases}$$

where $s = \sqrt{1-4z}$, $L = \ln(1+s) - \frac{1}{2}\ln 4z$, $t = \sqrt{4z-1}$, **and** $A = \arctan(1/t)$.

Summary

- **The dominant NNLO corrections to $\mathcal{B}_{s\gamma}$ are now known not only in the large m_c limit, but also at $m_c = 0$. However, no reduction of uncertainties with respect to the 2006 estimate is possible, except for the parametric one.**

- **Updated predictions:**

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.36 \pm 0.23) \times 10^{-4}$$

$$\mathcal{B}_{d\gamma}^{\rm SM} = \left(1.73^{+0.12}_{-0.22}\right) \times 10^{-5}$$

$$R_{\gamma}^{\rm SM} = (3.31 \pm 0.22) \times 10^{-3}$$

- **Completing the calculation of $K_{27}^{(2)}$ for arbitrary $z = m_c^2/m_b^2$ is necessary to further reduce the perturbative uncertainties.**