Tau Physics

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« New Physics at High Energy and High Precision »
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Outline:

1. Introduction and Motivation
2. Hadronic $\tau$-decays
3. LFV tau decays
4. CP violation in tau decays
5. Conclusion and outlook

NB: several topics not covered: Lepton Universality, g-2, EDM, etc...
1. Introduction and Motivation
The $\tau$ lepton

- $\tau$ lepton discovered in 1976 by M. Perl et al. (SLAC-LBL group)
  - Mass: $m_\tau = 1.77682(16)\text{ GeV}$
  - Lifetime: $\tau_\tau = 2.096(10) \cdot 10^{-13}\text{ s}$

- Enormous progress in tau physics since then (CLEO, LEP, Babar, Belle, BES, VEPP-2M, neutrino experiments, ...)
  - Early years: consolidate $\tau$ as a standard lepton no invisible decays and standard couplings
  - Better data: determination of fundamental SM parameters and QCD studies

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Number of $\tau$ pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP</td>
<td>$\sim 3 \times 10^5$</td>
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The \( \tau \) lepton

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- Mass: \[ m_\tau = 1.77682(16) \text{ GeV} \]
- Lifetime: \[ \tau_\tau = 2.096(10) \cdot 10^{-13} \text{ s} \]

- Enormous progress in tau physics since then (CLEO, LEP, Babar, Belle, BES, VEPP-2M, neutrino experiments,...)
  - More recently: huge number of tau at the B factories: BaBar, Belle:
    - Tool to search for NP: rare decays, final states in hadron colliders
    - Precision physics: \( \alpha_S \), \( |V_{us}| \) etc

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2. Hadronic $\tau$-decays
2.1 Introduction

- Tau, the only lepton heavy enough to decay into hadrons

- \( m_\tau \sim 1.77 \text{GeV} > \Lambda_{\text{QCD}} \)

- Inclusive \( \tau \) decays: \( \tau \to (\bar{u}d, \bar{u}s)\nu_\tau \) use perturbative tools: OPE...

- We consider

  \[
  \Gamma(\tau^- \to \nu_\tau \text{ hadrons}_{s=0})
  \]

  \[
  \Gamma(\tau^- \to \nu_\tau \text{ hadrons}_{s\neq0})
  \]

- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system huge QCD activity!

- Observable studied:

  \[
  R_\gamma \equiv \frac{\Gamma(\tau^- \to \nu_\tau \text{ hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}
  \]
2.2 Theory

- \( R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C \)
  
  parton model prediction

- \( R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C \)

- \( \frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} \rightarrow |V_{us}| \)
2.2 Theory

- \( R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C \) 
  - parton model prediction

- \( R_\tau = R^{NS}_\tau + R^S_\tau \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C \)

- Experimentally:
  \[
  R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086
  \]

\( \Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons}) \approx N_C \)

\( \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) \approx N_C \) 

\( R_\tau \text{ switch} \) 

\( \alpha_s \neq 0 \) 

González Alonso'13
2.2 Theory

- \( R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C \)  
  parton model prediction

- \( R_\tau = R_\tau^{NS} + R_\tau^S \approx \left| V_{ud} \right|^2 N_C + \left| V_{us} \right|^2 N_C \)

- Experimentally: \( R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086 \)

- Due to QCD corrections: \( R_\tau = \left| V_{ud} \right|^2 N_C + \left| V_{us} \right|^2 N_C + O(\alpha_s) \)
2.3 Theory

- From the measurement of the spectral functions, extraction of $\alpha_S$, $|V_{us}|$

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^-\bar{\nu}_e)} \approx N_C$$

naïve QCD prediction

- Extraction of the strong coupling constant:

$$R^{NS}_\tau = |V_{ud}|^2 N_C + O(\alpha_S) \rightarrow \alpha_S$$

measured calculated

- Determination of $V_{us}$:

$$|V_{us}|^2 = \frac{R^S_\tau}{R^{NS}_\tau} + O(\alpha_S)$$

- Aim: compute the QCD corrections with the best accuracy

González Alonso'13
2.4 Calculation of the QCD corrections

- Calculation of $R_{\tau}$:

$$R_{\tau}(m_{\tau}^2) = 12\pi S_{EW} \int_0^{m_{\tau}^2} ds \left( 1 \frac{s}{m_{\tau}^2} \right)^2 \left[ 1 + 2 \frac{s}{m_{\tau}^2} \right] \text{Im} \Pi^{(1)}(s + i\epsilon) + \text{Im} \Pi^{(0)}(s + i\epsilon)$$

$$\text{Im} \Pi^{(1)}_{\bar{u}d, V/A}(s) = \frac{1}{2\pi} v_1/a_1(s)$$
2.4 Calculation of the QCD corrections

- Calculation of $R_\tau$:

\[
R_\tau (m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left( 1 - \frac{s}{m_\tau^2} \right)^2 \left[ \left( 1 + 2 \frac{s}{m_\tau^2} \right) \text{Im} \Pi^{(1)} (s + i\epsilon) + \text{Im} \Pi^{(0)} (s + i\epsilon) \right]
\]

- We are in the **non-perturbative** region: we do not know how to compute!

- Trick: use the analytical properties of $\Pi$!
2.4 Calculation of the QCD corrections

- Calculation of $R_{\tau}$:
  \[
  R_{\tau}(m_{\tau}^2) = 12\pi S_{EW} \int_0^{m_{\tau}^2} \frac{ds}{m_{\tau}^2} \left( 1 - \frac{s}{m_{\tau}^2} \right)^2 \left[ \left( 1 + 2 \frac{s}{m_{\tau}^2} \right) \text{Im} \Pi^{(1)}(s + i\epsilon) + \text{Im} \Pi^{(0)}(s + i\epsilon) \right]
  \]

- Analyticity: $\Pi$ is analytic in the entire complex plane except for $s$ real positive

  Cauchy Theorem

\[
R_{\tau}(m_{\tau}^2) = 6i\pi S_{EW} \oint_{|s|=m_{\tau}^2} \frac{ds}{m_{\tau}^2} \left( 1 - \frac{s}{m_{\tau}^2} \right)^2 \left[ \left( 1 + 2 \frac{s}{m_{\tau}^2} \right) \Pi^{(1)}(s) + \Pi^{(0)}(s) \right]
\]

- We are now at sufficient energy to use OPE:

  \[
  \Pi^{(J)}(s) = \sum_{D=0,2,4,...} \frac{1}{(-s)^{D/2}} \sum_{\text{dim} O=D} C^{(J)}(s, \mu) \langle O_D(\mu) \rangle
  \]

  Wilson coefficients
  Operators

$\mu$: separation scale between short and long distances

Braaten, Narison, Pich’92
2.4 Calculation of the QCD corrections

- Calculation of $R_\tau$:

$$R_\tau \left( m_\tau^2 \right) = N_C \ S_{EW} \left( 1 + \delta_P + \delta_{NP} \right)$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$

Braaten, Narison, Pich’92

Marciano & Sirlin’88, Braaten & Li’90, Erler’04
2.4 Calculation of the QCD corrections

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  *Marciano & Sirlin’88, Braaten & Li’90, Erler’04*

- Perturbative part (D=0):

$$\delta_p = a_\tau + 5.20 \, a_\tau^2 + 26 \, a_\tau^3 + 127 \, a_\tau^4 + ... \approx 20\%$$

  $a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$

  *Baikov, Chetyrkin, Kühn’08*
2.4 Calculation of the QCD corrections

- Calculation of $R_\tau$:
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- D=2: quark mass corrections, \textit{neglected} for $R_\tau^{NS} \propto m_u, m_d$ but not for $R_\tau^S \propto m_s$
2.4 Calculation of the QCD corrections

- **Calculation of $R_\tau$:**
  
  \[
  R_\tau \left( m_\tau^2 \right) = N_C \cdot S_{EW} \left( 1 + \delta_p + \delta_{NP} \right)
  \]

- **Electroweak corrections:** \( S_{EW} = 1.0201(3) \) \( \text{Marciano \\& Sirlin'88, Braaten \\& Li'90, Erler'04} \)

- **Perturbative part (D=0):** \( \delta_p = a_\tau + 5.20 \ a_\tau^2 + 26 \ a_\tau^3 + 127 \ a_\tau^4 + ... \approx 20\% \)
  
  \[ a_\tau = \frac{\alpha_s(m_\tau)}{\pi} \] \( \text{Baikov, Chetyrkin, Kühn’08} \)

- **D=2:** quark mass corrections, *neglected* for $R_\tau^{NS}$ \( (\propto m_u, m_d) \) but not for $R_\tau^S$ \( (\propto m_s) \)

- **D \geq 4:** Non perturbative part, not known, *fitted from the data*

- Use of weighted distributions

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2.4 Calculation of the QCD corrections

- D ≥ 4: Non perturbative part, not known, fitted from the data
  - Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

\[ R^{k\ell}_{\tau, U}(s_0) = \int_{0}^{s_0} ds \left( 1 - \frac{s}{s_0} \right)^k \left( \frac{s}{s_0} \right)^\ell \frac{dR_{\tau, U}(s_0)}{ds} \]

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Le Diberder & Pich’92

Zhang’Tau14

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### 2.4 Calculation of the QCD corrections

- **Calculation of $R_\tau$:**
  
  $$ R_\tau \left( m_\tau^2 \right) = N_C \ S_{EW} \left( 1 + \delta_P + \delta_{NP} \right) $$

- **Electroweak corrections:** $S_{EW} = 1.0201(3)$

- **Perturbative part (D=0):** $\delta_P \approx 20\%$

- **D=2:** quark mass corrections, *neglected*

- **D $\geq$ 4:** Non perturbative part, not known, *fitted from the data*

  Use of weighted distributions

  $\delta_{NP} = -0.0064 \pm 0.0013$

- **Small unknown NP part** ($\delta_{NP} : 3\% \delta_P$) very precise extraction of $\alpha_S$!
## 2.5 Results and determination of $\alpha_S$

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>$\delta_{NP}$</th>
<th>$\delta_P$</th>
<th>$\alpha_s(m_\tau)$</th>
<th>$\alpha_s(m_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baikov et al</td>
<td>CIPT, FOPT</td>
<td>0.1998 (43)</td>
<td>0.332 (16)</td>
<td>0.1202 (19)</td>
<td></td>
</tr>
<tr>
<td><strong>Davier et al'14</strong></td>
<td><strong>CIPT, FOPT</strong></td>
<td><strong>– 0.0064 (13)</strong></td>
<td>–</td>
<td><strong>0.332 (12)</strong></td>
<td><strong>0.1199 (15)</strong></td>
</tr>
<tr>
<td>Beneke-Jamin</td>
<td>BSR + FOPT</td>
<td>– 0.007 (3)</td>
<td>0.2042 (50)</td>
<td>0.316 (06)</td>
<td>0.1180 (08)</td>
</tr>
<tr>
<td>Maltman-Yavin</td>
<td>PWM + CIPT</td>
<td>+ 0.012 (18)</td>
<td>–</td>
<td>0.321 (13)</td>
<td>0.1187 (16)</td>
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<tr>
<td>Menke</td>
<td>CIPT, FOPT</td>
<td>0.2042 (50)</td>
<td>0.342 (11)</td>
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<tr>
<td>Narison</td>
<td>CIPT, FOPT</td>
<td>–</td>
<td>0.324 (08)</td>
<td>0.1192 (10)</td>
<td></td>
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<tr>
<td>Caprini-Fischer</td>
<td>BSR + CIPT</td>
<td>0.2037 (54)</td>
<td>0.322 (16)</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Abbas et al</td>
<td>IFOPT</td>
<td>0.2037 (54)</td>
<td>0.338 (10)</td>
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<tr>
<td>Cvetič et al</td>
<td>$\beta_{exp}$ + CIPT</td>
<td>0.2040 (40)</td>
<td>0.341 (08)</td>
<td>0.1211 (10)</td>
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<tr>
<td>Boito et al</td>
<td>CIPT, DV</td>
<td>– 0.002 (12)</td>
<td>–</td>
<td>0.347 (25)</td>
<td>0.1216 (27)</td>
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<tr>
<td></td>
<td>FOPT, DV</td>
<td>– 0.004 (12)</td>
<td></td>
<td>0.325 (18)</td>
<td>0.1191 (22)</td>
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<td>Pich’14</td>
<td>CIPT</td>
<td>– 0.0064 (13)</td>
<td>0.2014 (31)</td>
<td>0.342 (13)</td>
<td>0.1213 (14)</td>
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<td></td>
<td>FOPT</td>
<td></td>
<td></td>
<td>0.320 (14)</td>
<td>0.1187 (17)</td>
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<td><strong>0.332 (13)</strong></td>
<td><strong>0.1202 (15)</strong></td>
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</table>

CIPT: Contour-improved perturbation theory  
FOPT: Fixed-order perturbation theory  
BSR: Borel summation of renormalon series  
IFOPT: Improved FOPT  
$\beta_{exp}$: Expansion in derivatives of $\alpha_s$ ($\beta$ function)  
PVM: Pinched-weight moments  
CIPTm: Modified CIPT (conformal mapping)  
DV: Duality violation (OPAL only)
2.5 Results and determination of $\alpha_S$

$\alpha_S(m_\tau^2) = 0.332 \pm 0.013$

$\alpha_S(M_Z^2) = 0.1202 \pm 0.0015$

to be compared to

$\alpha_S(M_Z^2)_{Z\, width} = 0.1197 \pm 0.0028$

• Impressive test of the running of $\alpha_S$!
2.5 Results and determination of $\alpha_s$ 

- **Extraction of $\alpha_s$** from hadronic $\tau$ decays very **competitive**!

- If new data room for **improvement**!
  - Study of duality violation effects
  - Improve precision on non-perturbative determination: higher order condensates, etc
  - New physics?

---

$\tau$-decays
Lattice
DIS
$e^+e^-$ annihilation
Z pole fits

$\alpha_s(M_Z)$

$\alpha_s(Q)$

$\tau$ decays (N$^3$LO)
Lattice QCD (NNLO)
DIS jets (NLO)
Heavy Quarkonia (NLO)
$e^+e^-$ jets & shapes (res. NNLO)
Z pole fit (N$^3$LO)
$p\bar{p} \rightarrow$ jets (NLO)

QCD $\alpha_s(M_Z) = 0.1185 \pm 0.0006$
2.6 Inclusive determination of $V_{us}$

- With QCD on:
  $$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R^S_{\tau}}{R_{\tau}^{NS}} + O(\alpha_s)$$

- Use OPE:
  $$R_{\tau}^{NS}(m_{\tau}^2) = N_C S_{EW} |V_{ud}|^2 (1 + \delta_p + \delta_{ud}^{NP})$$
  $$R_{\tau}^S(m_{\tau}^2) = N_C S_{EW} |V_{us}|^2 (1 + \delta_p + \delta_{NP}^{us})$$

- SU(3) breaking quantity, strong dependence in $m_s$ computed from OPE (L+T) + phenomenology
  $$\delta R_{\tau,th} = 0.0239(30) \quad \text{Gamiz et al'07, Maltman'11}$$

- HFAG'14
  $$\begin{align*}
  |V_{us}|^2 &= \frac{R_{\tau,S}}{R_{\tau,NS}^{2}} - \delta R_{\tau,th} \\
  |V_{us}| &= 0.2176 \pm 0.0019_{exp} \pm 0.0010_{th} \\
  R_{\tau,S} &= 0.1615(28) \\
  R_{\tau,NS} &= 3.4650(84) \\
  |V_{ud}| &= 0.97425(22)
  \end{align*}$$

3.4σ away from unitarity!
Kaon and hyperon decays

- $K_{l3}$ decays ($+ f_{+(0)}$)
- $K_{l2}/\pi_{l2}$ decays ($+ f_K/f_{\pi}$)

Hyperon decays

$\tau$ decays

- $\tau \rightarrow s$ inclusive
- $\tau \rightarrow K_{\nu}$ absolute ($+ f_{K}$)
- $\tau$ branching fraction ratio
- $\tau \rightarrow K_{\nu} / \tau \rightarrow \pi_{\nu}$ ($+ f_{K}/f_{\pi}$)

Our result from Belle BR
- $\tau \rightarrow K_{\pi_{\nu}}$ decays ($+ f_{+(0)}$, FLAG)

NB: BRs measured by B factories are systematically smaller than previous measurements
3. Charged Lepton-Flavour Violation
3.1 Introduction and Motivation

- Lepton Flavour Violation is an « accidental » symmetry of the SM \((m_\nu=0)\)

- In the \(SM\) with massive neutrinos effective CLFV vertices are tiny due to GIM suppression \(\rightarrow\) \textit{unobservably small rates!}

E.g.:

\[
\mu \rightarrow e\gamma
\]

\[
Br(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U_{\mu i}^* U_{ei} \frac{\Delta m^2_{1i}}{M_W^2} \right|^2 < 10^{-54}
\]

\(\text{Petcov’77, Marciano & Sanda’77, Lee & Shrock’77…}\)

\[
\left[ Br(\tau \rightarrow \mu\gamma) < 10^{-40} \right]
\]

- Extremely \textit{clean probe of beyond SM physics}
3.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

- But the sensitivity of particular modes to CLFV couplings is model dependent

- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic
3.2 CLFV processes: tau decays

- Several processes: $\tau \rightarrow \ell \gamma$, $\tau \rightarrow \ell_\alpha \bar{\ell}_\beta \ell_\beta$, $\tau \rightarrow \ell Y$ → $P, S, V, P\bar{P}, ...$

- 48 LFV modes studied at Belle and BaBar
3.2 CLFV processes: tau decays

- Several processes: $\tau \rightarrow \ell \gamma$, $\tau \rightarrow \ell_\alpha \bar{\ell}_\beta \ell_\beta$, $\tau \rightarrow \ell Y$
- Expected sensitivity $10^{-9}$ or better at $LHCb, Belle II$
3.3 Effective Field Theory approach

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C^{(6)}_i}{\Lambda^2} O^{(6)}_i + \ldots \]

- Build all D>5 LFV operators:
  - **Dipole:**
    \[ \mathcal{L}^D_{\text{eff}} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu} \]
  - **Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):**
    \[ \mathcal{L}^S_{\text{eff}} \supset -\frac{C_{SV}}{\Lambda^2} m_\tau m_q G_F \bar{\mu} \Gamma P_{L,R} \tau \bar{q} \Gamma q \]
  - **Lepton-gluon (Scalar, Pseudo-scalar):**
    \[ \mathcal{L}^G_{\text{eff}} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau \Gamma G_a \Gamma_{\mu\nu} \]
  - **4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector):**
    \[ \mathcal{L}^{4\ell}_{\text{eff}} \supset -\frac{C^{4\ell}_{SV}}{\Lambda^2} \bar{\mu} \Gamma P_{L,R} \tau \bar{\mu} \Gamma P_{L,R} \mu \]

- Each UV model generates a **specific pattern** of them

See e.g.
Black, Han, He, Sher’02
Brignole & Rossi’04
Dassinger et al.’07
Matsuzaki & Sanda’08
Giffels et al.’08
Crivellin, Najjari, Rosiek’13
Petrov & Zhuridov’14
Cirigliano, Celis, E.P.’14

\[ \Gamma \equiv 1, \gamma^\mu \]
3.4 Model discriminating power of Tau processes

- **Summary table:**

<table>
<thead>
<tr>
<th>Process</th>
<th>$\tau \to 3\mu$</th>
<th>$\tau \to \mu\gamma$</th>
<th>$\tau \to \mu\pi^+\pi^-$</th>
<th>$\tau \to \mu K\bar{K}$</th>
<th>$\tau \to \mu\pi$</th>
<th>$\tau \to \mu\eta^{(i)}$</th>
</tr>
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<tbody>
<tr>
<td>$O_{SV}^{4\ell}$</td>
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<td>✓</td>
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<tr>
<td>$O_{GG}$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>✓</td>
<td>✓</td>
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</tbody>
</table>

- The notion of “**best probe**” (process with largest decay rate) is **model dependent**

- If observed, compare rate of processes ▶ key handle on **relative strength** between operators and hence on the **underlying mechanism**
3.4 Model discriminating power of Tau processes

- **Summary table:**

<table>
<thead>
<tr>
<th></th>
<th>$\tau \rightarrow 3\mu$</th>
<th>$\tau \rightarrow \mu\gamma$</th>
<th>$\tau \rightarrow \mu\pi^+\pi^-$</th>
<th>$\tau \rightarrow \mu K\bar{K}$</th>
<th>$\tau \rightarrow \mu\pi$</th>
<th>$\tau \rightarrow \mu\eta^{(i)}$</th>
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<td>$O_{S,V}^{4}$</td>
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<td>✓</td>
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<td>-</td>
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<tr>
<td>$O_{V}^{q}$</td>
<td>-</td>
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<td>✓ (I=0,1)</td>
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<tr>
<td>$O_{S}^{q}$</td>
<td>-</td>
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<td>✓ (I=0,1)</td>
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<td>$O_{G}^{q}$</td>
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<td>-</td>
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<tr>
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<td>✓ (I=0)</td>
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<td>✓ (I=0)</td>
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</tr>
<tr>
<td>$O_{G\bar{G}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓</td>
</tr>
</tbody>
</table>

- In addition to leptonic and radiative decays, **hadronic decays** are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part: **form factors** and **decay constants** (e.g. $f_\eta$, $f_\eta'$)
3.4 Model discriminating power of Tau processes

- Summary table:

<table>
<thead>
<tr>
<th>$\tau \rightarrow 3\mu$</th>
<th>$\tau \rightarrow \mu\gamma$</th>
<th>$\tau \rightarrow \mu\pi^+\pi^-$</th>
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<th>$\tau \rightarrow \mu\eta^{(i)}$</th>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>$O_D$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>$O_{\pi}^{0}$</td>
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<td>✓ (I=0,1)</td>
<td>—</td>
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<tr>
<td>$O_{\pi}^{0}$</td>
<td>—</td>
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<td>✓ (I=0,1)</td>
<td>—</td>
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</tr>
<tr>
<td>$O_{GG}$</td>
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<td>✓</td>
<td>—</td>
<td>—</td>
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<tr>
<td>$O_{AA}$</td>
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<td>✓ (I=0)</td>
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<tr>
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<td>✓ (I=0)</td>
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<tr>
<td>$O_{G\bar{G}}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

- Recent progress in $\tau \rightarrow \mu(e)\pi\pi$ using dispersive techniques

- Hadronic part:

$$H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right)e^{i\theta_{QCD}} | 0 \rangle = \left(\text{Lorentz struct.}\right)_\mu \mathcal{F}_i(s)$$

with $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$

- Form factors determined by solving 2-channel unitarity condition, with I=0 s-wave $\pi\pi$ and KK scattering data as input

$$\text{Im} \mathcal{F}_n(s) = \sum_{m=1}^{2} T_{nm}^*(s)\sigma_m(s)F_m(s)$$

$n = \pi\pi, K\bar{K}$
3.5 Model discriminating power of Tau processes

- Two handles:
  - Branching ratios: \( R_{F,M} \equiv \frac{\Gamma(\tau \rightarrow F)}{\Gamma(\tau \rightarrow F_M)} \) with \( F_M \) dominant LFV mode for model M

- Spectra for > 2 bodies in the final state:

\[
\frac{dBR(\tau \rightarrow \mu\pi^+\pi^-)}{d\sqrt{s}} \quad \text{and} \quad \frac{dR_{\pi^+\pi^-}}{d\sqrt{s}} \equiv \frac{1}{\Gamma(\tau \rightarrow \mu\gamma)} \frac{d\Gamma(\tau \rightarrow \mu\pi^+\pi^-)}{d\sqrt{s}}
\]
3.6 Model discriminating of BRs

Studies in specific models

Buras et al.’10

<table>
<thead>
<tr>
<th>ratio</th>
<th>LHT</th>
<th>MSSM (dipole)</th>
<th>MSSM (Higgs)</th>
<th>SM4</th>
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<tr>
<td>(\frac{\text{Br}(\mu^- \to e^- e^+ e^-)}{\text{Br}(\mu \to e \gamma)})</td>
<td>0.02...1</td>
<td>(\sim 6 \cdot 10^{-3})</td>
<td>(\sim 6 \cdot 10^{-3})</td>
<td>0.06...2.2</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^- \to e^- e^+ e^-)}{\text{Br}(\tau \to e \gamma)})</td>
<td>0.04...0.4</td>
<td>(\sim 1 \cdot 10^{-2})</td>
<td>(\sim 1 \cdot 10^{-2})</td>
<td>0.07...2.2</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\text{Br}(\tau \to \mu \gamma)})</td>
<td>0.04...0.4</td>
<td>(\sim 2 \cdot 10^{-3})</td>
<td>0.06...0.1</td>
<td>0.06...2.2</td>
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<tr>
<td>(\frac{\text{Br}(\tau^- \to e^- \mu^+ \mu^-)}{\text{Br}(\tau \to e \gamma)})</td>
<td>0.04...0.3</td>
<td>(\sim 2 \cdot 10^{-3})</td>
<td>0.02...0.04</td>
<td>0.03...1.3</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^- \to \mu^- e^+ e^-)}{\text{Br}(\tau \to \mu \gamma)})</td>
<td>0.04...0.3</td>
<td>(\sim 1 \cdot 10^{-2})</td>
<td>(\sim 1 \cdot 10^{-2})</td>
<td>0.04...1.4</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^- \to e^- e^+ e^-)}{\text{Br}(\tau^- \to e^- \mu^+ \mu^-)})</td>
<td>0.8...2</td>
<td>(\sim 5)</td>
<td>0.3...0.5</td>
<td>1.5...2.3</td>
</tr>
<tr>
<td>(\frac{\text{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\text{Br}(\tau^- \to \mu^- e^+ e^-)})</td>
<td>0.7...1.6</td>
<td>(\sim 0.2)</td>
<td>5...10</td>
<td>1.4...1.7</td>
</tr>
<tr>
<td>(\frac{\text{R}(\mu \to e \gamma)}{\text{Br}(\mu \to e \gamma)})</td>
<td>(10^{-3} \ldots 10^2)</td>
<td>(\sim 5 \cdot 10^{-3})</td>
<td>0.08...0.15</td>
<td>(10^{-12} \ldots 26)</td>
</tr>
</tbody>
</table>

Disentangle the underlying dynamics of NP
3.7 Model discriminating of Spectra: $\tau \rightarrow \mu\pi\pi$

Very different distributions according to the final hadronic state!

NB: See also Dalitz plot analyses for $\tau \rightarrow \mu\mu\mu$

Celis, Cirigliano, E.P.’14

Dassinger et al.’07
4. Charged Lepton-Flavour Violation and Higgs Physics
4.1 Non standard LFV Higgs coupling

\[ \Delta L_y = -\frac{\lambda_{ij} \bar{f}^i_L f^j_R H}{\Delta^2} H^+ H \]

- High energy: LHC

- Low energy: D, S operators

In the SM:

\[ Y^h_{ij} = \frac{m_i}{v} \delta_{ij} \]

Hadronic part treated with perturbative QCD

Goudelis, Lebedev, Park’11
Davidson, Grenier’10
Harnik, Kopp, Zupan’12
Blankenburg, Ellis, Isidori’12
McKeen, Pospelov, Ritz’12
Arhrib, Cheng, Kong’12
4.1 Non standard LFV Higgs coupling

\[ \Delta L_y = -\frac{\lambda_{ij}}{\Lambda^2} \left( \bar{f}_L^i f_R^j H \right) H^+ H \]

- **High energy**: LHC

- **Low energy**: D, S, G operators

In the SM:

\[ Y_{ij}^{h\text{SM}} = \frac{m_i}{v} \delta_{ij} \]

Hadronic part treated with perturbative QCD

Reverse the process

Hadronic part treated with non-perturbative QCD

**References**

Goudelis, Lebedev, Park’11
Davidson, Grenier’10
Harnik, Kopp, Zupan’12
Blankenburg, Ellis, Isidori’12
McKeen, Pospelov, Ritz’12
Arhrib, Cheng, Kong’12
4.2 Constraints in the $\tau\mu$ sector

- At low energy

  $\tau \rightarrow \mu \pi \pi$

  Dominated by

  $\rho(770)$ (photon mediated)

  $f_0(980)$ (Higgs mediated)
4.3 Constraints in the $\tau\mu$ sector

- **Constraints from LE:**
  - $\tau \rightarrow \mu\gamma$: best constraints but loop level
  - $\tau \rightarrow \mu\pi\pi$: tree level diagrams
    - sensitive to UV completion of the theory
  - $\tau \rightarrow \mu\pi\pi$: robust handle on LFV

- **Constraints from HE:**
  - $LHC$ wins for $\tau\mu$!
  - Opposite situation for $\mu e$!
  - For LFV Higgs and nothing else: LHC bound

- Plots from *Harnik, Kopp, Zupan’12*
  - Updated by *CMS*
  - *Talk by S. Bressler*
4.4 Hint of New Physics in $h \rightarrow \tau \mu$?

CMS’14

See talk by S. Bressler
4.5 Interplay between LHC & Low Energy

- If real what type of NP?
- If $h \rightarrow \tau \mu$ due to loop corrections:
  - extra charged particles necessary
  - $\tau \rightarrow \mu \gamma$ too large
- $h \rightarrow \tau \mu$ possible to explain if extra scalar doublet: 2HDM of type III
- Constraints from $\tau \rightarrow \mu \gamma$ important! Belle II

Dorsner et al.’15
4.5 Interplay between LHC & Low Energy

- **2HDMs** with gauged $L_\mu - L_\tau$:
  - $Z'$, explain anomalies for
    - $h \rightarrow \tau \mu$
    - $B \rightarrow K^* \mu \mu$
    - $R_K = B \rightarrow K \mu \mu / B \rightarrow K \mu \mu$

- Constraints from $\tau \rightarrow 3\mu$
  - crucial $\Rightarrow$ Belle II, LHCb

- See also:
  - Aristizabal-Sierra & Vicente’14,
  - Lima et al’15,
  - Omhura, Senaha, Tobe ’15
5. CPV in tau decays
5.1 $\tau \to K\pi \nu_\tau$ CP violating asymmetry

- $A_Q = \frac{\Gamma(\tau^+ \to \pi^+ K^0_S \bar{\nu}_\tau) - \Gamma(\tau^- \to \pi^- K^0_S \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K^0_S \bar{\nu}_\tau) + \Gamma(\tau^- \to \pi^- K^0_S \nu_\tau)}$

$= |p|^2 - |q|^2 \approx (0.36 \pm 0.01)\%$ in the SM

Bigi & Sanda’05
Grossman & Nir’11

- Experimental measurement: $A_{Q_{\text{exp}}} = (-0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}})\%$ → $2.8\sigma$ from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

$A_D = \frac{\Gamma(D^+ \to \pi^+ K^0_S) - \Gamma(D^- \to \pi^- K^0_S)}{\Gamma(D^+ \to \pi^+ K^0_S) + \Gamma(D^- \to \pi^- K^0_S)} = (-0.54 \pm 0.14)\%$

Belle, Babar, CLOE, FOCUS
5.1 $\tau \rightarrow K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, $W_L$-$W_R$ mixings, leptoquarks, tensor interactions (Devi, Dhargyal, Sinha’14)?

- Problem with this measurement? It would be great to have other experimental measurements from Belle, BES III or Tau-Charm factory

- Measurement of the direct contribution of NP in the angular CP violating asymmetry done by CLEO and Belle

  *Belle does not see any asymmetry at the 0.2 - 0.3% level*
5.2 Three body CP asymmetries

- Ex: \( \tau \rightarrow K\pi\pi\nu_\tau \)

- A variety of CPV observables can be studied: 
  \( \tau \rightarrow K\pi\pi\nu_\tau, \tau \rightarrow \pi\pi\pi\nu_\tau \) rate, angular asymmetries, triple products,....

Same principle as in charm, see Bevan’15

Difficulty: Treatment of the hadronic part
Hadronic final state interactions have to be taken into account!
Disentangle weak and strong phases

- More form factors, more asymmetries to build but same principles as for 2 bodies
6. Conclusion and outlook
6.1 Conclusion

• Hadronic $\tau$-decays very interesting to study
  – Very precise determination of $\alpha_s$
    But error assignment and treatment of the NP part and new data needed
  – Extraction of $V_{us}$: the $\tau$ could give a very precise determination of $V_{us}$ but difference between inclusive/exclusive modes:
    Data normalization, unmeasured modes? New Physics?

• Charged LFV are a very important probe of new physics
  – Extremely small SM rates
  – Experimental results at low energy are very precise $\Rightarrow$ very high scale sensitivity
  – Excellent model discriminating tools:
    ➢ BRs
    ➢ Decay distributions
    *Hadronic decays* such as $\tau \rightarrow \mu \pi \pi$ important!

• CPV asymmetry in $\tau \rightarrow K\pi\nu_\tau$
  BaBar result does not agree with SM expectation (2.8$\sigma$)
6.1 Conclusion

- Hadronic $\tau$-decays very interesting to study
  - Very precise determination of $\alpha_s$
  - Extraction of $V_{us}$

- Charged LFV are a very important probe of new physics

- CPV asymmetry in $\tau \rightarrow K\pi\nu_{\tau}$
  BaBar result does not agree with SM expectation (2.8σ)

- Several topics extremely interesting to study that I did not address:
  - Lepton universality tests, Michel parameters...
  - EDM and g-2 of the tau

- A lot of very interesting physics remains to be done in the tau sector!
6.2 Prospects: Belle II Theory Interface Platform

- Initiative to coordinate a *joint theory-experiment* effort to study the potential impacts of the Belle II program

- *Tau, EW and low multiplicity working group*

- Meetings twice a year until 2016 gathering theory experts and Belle II members

- Next meeting: *October 28-29, 2015* @ KEK, merged with KEK-FF meeting (October 26-27)

- Visit: [https://belle2.cc.kek.jp/~twiki/bin/view/B2TiP](https://belle2.cc.kek.jp/~twiki/bin/view/B2TiP)
7. Back-up
Comparison with Other Determinations

- Maltman, Yavin, PRD78 (2008) 094020, [0807.0650]
- Menke, 0904.1796
- Caprini, Fischer, EPJC64 (2009) 35, [0906.5211]
- Cvetic, Loewe, Martinez, Valenzuela, PRD82 (2010) 093007, [1005.4444]
- Boito et al., PRD84 (2011) 113006, [1110.1127]; PRD85 (2012) 093015, [1203.3146]
- Beneke, Boito, Jamin, JHEP 1301 (2013) 125, [1210.8038]

* experimental uncertainty when available is shown as inner error bar

Zhiqing Zhang (zhang@lal.in2p3.fr, LAL, Orsay)
2.4 Operator Product Expansion

\[ R_{\tau,V+A}(s_0) = N_C S_{EW} |V_{ud}|^2 (1 + \delta_p + \delta_{NP}) \]

\[ S_{EW} = 1.0201(3) \]

Marciano & Sirlin’88, Braaten & Li’90, Erler’04

Perturbative part \((m_q=0)\)

\[-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0}^{\infty} K_n \left( \frac{\alpha_s(-s)}{\pi} \right)^n \]

\[ K_0 = K_1 = 1, \quad K_2 = 1.63982, \quad K_3 = 6.37101 \]

\[ K_4 = 49.07570 \quad \text{Baikov, Chetyrkin, Kühn’08} \]

\[ \delta_p = \sum_{n=1} K_n A^n(\alpha_s) = a_\tau + 5.20 \ a_\tau^2 + 26 \ a_\tau^3 + 127 \ a_\tau^4 + \ldots \]

with \( A^n(\alpha_s) = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left( 1 - 2 \frac{s}{m_\tau^2} + 2 \frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right) \left( \frac{\alpha_s(-s)}{\pi} \right)^n \)

\[ a_\tau = \frac{\alpha_s(m_\tau)}{\pi} \]

\[ \delta_p \approx 20\% \quad (\delta_p = 0.2066 \pm 0.0070) \quad \text{Davier et al’08} \]
CIPT vs. FOPT

\[ -s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0}^{\infty} K_n \left( \frac{\alpha_s(-s)}{\pi} \right)^n \]

\[ A''(\alpha_s) = \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} \left( 1 - 2x + 2x^3 - x^4 \right) \left( \frac{\alpha_s(-xm^2_\tau)}{\pi} \right)^n = a_\tau + ... \]

\[ \delta_p = \sum_{n=1}^{\infty} K_n A^n(\alpha_s) = \sum_{n=0}^{\infty} r_n \left( \frac{\alpha_s(-s)}{\pi} \right)^n \]

\[ r_n = K_n + g_n \]

\[ a_\tau = \frac{\alpha_s(m_\tau)}{\pi} \]

<table>
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<td>26.3659</td>
<td>127.079</td>
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</table>

The dominant corrections come from the contour integration

**Large running of \[\alpha_s\] along the circle \[\alpha = m_\tau^2 e^{i\phi} \quad \phi \in [0, 2\pi]\]**

Pich Tau’10

LeDiberder & Pich’’92
Perturbative \((m_q=0)\)

\[
-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \left( \frac{\alpha_s(-s)}{\pi} \right)^n
\]

\[K_0 = K_1 = 1, \quad K_2 = 1.63982, \quad K_3 = 6.37101, \quad K_4 = 49.07570\]  

Baikov-Chetyrkin-Kühn '08

\[
\delta_p = \sum_{n=1} K_n A^{(n)}(\alpha_s) = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \cdots
\]

Le Diberder- Pich '92

\[
A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} \left( 1 - 2x + 2x^3 - x^4 \right) \left( \frac{\alpha_s(-s)}{\pi} \right)^n = a_\tau^n + \cdots ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi
\]

Power Corrections

\[
\Pi_{\text{OPE}}^{(0+1)}(s) \approx \frac{1}{4\pi^2} \sum_{n\geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-s)^n}
\]

Braaten-Narison-Pich '92

\[
\delta_{\text{NP}} \approx \frac{-1}{2\pi i} \oint_{|x|=1} dx \left( 1 - 3x^2 + 2x^3 \right) \sum_{n\geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-x m_\tau^2)^n} = -3 \frac{C_6 \langle O_6 \rangle}{m_\tau^6} - 2 \frac{C_8 \langle O_8 \rangle}{m_\tau^8}
\]

Suppressed by \(m_\tau^6\)  

[additional chiral suppression in \(C_6 \langle O_6 \rangle^{V+A}\)]
Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n a(-s)^n$$

$$\delta_p = \sum_{n=1} K_n A^{(n)}(\alpha_s) = \sum_{n=0} r_n a_\tau^n$$

CIPT  FOPT

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4) a_\tau (-x m_\tau^2)^n = a_\tau^n + \cdots; \quad a_\tau \equiv \alpha_s(m_\tau) / \pi$$

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The dominant corrections come from the contour integration

Le Diberder- Pich 1992

Large running of $\alpha_s$ along the circle $s = m_\tau^2 e^{i\phi}, \quad \phi \in [0, 2\pi]$
3.4.4 Determination of the form factors: $\Gamma_\pi(s), \Delta_\pi(s), \theta_\pi(s)$

- No experimental data for the other FFs up to $\sqrt{s}\sim 1.4$ GeV
  - Coupled channel analysis
    - Donoghue, Gasser, Leutwyler’90
    - Moussallam’99
    - Daub et al’13
  - Inputs: $I=0$, S-wave $\pi\pi$ and KK data

- Unitarity:
  \[ \text{disc} \left[ \begin{array}{c} \pi \\ \pi \end{array} \right] = \pi \pi + K \bar{K} \]
  \[ \text{Im} F_n(s) = \sum_{m=1}^{2} T_{nm}^*(s) \sigma_m(s) F_m(s) \]
  \[ n = \pi\pi, K\bar{K} \]
3.4.4 Determination of the form factors: $\Gamma_\pi(s), \Delta_\pi(s), \theta_\pi(s)$

- Inputs: $\pi\pi \rightarrow \pi\pi, KK$

A large number of theoretical analyses (Descotes-Genon et al’01, Kaminsky et al’01, Buttiker et al’03, Garcia-Martin et al’09, Colangelo et al.’11) and all agree

- 3 inputs: $\delta_\pi(s), \delta_K(s), \eta$ from B. Moussallam → reconstruct $T$ matrix
3.4.4 Determination of the form factors: $\Gamma_\pi(s), \Delta_\pi(s), \theta_\pi(s)$

- General solution:

$$
\begin{pmatrix}
F_\pi(s) \\
\frac{2}{\sqrt{3}} F_K(s)
\end{pmatrix}
= 
\begin{pmatrix}
C_1(s) & D_1(s) \\
C_2(s) & D_2(s)
\end{pmatrix}
\begin{pmatrix}
P_F(s) \\
Q_F(s)
\end{pmatrix}
$$

- Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

$$\Im X_n^{(N+1)}(s) = \sum_{m=1}^{2} \Re \{ T^*_n \sigma_m(s) X_m^{(N)} \}$$

$$\Re X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m^2_\pi}^{\infty} \frac{ds'}{s' - s} \Im X_n^{(N+1)}$$
Determination of the polynomial

- General solution

\[
\begin{pmatrix}
F_\pi(s) \\
\frac{2}{\sqrt{3}} F_K(s)
\end{pmatrix}
= 
\begin{pmatrix}
C_1(s) & D_1(s) \\
C_2(s) & D_2(s)
\end{pmatrix}
\begin{pmatrix}
P_F(s) \\
Q_F(s)
\end{pmatrix}
\]

- Fix the polynomial with requiring \( F_p(s) \rightarrow 1/s \) (Brodsky & Lepage) + ChPT:

Feynman-Hellmann theorem:

\[
\Gamma_P(0) = \left( m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2
\]

\[
\Delta_P(0) = \left( m_s \frac{\partial}{\partial m_s} \right) M_P^2
\]

- At LO in ChPT:

\[
M_{\pi^+}^2 = (m_u + m_d) B_0 + O(m^2)
\]

\[
M_{K^+}^2 = (m_u + m_s) B_0 + O(m^2)
\]

\[
M_{K^0}^2 = (m_d + m_s) B_0 + O(m^2)
\]
Determination of the polynomial

- General solution

\[
\begin{pmatrix}
F_\pi(s) \\
\frac{2}{\sqrt{3}} F_K(s)
\end{pmatrix} =
\begin{pmatrix}
C_1(s) & D_1(s) \\
C_2(s) & D_2(s)
\end{pmatrix}
\begin{pmatrix}
P_F(s) \\
Q_F(s)
\end{pmatrix}
\]

- At LO in ChPT:

\[
\begin{align*}
M_{\pi^+}^2 &= (m_u + m_d) B_0 + O(m^2) \\
M_{K^+}^2 &= (m_u + m_s) B_0 + O(m^2) \\
M_{K^0}^2 &= (m_d + m_s) B_0 + O(m^2)
\end{align*}
\]

\[
\begin{align*}
P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\
Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\
P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\
Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2\right) + \cdots
\end{align*}
\]

- Problem: large corrections in the case of the kaons! Use lattice QCD to determine the SU(3) LECs

\[
\begin{align*}
\Gamma_K(0) &= (0.5 \pm 0.1) M_\pi^2 \\
\Delta_K(0) &= 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2 M_\pi^2\right)
\end{align*}
\]

Dreiner, Hanart, Kubis, Meissner’13
Bernard, Descotes-Genon, Toucas’12
Determination of the polynomial

- General solution

\[
\begin{pmatrix}
F_\pi(s) \\
\frac{2}{\sqrt{3}} F_K(s)
\end{pmatrix} =
\begin{pmatrix}
C_1(s) & D_1(s) \\
C_2(s) & D_2(s)
\end{pmatrix}
\begin{pmatrix}
P_F(s) \\
Q_F(s)
\end{pmatrix}
\]

- For $\theta_p$ enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

  Relax the constraints and match to ChPT

\[
\begin{align*}
P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s \\
Q_\theta(s) &= \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s
\end{align*}
\]
Dispersion relations:
Model-independent method, based on first principles that extrapolates ChPT based on data
• ChPT, EFT only valid at low energy for $E = \infty$. 

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3.4.3 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements
    \[ e^+ e^- \rightarrow \pi^+ \pi^- \text{ and } \tau^- \rightarrow \pi^0 \pi^- \nu_\tau \] (isospin rotation)
  - Theoretically: Dispersive parametrization for $F_V(s)$
    \[
    F_V(s) = \exp \left[ \frac{\lambda_V}{m^2_\pi} \frac{s}{m^2_\pi} + \frac{1}{2} \left( \frac{\lambda''_V - \lambda'^2_V}{m^2_\pi} \right) \left( \frac{s}{m^2_\pi} \right)^2 + \frac{s^3}{\pi} \int_{4m^2_\pi}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s' - s - i\varepsilon)} \right]
    \]
    
    Extracted from a model including 3 resonances $\rho(770)$, $\rho'(1465)$ and $\rho''(1700)$ fitted to the data
  
- Subtraction polynomial + phase determined from a fit to the Belle data
  \[ \tau^- \rightarrow \pi^0 \pi^- \nu_\tau \]
3.4.3 Determination of $F_V(s)$

Determination of $F_V(s)$ thanks to precise measurements from Belle!
3.5 Results

Bound:

\[ \sqrt{|Y_{\mu\tau}^h|^2 + |Y_{\tau\mu}^h|^2} \leq 0.13 \]

Less stringent but more robust handle on LFV Higgs couplings

| Process          | (BR $\times 10^8$) 90% CL | $\sqrt{|Y_{\mu\tau}^h|^2 + |Y_{\tau\mu}^h|^2}$ | Operator(s)          |
|------------------|---------------------------|---------------------------------------------|----------------------|
| $\tau \to \mu\gamma$ | $< 4.4$ [88]              | $< 0.016$                                   | Dipole               |
| $\tau \to \mu\mu\mu$ | $< 2.1$ [89]              | $< 0.24$                                    | Dipole               |
| $\tau \to \mu\pi^+\pi^-$ | $< 2.1$ [86]              | $< 0.13$                                    | Scalar, Gluon, Dipole|
| $\tau \to \mu\rho$    | $< 1.2$ [85]               | $< 0.13$                                    | Scalar, Gluon, Dipole|
| $\tau \to \mu\pi^0\pi^0$ | $< 1.4 \times 10^3$ [87] | $< 6.3$                                     | Scalar, Gluon         |
2.5 Model discriminating power of Tau processes

- Depending on the UV model different correlations between the BRs

Interesting to study to determine the underlying dynamics of NP

*Blankenburg et al.’12*

*Buras et al.’10*
3.5 What if $\tau \rightarrow \mu(e)\pi\pi$ observed?

Reinterpreting Celis, Cirigliano, E.P’14

- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!
- $Y_{u,d,s}$ poorly bounded
- For $Y_{u,d,s}$ at their SM values:
  \[ Br(\tau \rightarrow \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, \quad Br(\tau \rightarrow \mu\pi^0\pi^0) < 4.6 \times 10^{-12} \]
  \[ Br(\tau \rightarrow e\pi^+\pi^-) < 2.3 \times 10^{-10}, \quad Br(\tau \rightarrow e\pi^0\pi^0) < 6.9 \times 10^{-11} \]
- But for $Y_{u,d,s}$ at their upper bound:
  \[ Br(\tau \rightarrow \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, \quad Br(\tau \rightarrow \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \]
  \[ Br(\tau \rightarrow e\pi^+\pi^-) < 4.3 \times 10^{-7}, \quad Br(\tau \rightarrow e\pi^0\pi^0) < 2.1 \times 10^{-7} \]
  below present experimental limits!
- If discovered among other things upper limit on $Y_{u,d,s}$!

Interplay between high-energy and low-energy constraints!
3.6 Prospects: $\tau$ strange $\text{Brs}$

- Experimental measurements of the strange spectral functions not very precise

- Before B-factories:
  - Smaller $\tau \rightarrow K$ branching ratios

  $$R^S_\tau \big|_{\text{old}} = 0.1686(47)$$

- With B-factories new measurements:
  - Smaller $R_{\tau,S}$
  - Smaller $V_{us}$

  $$V_{us} \big|_{\text{new}} = 0.2176 \pm 0.0019 \exp \pm 0.0010_{\text{th}}$$
  $$V_{us} \big|_{\text{old}} = 0.2214 \pm 0.0031 \exp \pm 0.0010_{\text{th}}$$

\[ \text{New measurements are needed!} \]
3.6 Prospects: $\tau$ strange $B$S

- **PDG 2014**: « Eighteen of the 20 $B$-factory branching fraction measurements are smaller than the non-$B$-factory values. The average normalized difference between the two sets of measurements is -1.30 » (-1.41 for the 11 Belle measurements and -1.24 for the 9 BaBar measurements)

- Measured modes by the 2 B factories:

<table>
<thead>
<tr>
<th>Mode</th>
<th>BaBar – Belle Normalized Difference ($#\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-\pi^+\pi^-\nu_\tau$ (ex. $K^0$)</td>
<td>+1.4</td>
</tr>
<tr>
<td>$K^-\pi^+\pi^-\nu_\tau$ (ex. $K^0$)</td>
<td>-2.9</td>
</tr>
<tr>
<td>$K^-K^+\pi^-\nu_\tau$</td>
<td>-2.9</td>
</tr>
<tr>
<td>$K^-K^+K^-\nu_\tau$</td>
<td>-5.4</td>
</tr>
<tr>
<td>$\eta K^-\nu_\tau$</td>
<td>-1.0</td>
</tr>
<tr>
<td>$\phi K^-\nu_\tau$</td>
<td>-1.3</td>
</tr>
</tbody>
</table>

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2.2 Experimental situation

• Observable studied

\[ R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^-\bar{\nu}_e)} \] and \[ \frac{dR_\tau}{ds} \]

• Decomposition as a function of observed and separated final states

\[ R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S} \]

\[ \nu_1/a_1[\tau^- \to V^-/A^-\nu_\tau] \propto \frac{\text{BR}[\tau^- \to V^-/A^-\nu_\tau]}{\text{BR}[\tau^- \to e^-\bar{\nu}_e\nu_\tau]} \times \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds} \frac{m^2}{(1-s/m^2)^2(1+2s/m^2)} \]

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