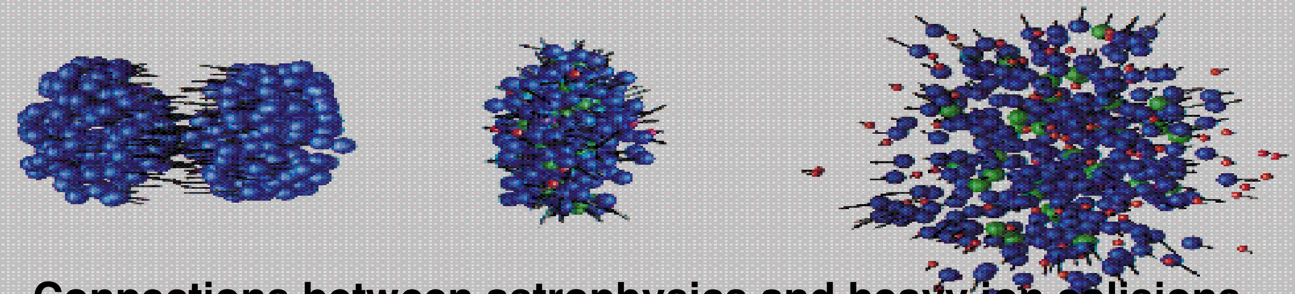
The Nuclear Equation-of-State and the Symmetry Energy



Connections between astrophysics and heavy ion colisions

Hermann Wolter, University of Munich, Germany

Science Day, Research Area G,
DFG Excellence Cluster *Origin and Structure of the Universe*
Garching, July 9, 2015

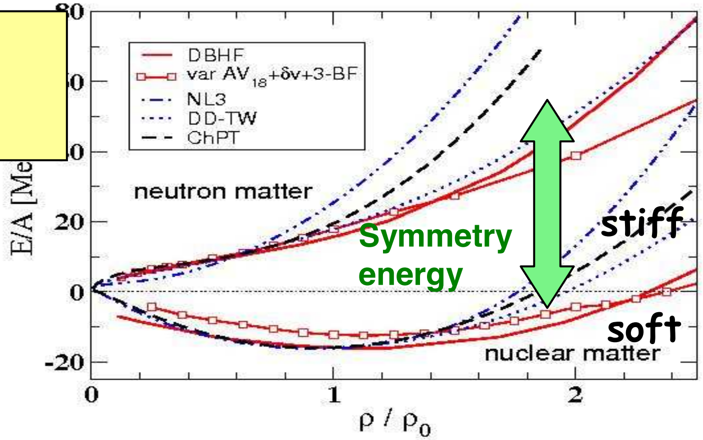
Equation-of-State and Symmetry Energy

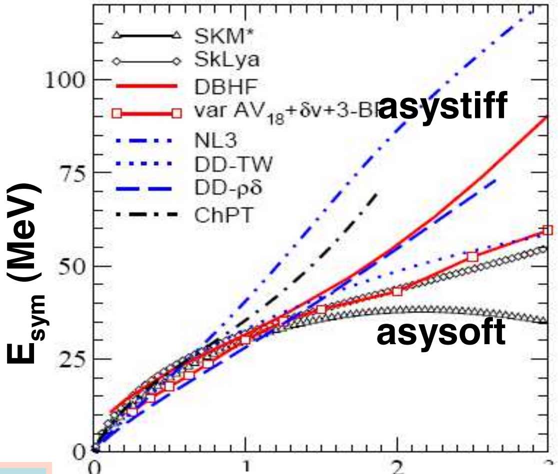
density-
asymmetry dep.
of nucl.matt.

$$E(\rho_B,\delta)/A = E_{nm}(\rho_B) + \boxed{E_{sym}(\rho_B)}\delta^2 + O(\delta^4) + \ldots \qquad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Many-Body calculations:

Rel, Brueckner
Variational
Rel. Mean field
Chiral perturb.

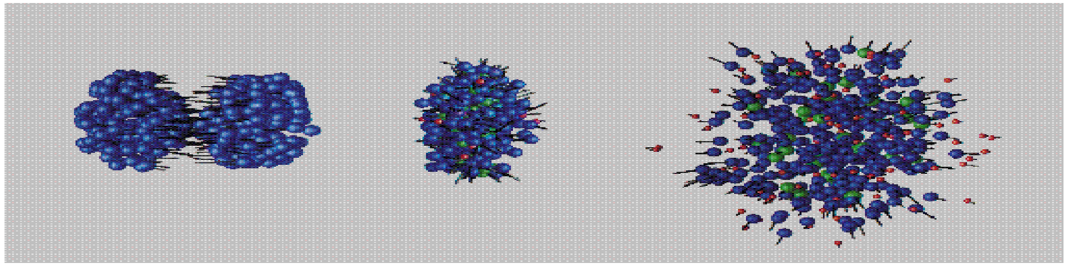


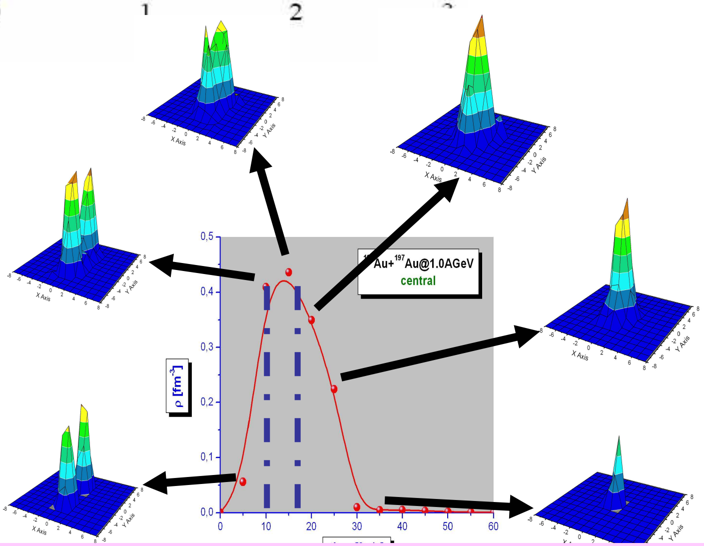


Fuchs, H.H. Wolter, EPJA 30(2006)5

Why is symmetry energy so uncertain??
→ Short range isovector tensor correlations; 3-body forces

Heavy ion collisions to investigate EoS in the laboratory



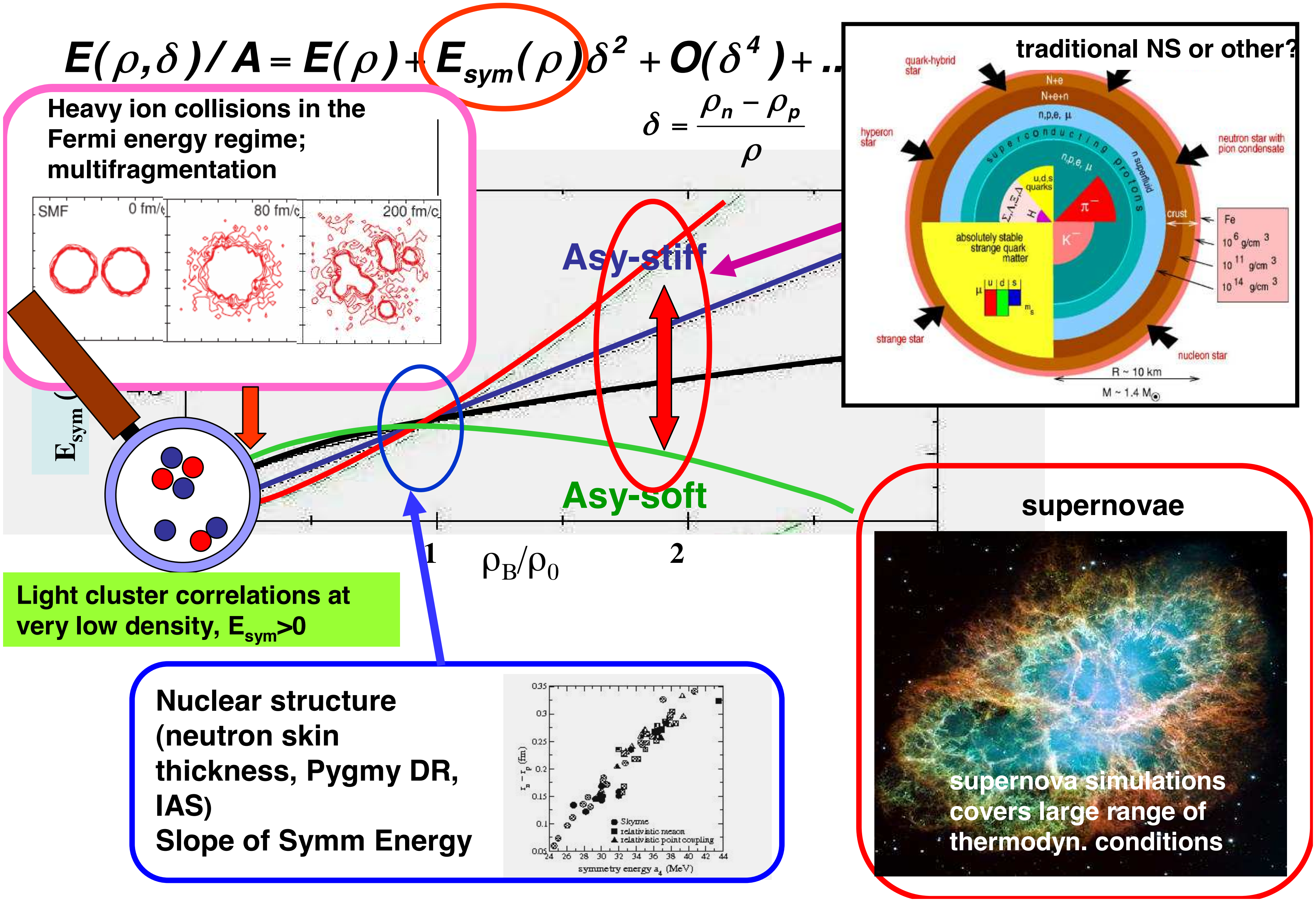


non-equilibrium, transport theory

Importance of the Nuclear Symmetry Energy in Nuclear and Astro-Physics

$$E(\rho,\delta)/A = E(\rho) + E_{sym}(\rho)\delta^2 + O(\delta^4) + \ldots$$

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$



Heavy ion collisions in the Fermi energy regime; multifragmentation

traditional NS or other?

Light cluster correlations at very low density, $E_{sym}>0$

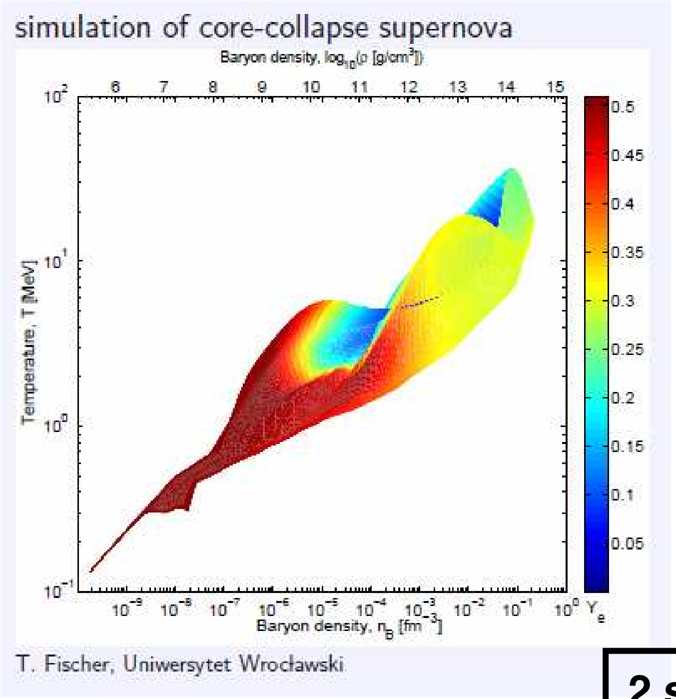
Nuclear structure (neutron skin thickness, Pygmy DR, IAS)
Slope of Symm Energy

supernovae

supernova simulations covers large range of thermodyn. conditions

Large range of thermodynamic conditions in astrophysical applications

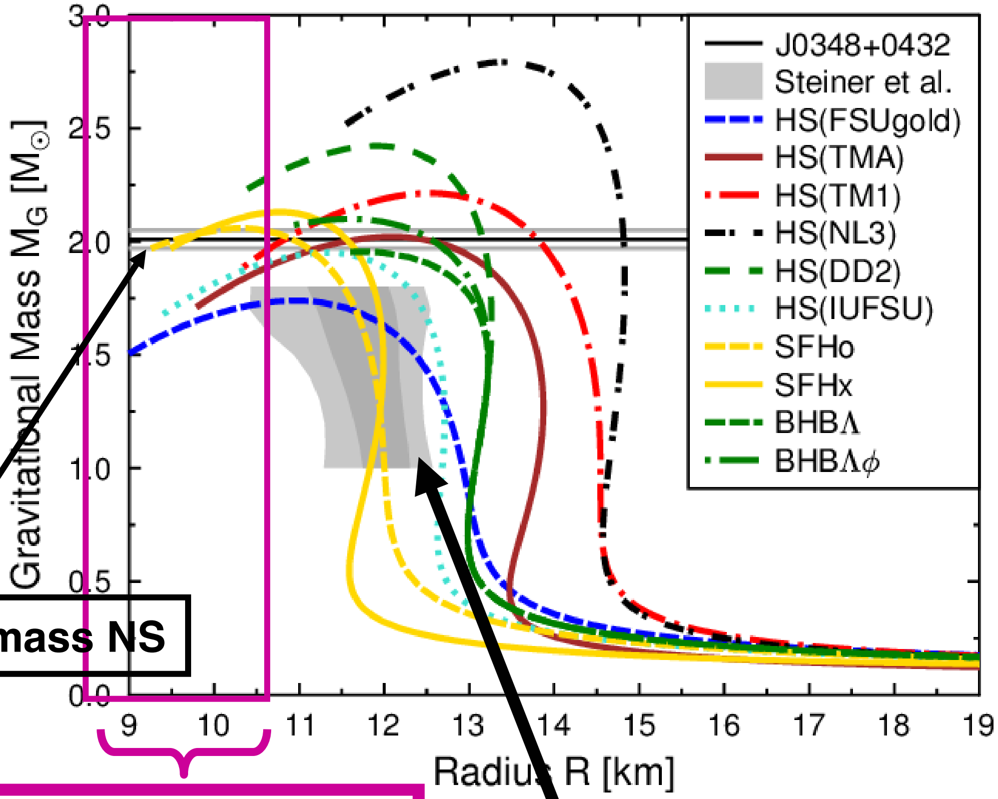
core collapse supernovae


simulation of core-collapse supernova

T. Fischer, Uniwersytet Wrocławski

$1/1000<\rho/\rho_0<2$-3
T=0-50 MeV
$0.1<x_p<0.5$

mass-radius relation of neutron stars determined uniquely for gives EoS



2 solar mass NS

analysis by S. Guillot, R. Rutledge,
APJ 772, 7G (2013)
R_{NS}=9.1±1.4km
(assume pure H atmospheres and identical radii for all NS)

analysis by A. Steiner, J. Latitmer, E. Brown, APJ Lett, 765, L5 (2013)
R_{NS}=10.4÷12.9 km
(Bayesian analysis of qLMXB‘s)

Recent reviews of the Nuclear Symmetry Energy


EPJ A

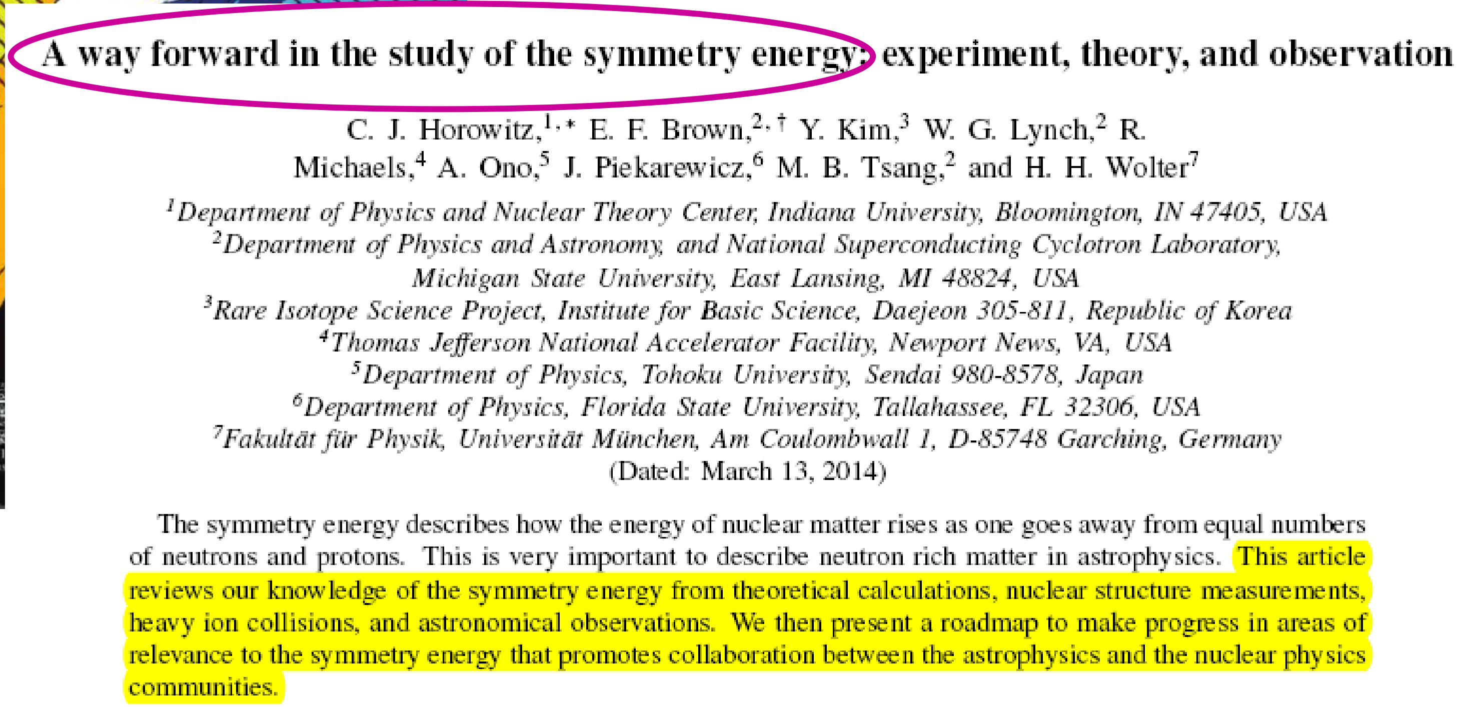
Recognized by European Physical Society

Hadrons and Nuclei

Topical Issue on Nuclear Symmetry Energy
edited by Bao-An Li, Àngels Ramos,
Giuseppe Verde and Isaac Vidaña

Vol. 50

J. Phys. G: Nucl. Part. Phys. 41 (2014) 093001


A way forward in the study of the symmetry energy: experiment, theory, and observation

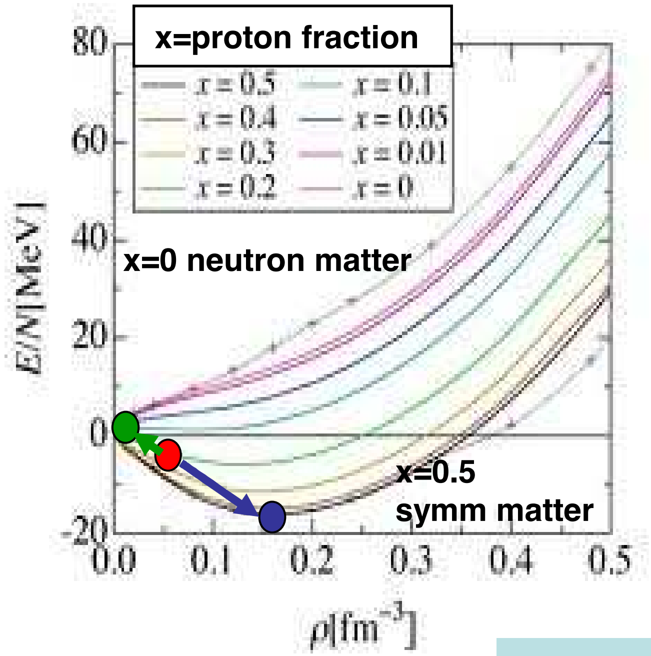
C. J. Horowitz,^{1,*} E. F. Brown,^{2,†} Y. Kim,^{3} W. G. Lynch,^{2} R. Michaels,^{4} A. Ono,^{5} J. Piekarewicz,^{6} M. B. Tsang,^{2} and H. H. Wolter^{7}

[1] *Department of Physics and Nuclear Theory Center, Indiana University, Bloomington, IN 47405, USA*
[2] *Department of Physics and Astronomy, and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA*
[3] *Rare Isotope Science Project, Institute for Basic Science, Daejeon 305-811, Republic of Korea*
[4] *Thomas Jefferson National Accelerator Facility, Newport News, VA, USA*
[5] *Department of Physics, Tohoku University, Sendai 980-8578, Japan*
[6] *Department of Physics, Florida State University, Tallahassee, FL 32306, USA*
[7] *Fakultät für Physik, Universität München, Am Coulombwall 1, D-85748 Garching, Germany*

(Dated: March 13, 2014)

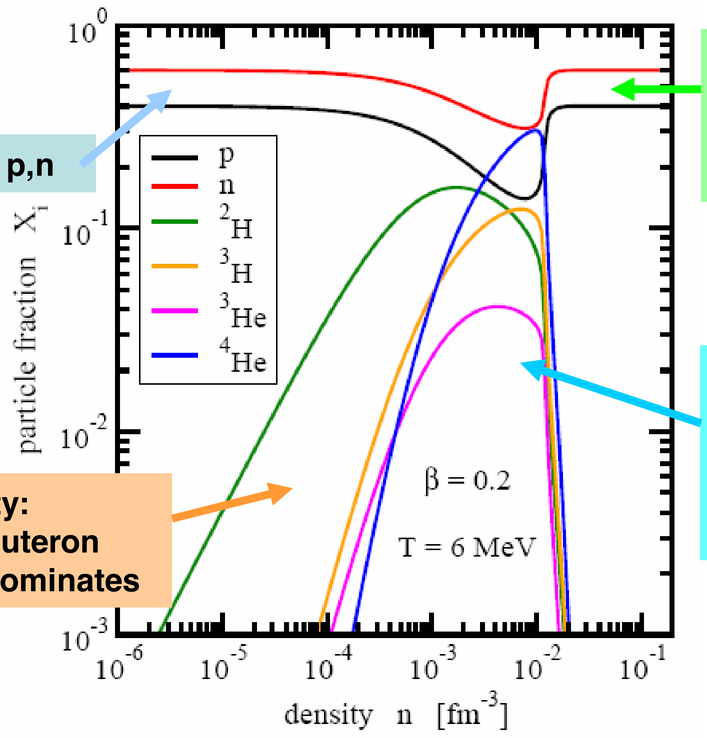
The symmetry energy describes how the energy of nuclear matter rises as one goes away from equal numbers of neutrons and protons. This is very important to describe neutron rich matter in astrophysics. This article reviews our knowledge of the symmetry energy from theoretical calculations, nuclear structure measurements, heavy ion collisions, and astronomical observations. We then present a roadmap to make progress in areas of relevance to the symmetry energy that promotes collaboration between the astrophysics and the nuclear physics communities.

Clustering of very dilute nuclear matter



decrease energy by inhomogeneity
→ fractionation into clusters at higher density
and neutron gas

composition as fct of density



very low density: p,n

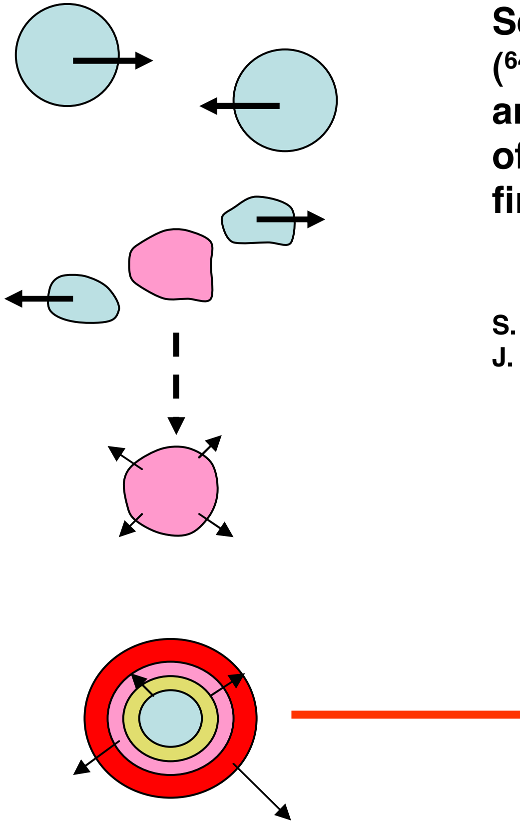
Mott density: clusters melt, homogeneous p,n matter;

here heavier nuclei (embedded into a gas) become important here

Increasing density: clusters arise: deuteron first, but then α dominates

Can this be cecked in heavy ion collisions? yes!

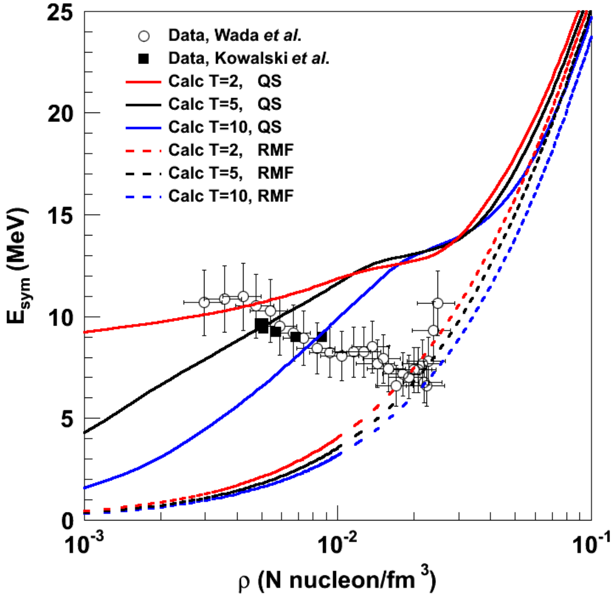
Investigation of warm, low density matter in heavy ion collisions

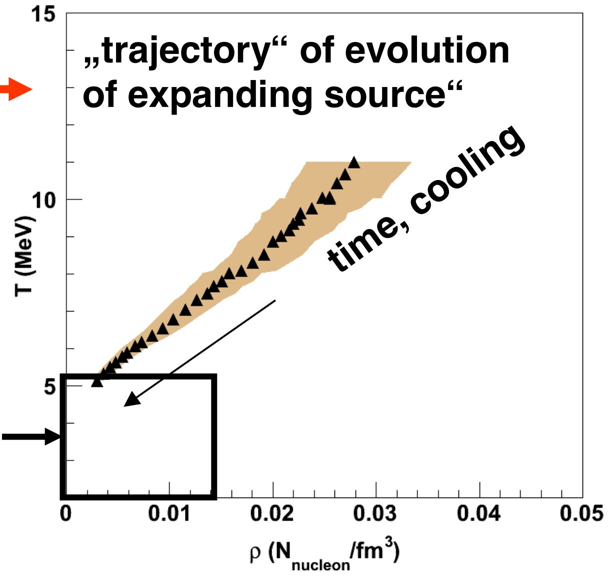


Semi-central heavy ion collisions, (^{64}Zn+^{92}Mo,^{197}Au at 35MeV/A) and time-resolved measurement of light fragments from decay of fireball

S. Kowalski, J. Natowitz, et al.,PRC75 014601 (2007)
J. Natowitz, G. Röpke,, … HHW, PRL 104, 202501 (2010)

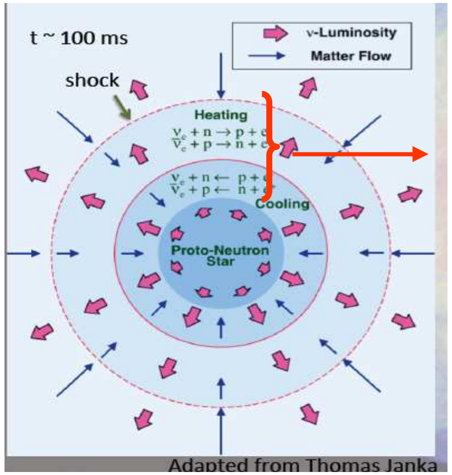
extracted symmetry energy in comp. with quantumstat. calculation of clusters in medium



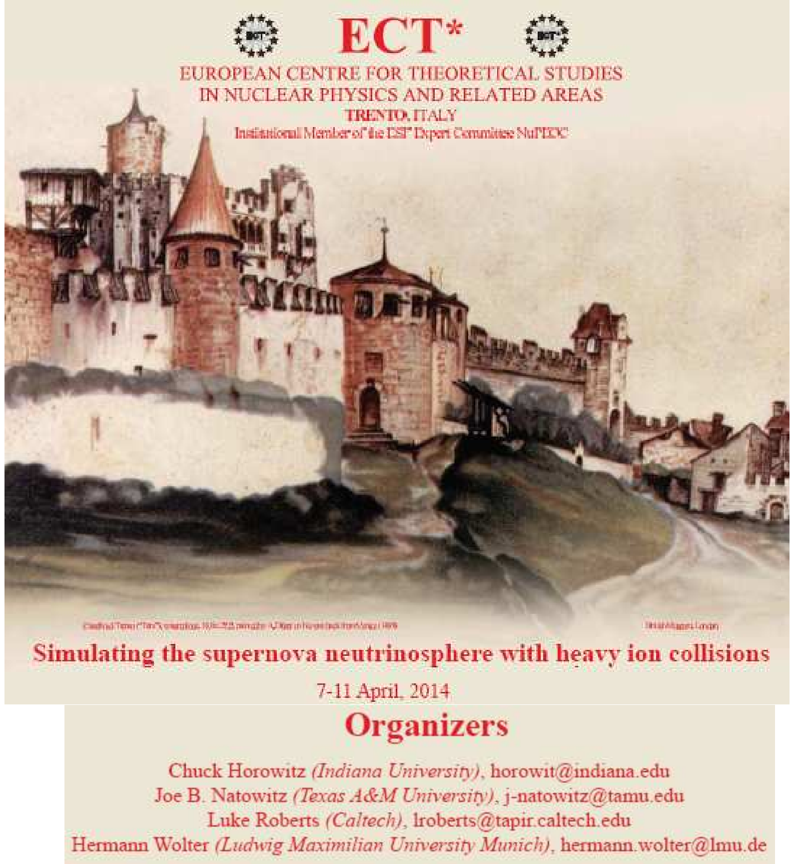


conditions of neutrinosphere:
densities 1/1000 to 1/10 ρ_o
temperature T=1-5 MeV
asymmetry Y_e=0.1 – 0.25

Results relevant for neutrino opacity
in ν-sphere in CC SNe
→ workshop in ECT*, Trento



ν−sphere



ECT*

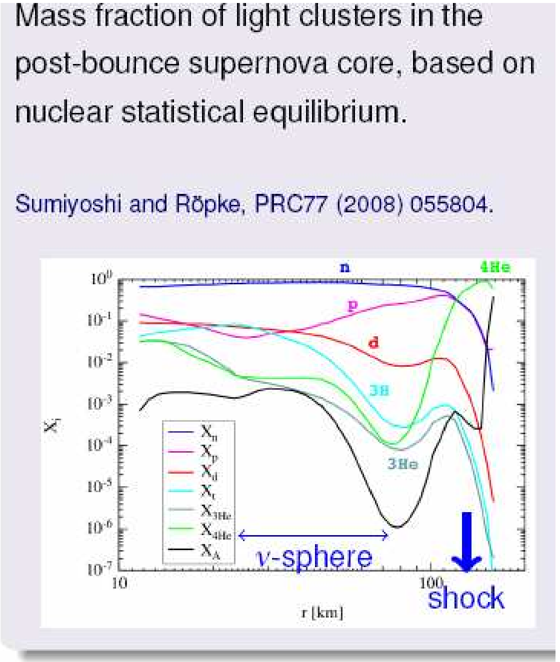
EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY
Institutional Member of the ESF Expert Committee NuPECC

Simulating the supernova neutrinosphere with heavy ion collisions

7-11 April, 2014

Organizers

Chuck Horowitz *(Indiana University)*, horowit@indiana.edu
Joe B. Natowitz *(Texas A&M University)*, j-natowitz@tamu.edu
Luke Roberts *(Caltech)*, lroberts@tapir.caltech.edu
Hermann Wolter *(Ludwig Maximilian University Munich)*, hermann.wolter@lmu.de



Mass fraction of light clusters in the post-bounce supernova core, based on nuclear statistical equilibrium.

Sumiyoshi and Röpke, PRC77 (2008) 055804.

ν-opacities important for ν-driven wind and r-process nucleosynthesis

Differential Scattering/Absorption Rate:

$$\frac{d\Gamma(E_1)}{d\cos\theta\; dq_0} = \frac{G_F^2}{4\pi^2}\,(E_1-q_0)^2\;\left[(1+\cos\theta)\;S_V^{\mathrm{RPA}}(q_0,q) + (3-\cos\theta)\;S_A^{\mathrm{RPA}}(q_0,q)\right]$$

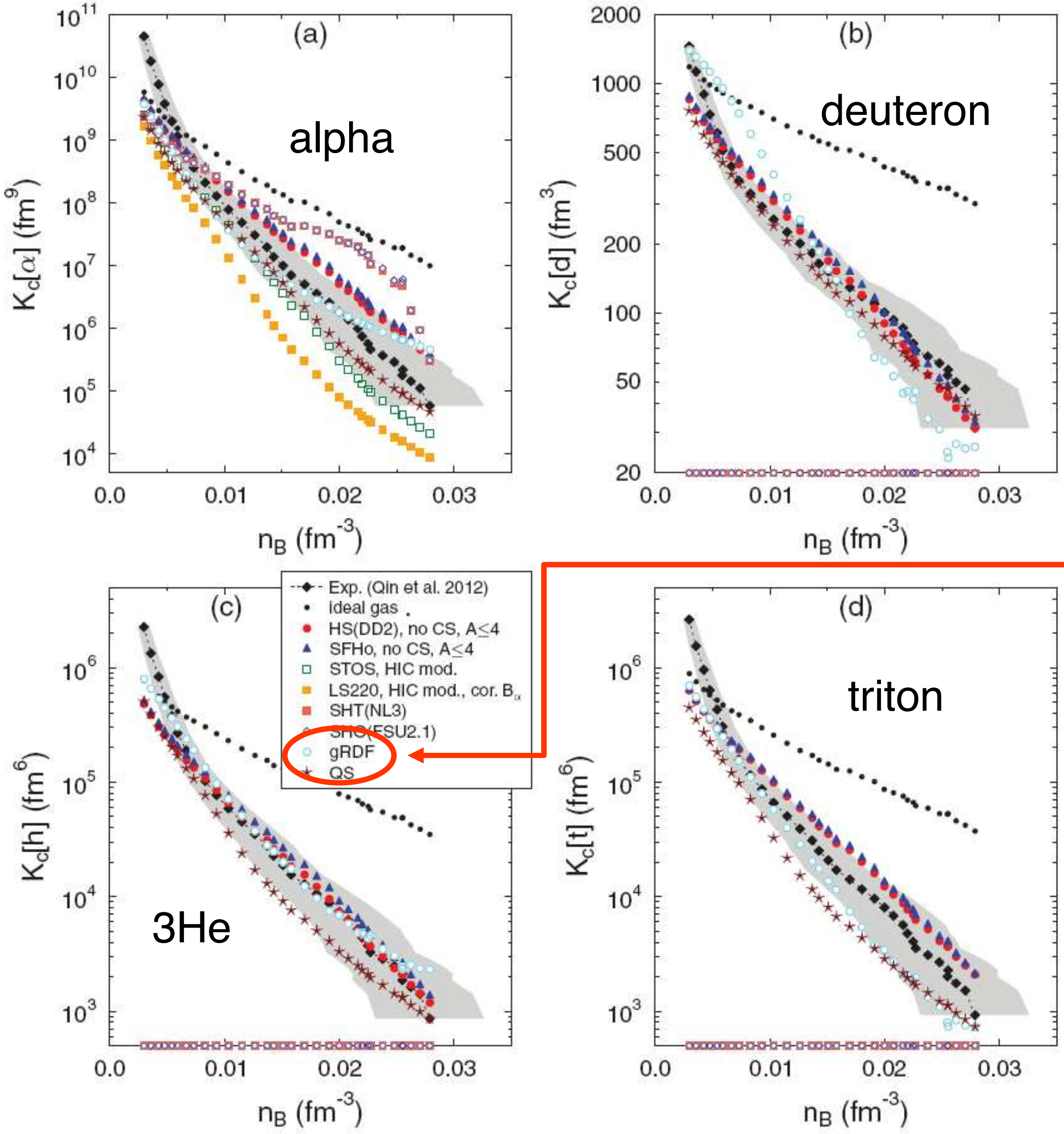
spectrum of density fluctuations

spectrum of spin fluctuations

direct interface with heavy ion physics,

Supernova ⟺ Femtonova (heavy ion collision)

Chemical Equilibrium Constants for light clusters in different clusterization models



comparison of equilibrium constants (rather than particle fractions)**, more robust**

$$K_c[i] = \frac{n_i}{n_n^{N_i} n_p^{Z_i}},$$

theories with medium modifications of light clusters work very well:
→QS (Quantumtat. Green fct. Röpke et al.),
→gRDF (gen. Rel. Dens. Funct., Typel etal,)
→many traditional SN EoS not so well

M.Hempel, et al., PRC 91 (2015)

Equation of State for Astrophysical Applications

range of variables

- temperature: $0.1\ \text{MeV} \leq T \leq 100\ \text{MeV} \Rightarrow$ 76 mesh points
- baryon density: $10^{-10}\ \text{fm}^{-3} \leq n_b \leq 1\ \text{fm}^{-3} \Rightarrow$ 251 mesh points
- hadronic charge fraction: $0.01 \leq Y_q \leq 0.60 \Rightarrow$ 60 mesh points

$\Rightarrow$ in total 1144560 mesh points

information on

- thermodynamic properties (pressure, entropy, energies, chemical potentials)
- chemical composition (nucleons, leptons, light and heavy clusters)
- microscopic quantities (mean-field potentials)

availability

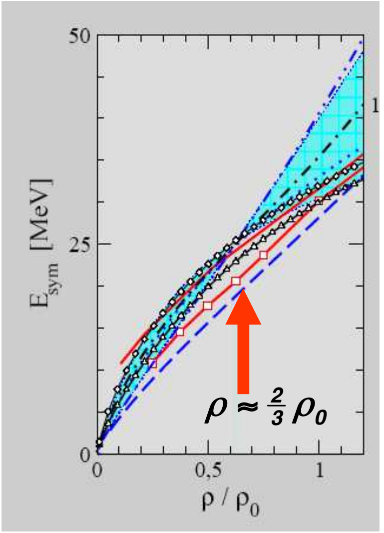
- data tables will be released on CompOSE website (`http://compose.obspm.fr`)

S. Typel (GSI), M. Oertel (LUTH Paris), T. Klähn (Wroclaw)

Correlation of parameters of Symmetry Energy and with experiment

e.g. expansion of Symmetry energy around saturation

$$E_{sym}(\rho) \equiv S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots$$

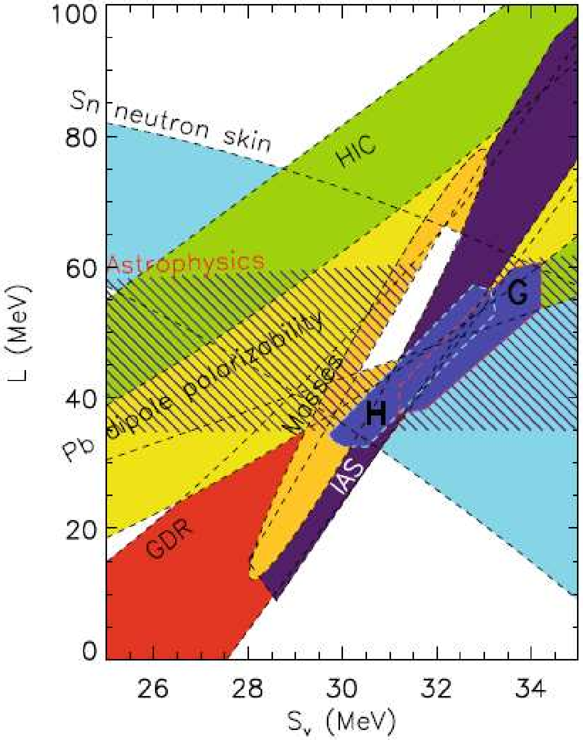


SE fitted to nuclear masses cross below saturation density, (average density of a finite nucleus)

→ induces a correlation between value and slope at ρ_0, eg. in lin. approx.

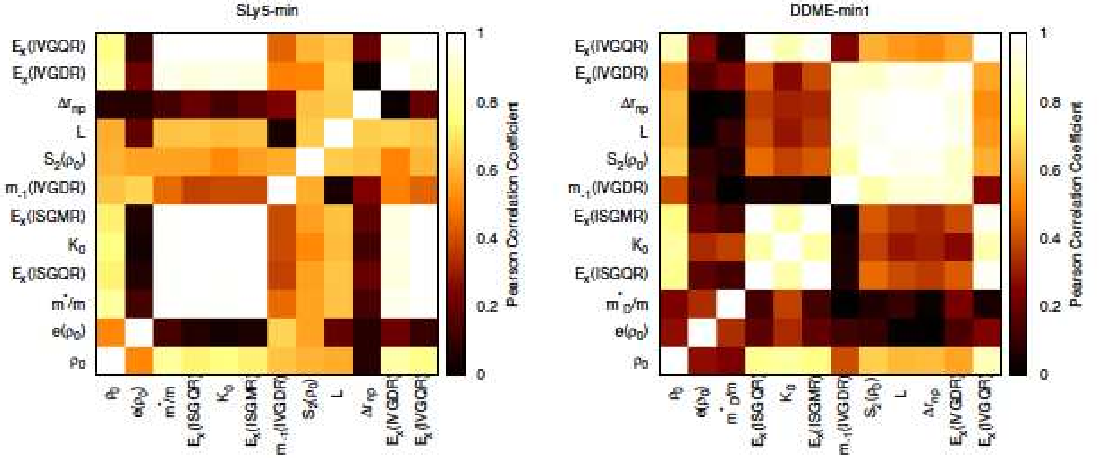
$$S(\rho \approx \tfrac{2}{3}\rho_0) = S_0 - \tfrac{1}{9}L$$

correlations derived from different observables may determine S_0,L



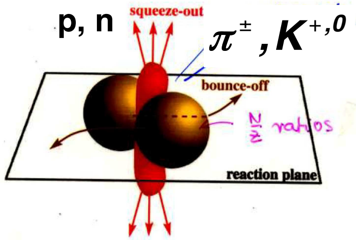
J.Lattimer, A. Steiner, EPJA 50, 40 (2014)

correlations between model parameters and between model parameters and observables in two models (SLy5, DDRMF, G. Colo, Nusym15)



Note: correlations are model dependent!

The Symmetry Energy at High Density



$\pi^{\pm}, K^{+,0}$ **Fourier analysis azimuthal distribution**

$$N(\Theta; y, p_t) = N_0(1 + \boxed{v_1}\cos\Theta + \boxed{v_2}\cos 2\Theta + \ldots)$$

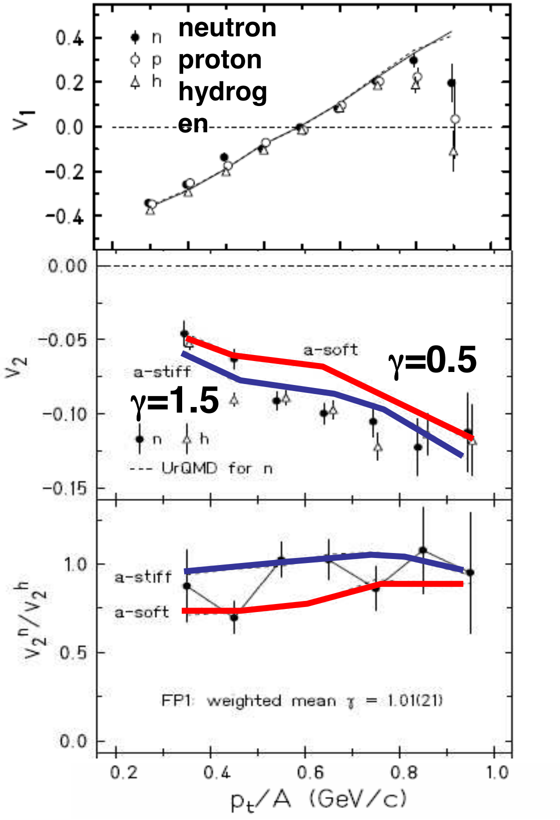
v_1: directed flow

v_2: Elliptic flow

Au+Au @ 400 AMeV, FOPI-LAND

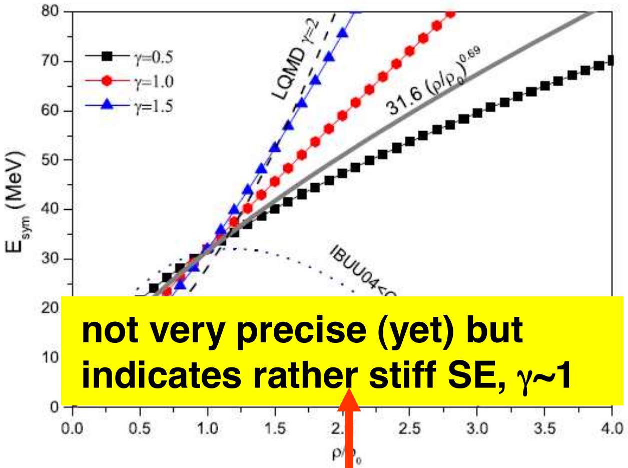
- Directed flow not very sensitive to SE (involves many different densities)

- Elliptic flow in this energy region probe of high density

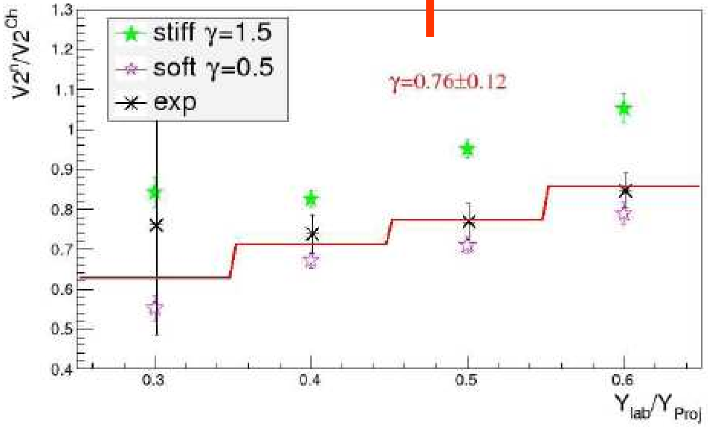


(Russotto, et al., PLB 697, 471 (11))

$$E_{sym}(\rho) = \tfrac{1}{3}\varepsilon_F\left(\frac{\rho}{\rho_0}\right)^{2/3} + C\left(\frac{\rho}{\rho_0}\right)^{\boxed{\gamma}}$$

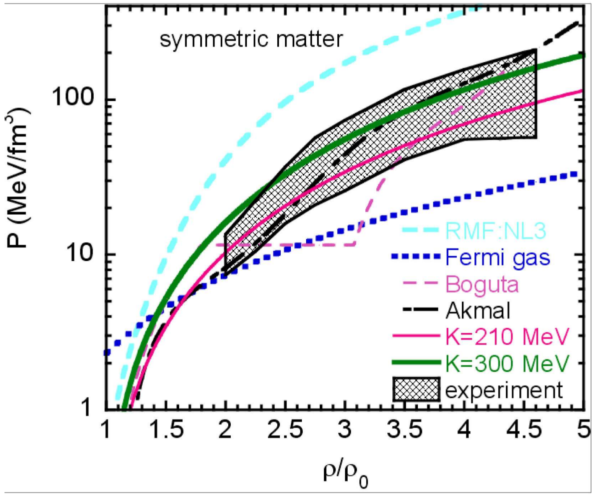


not very precise (yet) but indicates rather stiff SE, γ~1



preliminary result from new experiment ASY-EOS
(Russotto,NuSYM 2015, Krakow)

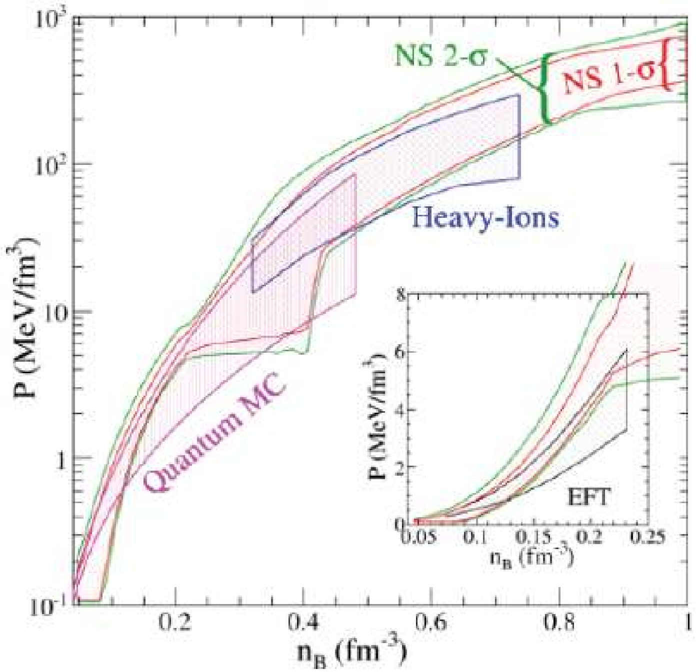
Constraints on EOS of symmetric nuclear matter

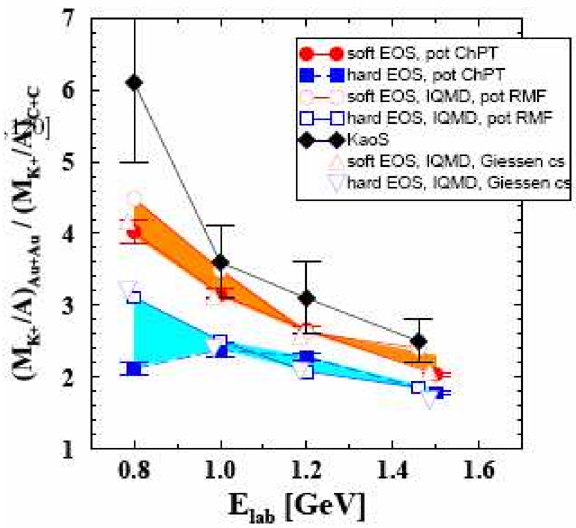


- pressure-density diagram
- analysis in a particular model, shaded area constraint in this model
- densities in the range from 2 – 4.5 ρ_0
- eliminates some extreme model

(P.Danielewicz, et al., Science 298(02)1592)

Limits on the EOS in β-equilibrium





ratio of K+ production in heavy (Au+Au, compression) relative to light system (C+C,small compression)

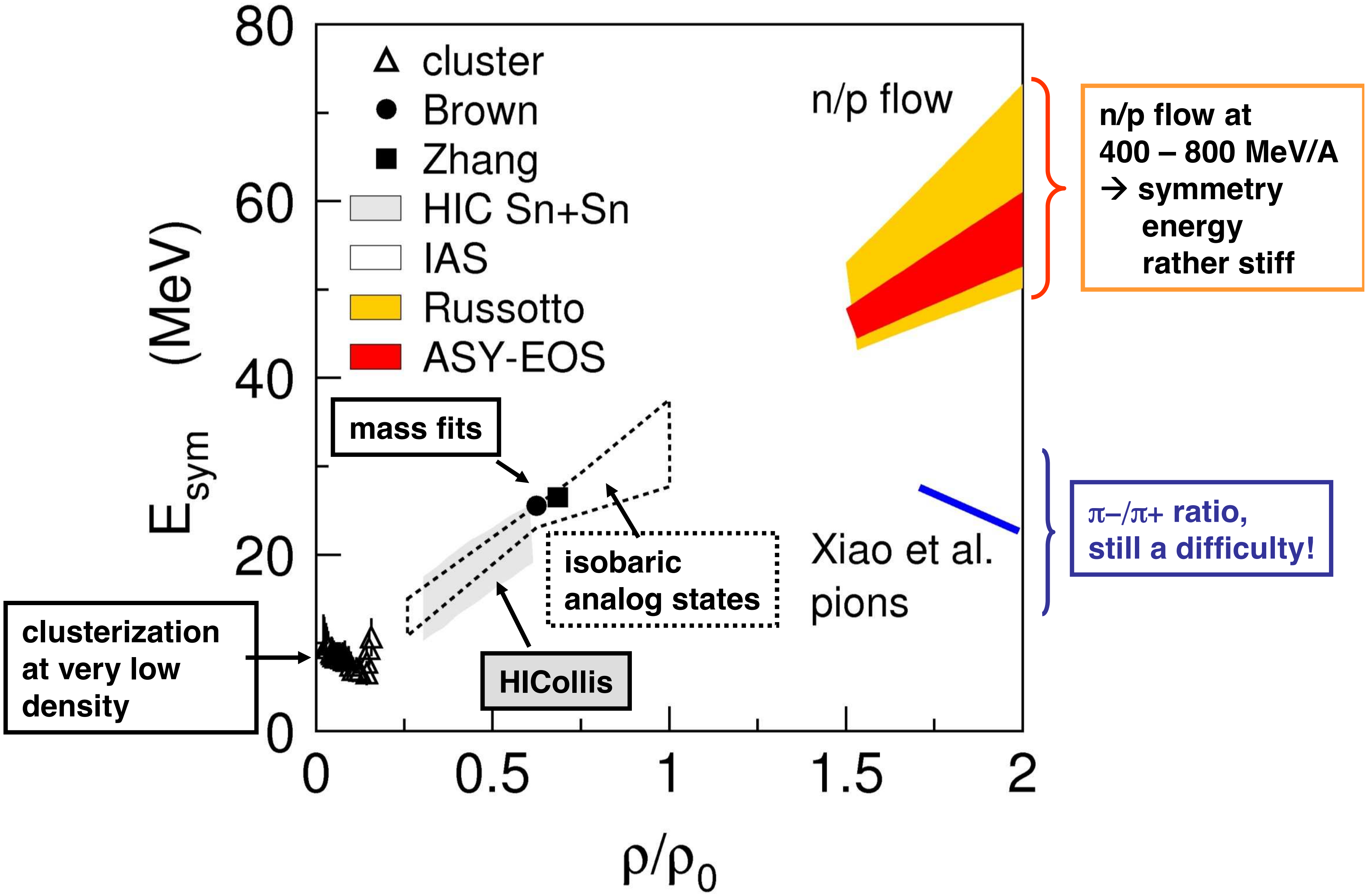
favors soft EOS

C. Fuchs, PPNP56(06)

constraints on EoS of symmetric nuclear matter from HIC, neutron stars, and microscopic calculations
→ seem to converge to soft EoS

A. Steiner, J. Lattimer, E.F.Brown, arXiv 1205.6871

Constraints on the Symmetry Energy



n/p flow at 400 – 800 MeV/A → symmetry energy rather stiff

π−/π+ ratio, still a difficulty!

SUMMARY:

Equation-of-State (EoS) of nuclear matter of interest in itself
and important imput for astrophysics (CC-SN, nucleosythesis, NS structure)

Investigation of EoS in the laboratory in Heavy Ion Collisions
(interpretation in complex transport models)

EoS of symmetric nuclear matter ($\rho_n=\rho_p$) fairly well determined,
but symmetry energy area of very active investigations experimentally
(new facilities) and theoretically:

→ constraints around ρ_0 rather stringent
→ cluster effects at very low densities, corresponding cond. of ν-sphere
→ few experiments for high density, biggest uncertainty

Thanks to my collaborators:

Heavy ion collisions:
Maria Colonna, Massimo Di Toro, Enzo Greco, Joseph Rizzo (Lab. Naz. del Sud, INFN, Catania),
Malgorzata Zielinska-Pfabe (Smith Coll. Mass, USA)
Theo Gaitanos (Univ. Thesaloniki), et al.,

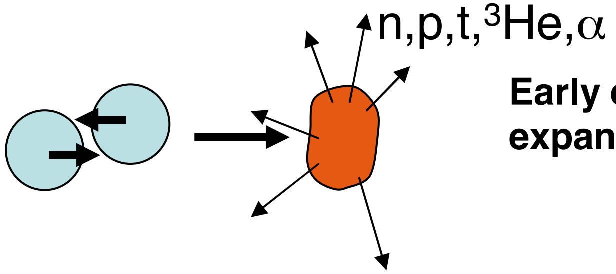
Clustering in dilute Matter, SN and NS EoS:
Stefan Typel (Navi, GSI)
Gerd Röpke (Univ. of Rostock)
David Blaschke, Thomas Klähn (Univ. of Wroclaw)

Discussions with Experiment:
Wolfgang Trautmann (GSI)
Betty Tsang, W, Lynch (MSU)
Abdou Chbihi (GANIL), and many other

and to → you for your attention

backup slides

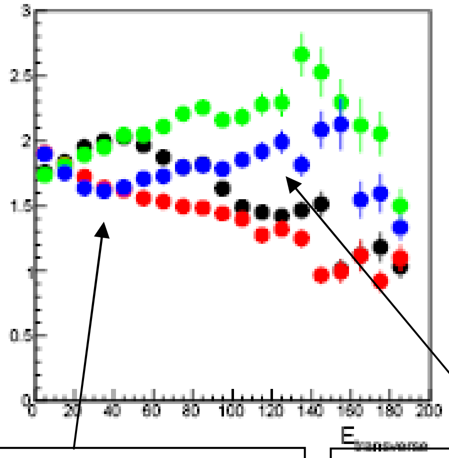
Example: Ratios of emitted pre-equilibrium particles in central collisions



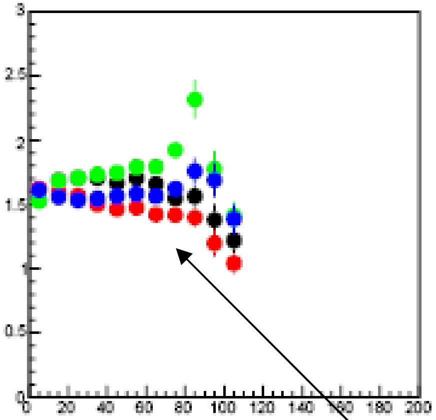
Early emitted Light Clusters reflect difference in potentials in expanded source, e.g. ratio Y(n)/Y(p).

^{136}Xe+^{124}Sn, 150 MeV

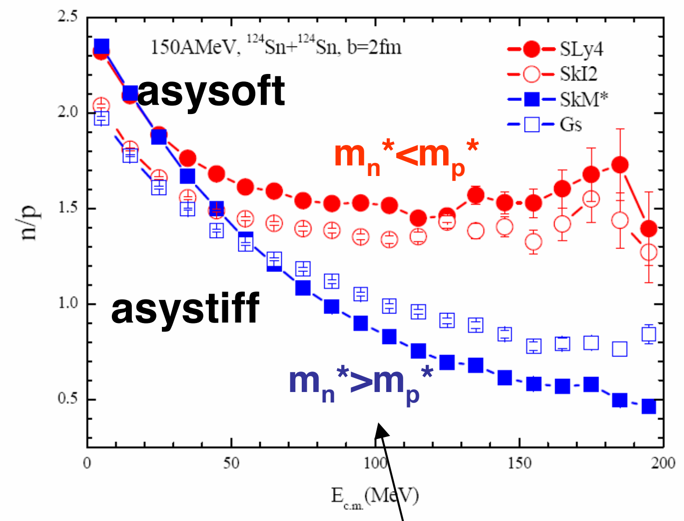
Y(n)/Y(p)



Y(t)/Y(^{3}He)



^{124}Sn+^{124}Sn, 150 MeV



Y. Zhang, M.B.Tsang,et al., PLB 732, 186 (2014)

Asy-EOS dominates for slow particles; asysoft has larger repulsion at lower densities

Effective mass dominates for fast particles; smaller eff. mass favors emission

Effect also exists for light clusters (easier to measure) but somewhat reduced

similar findings for Sn+Sn collisions (MSU)

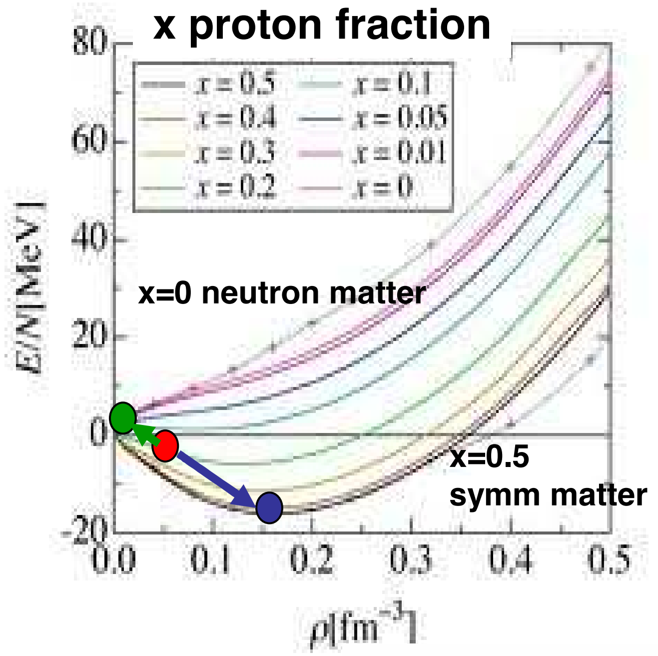
son: asysoft, $m_n^*>m_p^*$
stn: asystiff, $m_n^*>m_p^*$
sop: asysoft, $m_n^*<m_p^*$
stp: asystiff, $m_n^*<m_p^*$

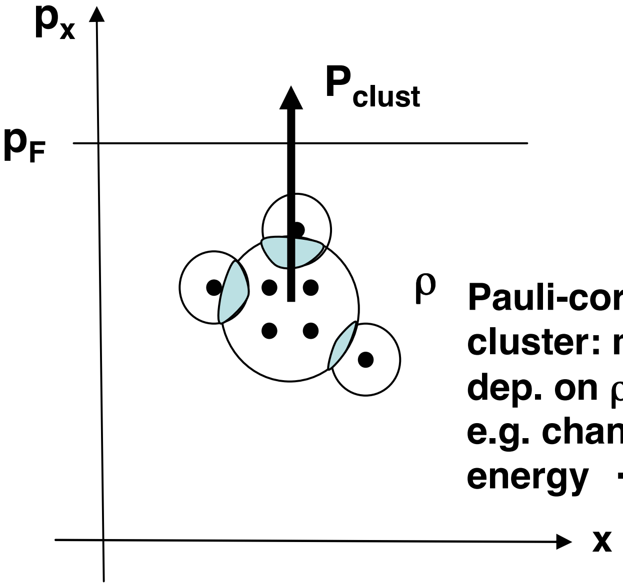
H.H. Wolter, et al.,
EPJ Web of Conf. &&, 03097 (2014)

role of clusters?

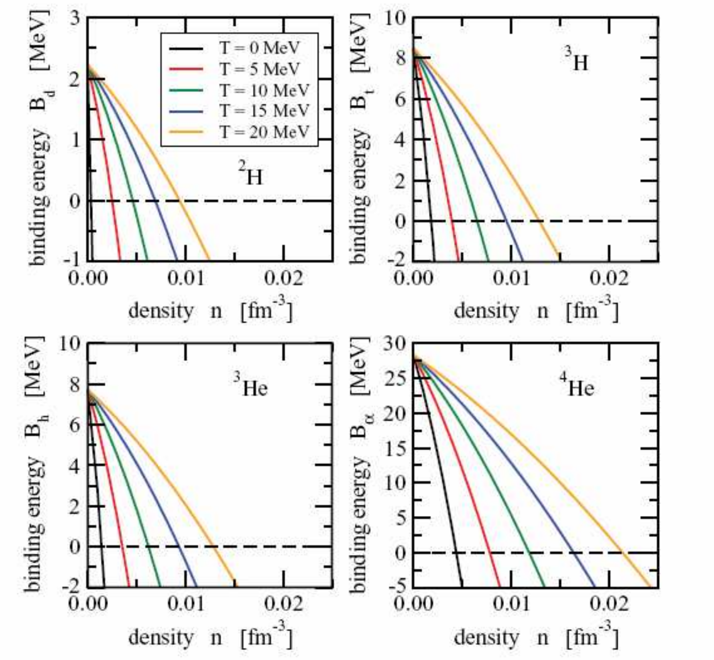
Composition of dilute matter

decrease energy by inhomogeneity
→ fractionation into clusters at
higher density and neutron gas





Pauli-correlations weakens cluster: medium effects dep. on ρ, T, P$_{clust,}$ e.g. change of binding energy



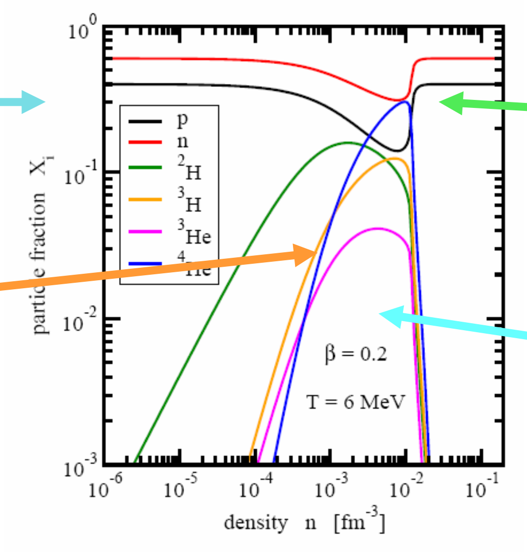
clusterization increases with ρ↘ ,T ↗, P$_{cm}$ ↗

Particle Fractions

very low density: p,n

Increasing density: clusters arise: deuteron first, but then α dominates

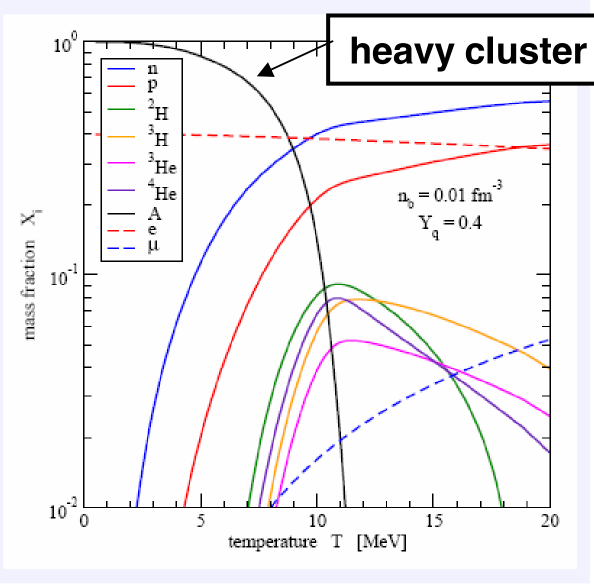
S.Typel, G. Röpke, et al., PRC 81 (2010)



Mott density: clusters melt, homogeneous p,n matter;

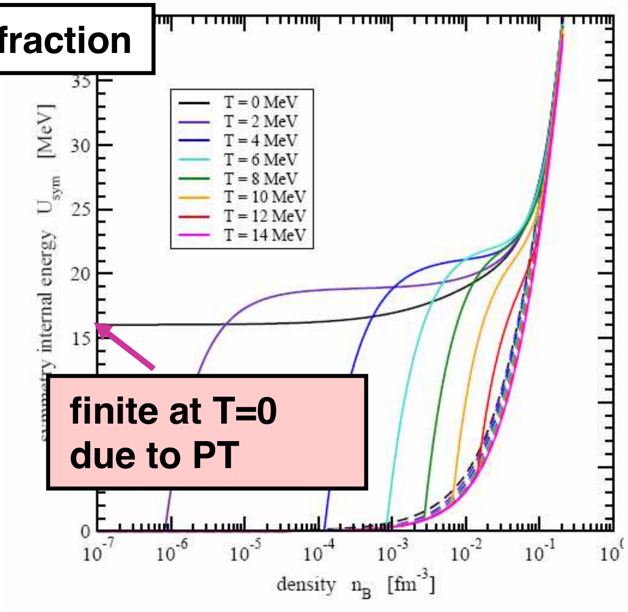
here heavier nuclei (embedded into a gas) become important here

composition as fct of Temp.



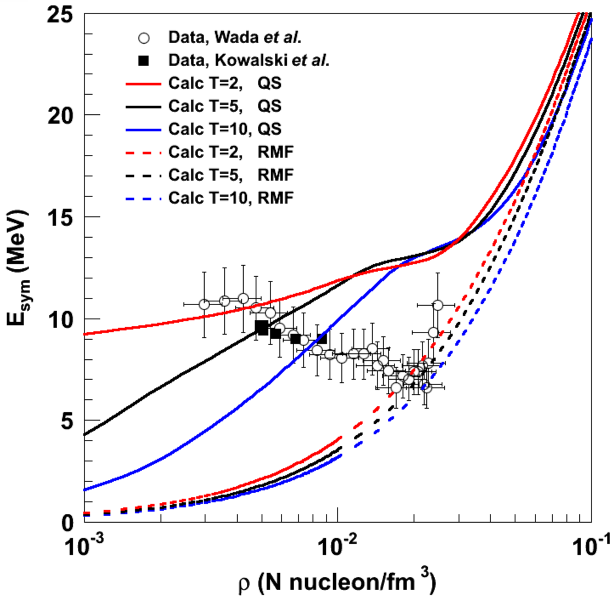
heavy cluster fraction

Symmetry Energy

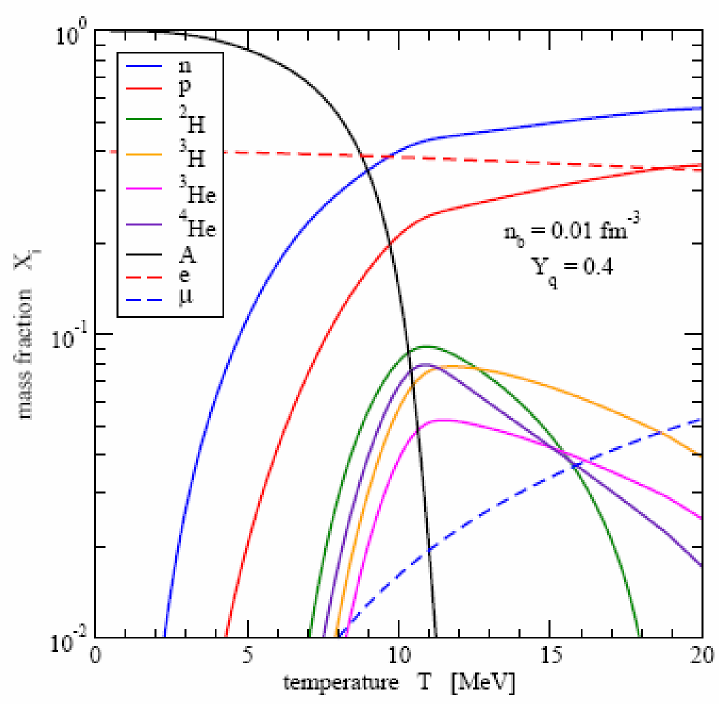


finite at T=0 due to PT

comparison to data from heavy ion collisions

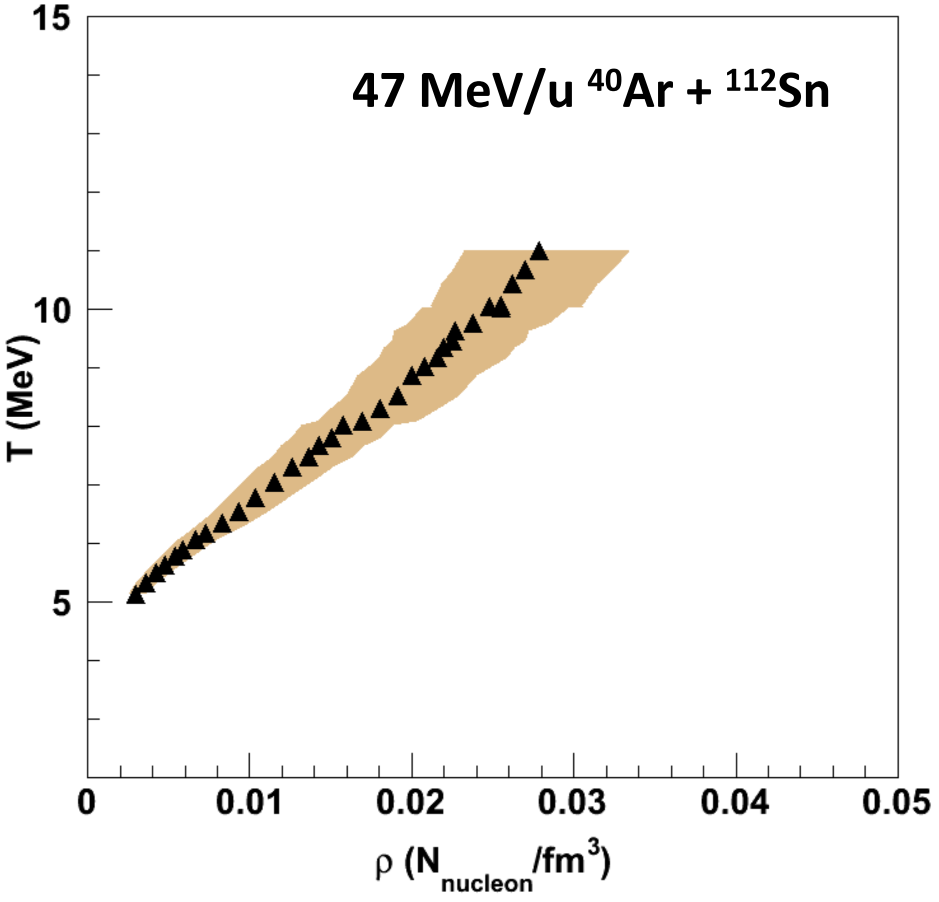


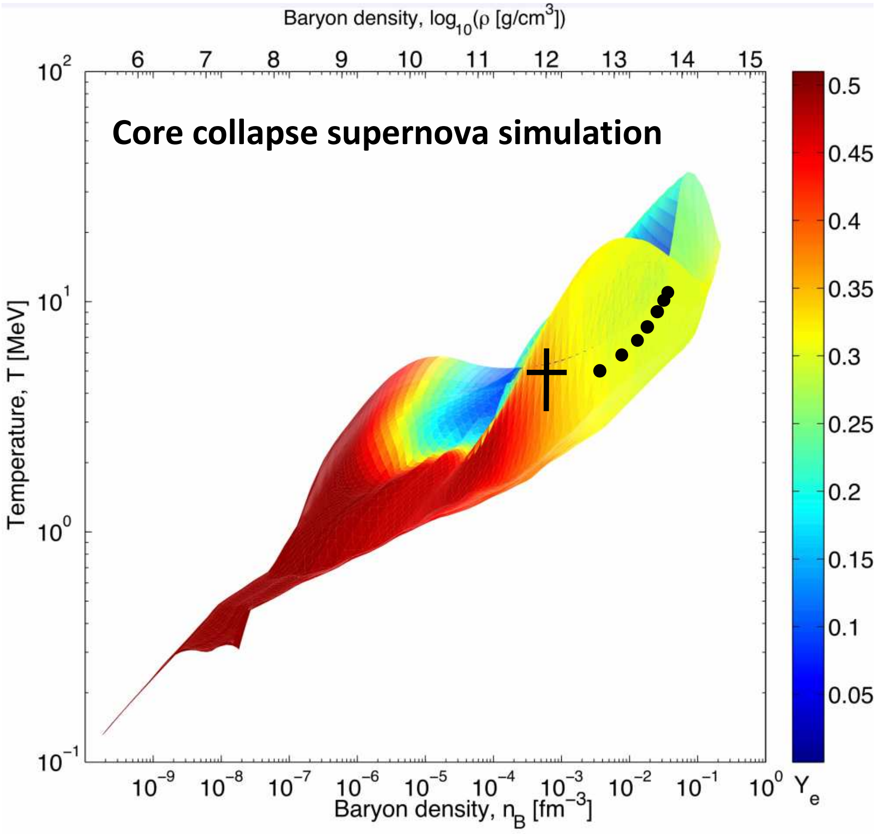
- mass fractions of nucleons, light and heavy nuclei, electrons, muons



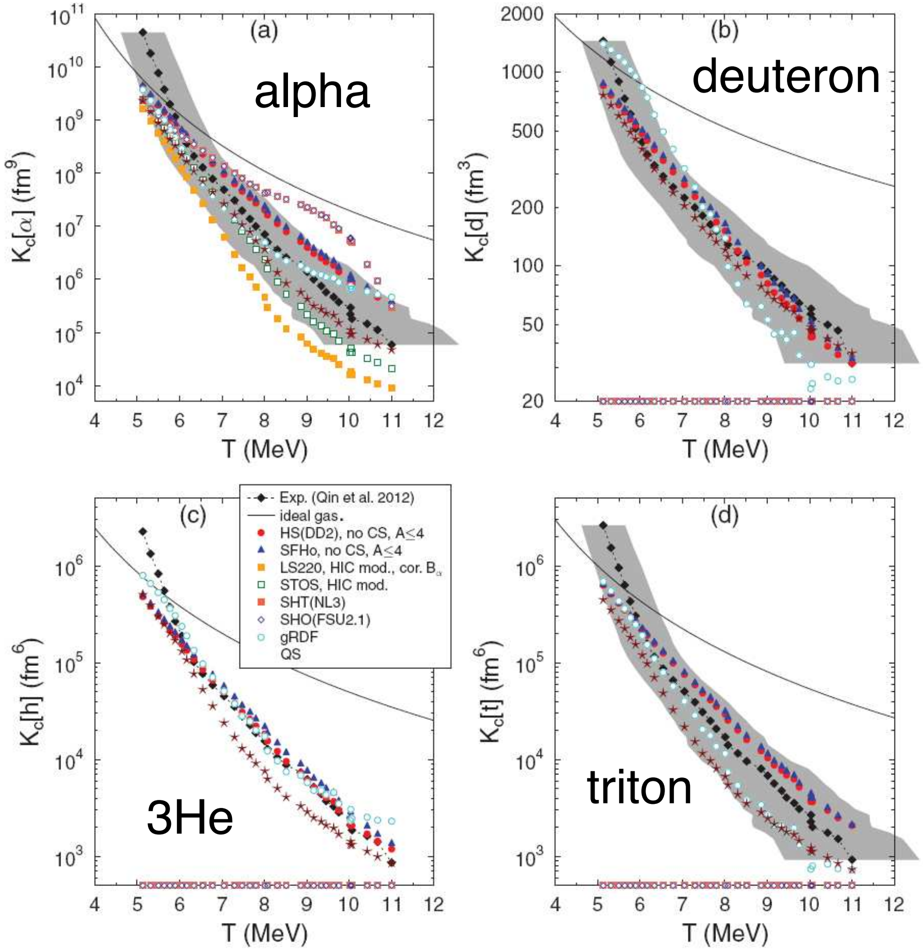
Temperatures and Densities

- Recall v_{surf} vs time calculation
- System starts hot and expands as it cools



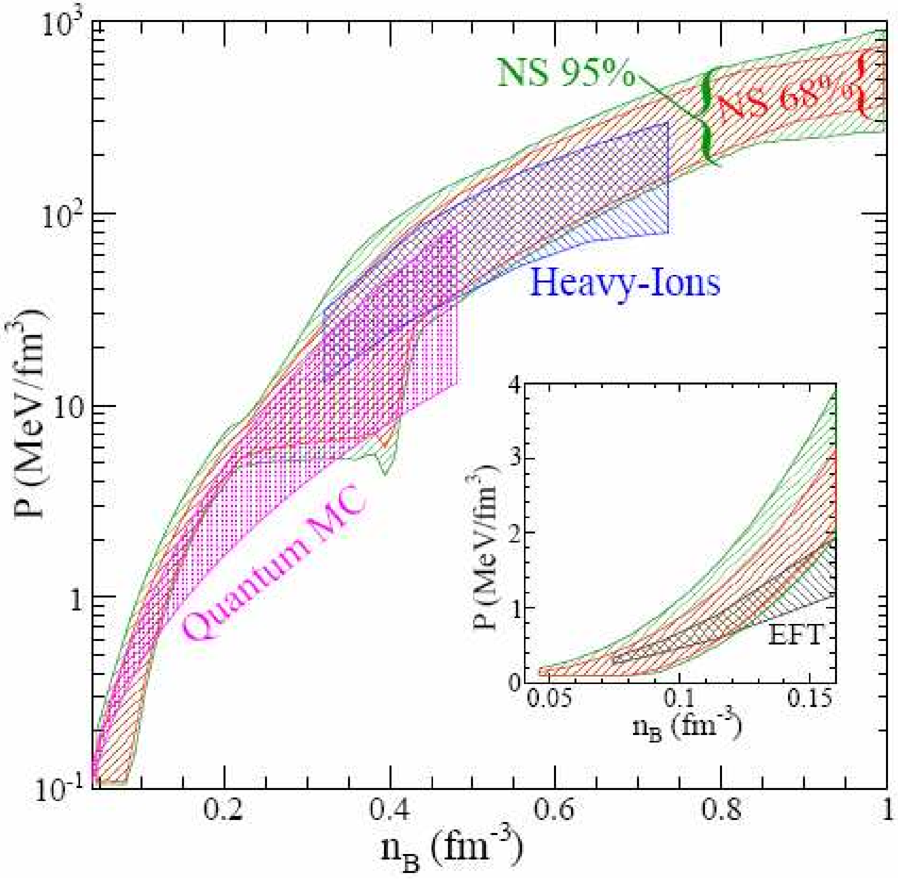


Chemical Equilibrium Constants for light clusters in different clusterization models



$$K_c[i] = \frac{n_i}{n_n^{N_i} n_p^{Z_i}},$$

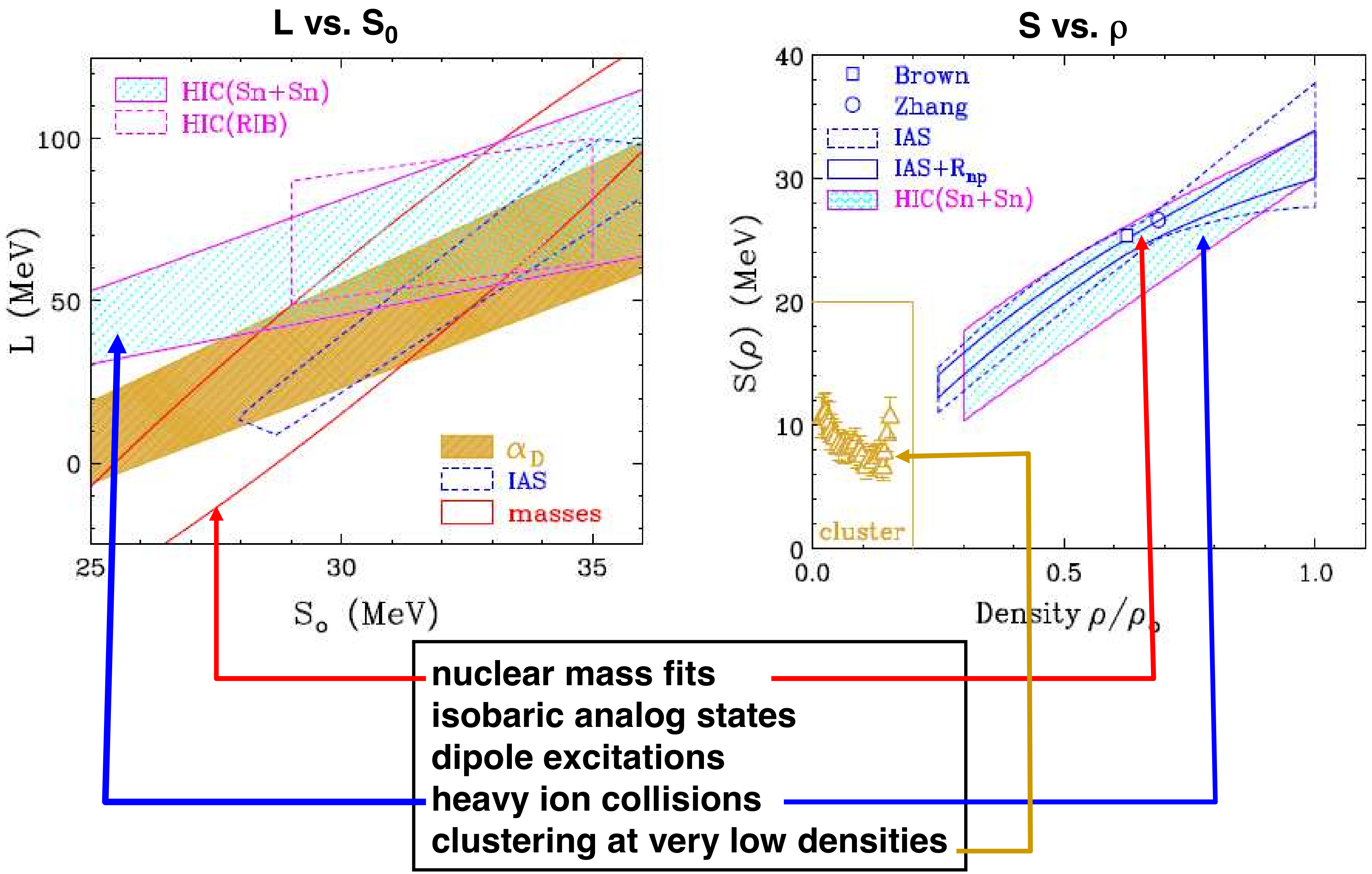
M.Hempel, et al., PRC 91 (2015)



A. Steiner, et al., ApJ 765, L5 (2013)

Status of determination of symmetry energy around saturation density:

$$E_{sym}(\rho) \equiv S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots$$



L vs. S_0

S vs. ρ

nuclear mass fits
isobaric analog states
dipole excitations
heavy ion collisions
clustering at very low densities

A consistent picture emerges!

Transport equations

Boltzmann-Ühling-Uhlenbeck (BUU)

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - \vec{\nabla}U(r)\,\vec{\nabla}^{(p)} f(\vec{r},\vec{p};t) = \int d\vec{v}_2\, d\vec{v}_{1'}\, d\vec{v}_{2'}\, v_{21}\,\sigma_{12}(\Omega)\,(2\pi)^3\,\delta(p_1 + p_2 - p_{1'} - p_{2'}) \left[f_{1'} f_{2'} (1-f_1)(1-f_2) - f_1 f_2 (1-f_{1'})(1-f_2')\right]$$

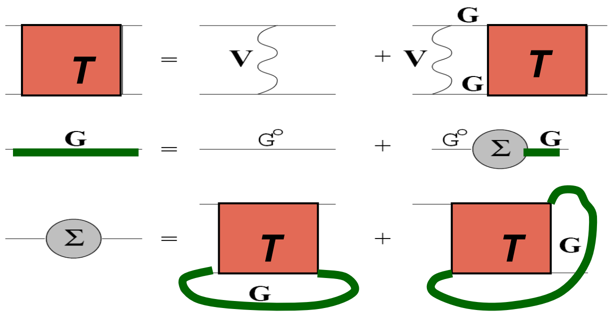
Can be derived:

→ Classically from the Liouville theorem

→ Semiclassically from THDF

$\Big\}$ collision term added (and fluctuations)

→ From non-equilibrium theory (Kadanoff-Baym)
collision term included
mean field and in-medium cross sections
consistent, e.g. from BHF



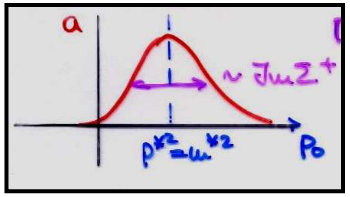
Spectral fcts, off-shell transport, quasi-particle approx.

$$A(x,p) \propto \frac{2\Gamma(x,p)}{(p^{*2} - m^{*2}) + \Gamma^2(x,p)}$$

$$\Gamma(x,p) = m^*\, Im\Sigma_s^+ - p_\mu{}^*\, Im\Sigma^{+\mu}$$

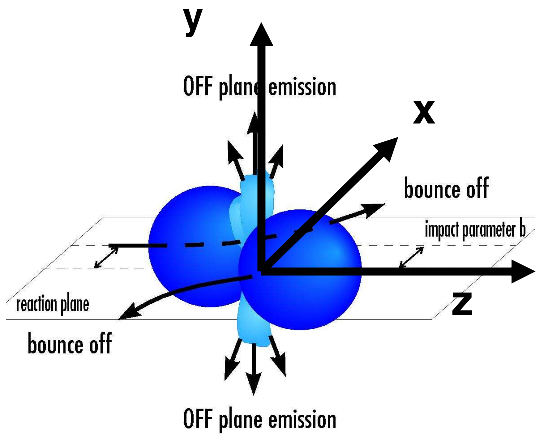
QPA

$$\propto \delta(p^{*2} - m^{*2})\,\Theta(p^{*0})$$



Transport theory is on a well defined footing, in principle

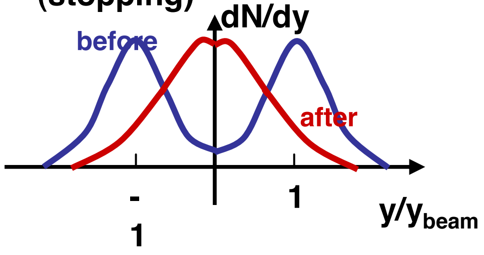
Flow Observables



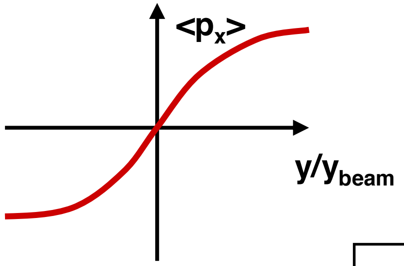
Measurement of momenta of emitted particles: i.e. momentum tensor.

Possible to measure for all particles: p,n, d, ..,α, fragments, mesons (π,K,η),..

Rapidity distribution (stopping)



Transverse flow (compression)



$$rapidity \qquad y = \frac{1}{2}\ln\frac{1+\beta_z}{1-\beta_z}$$

modern approach:

$$Flow \;:\; N(\Theta\,;y,p_t,b) = N_0(1+\boxed{v_1(y,p_t)}\cos\Theta + \boxed{v_2(y,p_t)}\cos 2\Theta + \ldots$$

v_1: Sideward flow

v_2: Elliptic flow

impact parameter selection:

Observable that is strongly crrelated to impact parameter, like mutiplicity, transvers energy, etc.

charged particle multipli city