All-sky reconstruction of the primordial scalar potential Φ

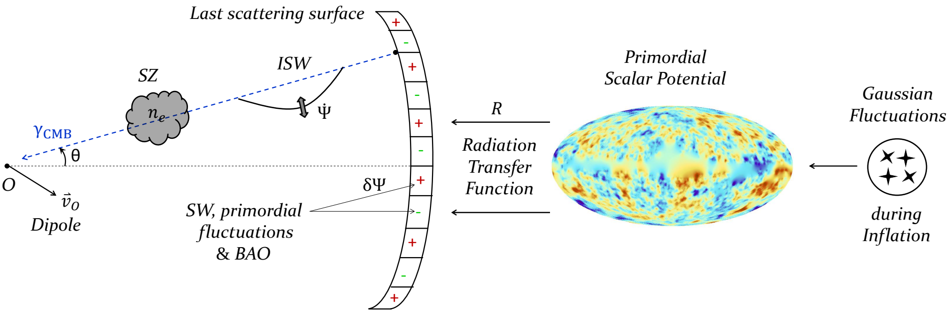
Sebastian Dorn

with Maksim Greiner and Torsten A. Enßlin

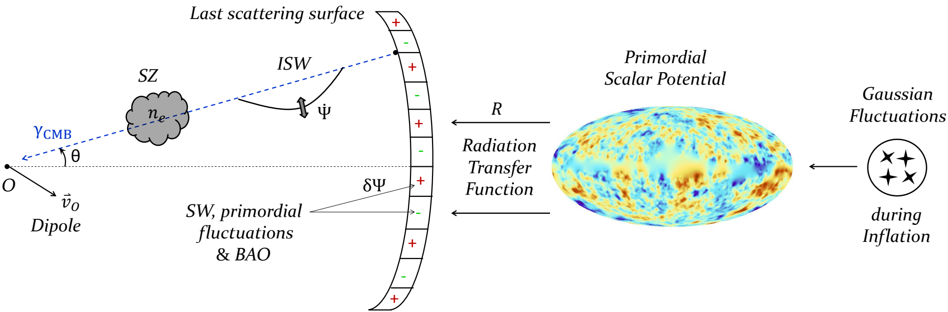
Max-Planck-Institut for Astrophysics

Research Area E Science Day | Garching | October 27, 2015

Motivation


Last scattering surface
ISW
SZ
n_e
$\dot{\Psi}$
γ_{CMB}
θ
O
$\vec{v}_O$
Dipole
δΨ
SW, primordial fluctuations & BAO
R
Radiation Transfer Function
Primordial Scalar Potential
Gaussian Fluctuations
during Inflation

Motivation



⇒ So why actually don't we infer these properties directly from Φ?

Data model: $d_{\mathrm{CMB}} = R\Phi + n, \qquad \Phi \in \mathbb{R}^3$

The naive ansatz: $\hat{\Phi} = F\left(d_{\mathrm{CMB}}\right)$

(F: Filter operation)

Data model: $d_{\mathrm{CMB}} = R\Phi + n, \qquad \Phi \in \mathbb{R}^3$

The naive ansatz: $\hat{\Phi} = F\,(d_{\mathrm{CMB}})$

(F: Filter operation)

The problem :

$$N_{\mathrm{pixel}} \begin{cases} (d_{\mathrm{CMB}}) \propto \mathcal{O}(10^7) \\ (\Phi) \propto \mathcal{O}(10^7)^{3/2} \\ (R) \propto \mathcal{O}(10^7)^3 \end{cases}$$

Data model: $d_{\mathrm{CMB}} = R\Phi + n, \qquad \Phi \in \mathbb{R}^3$

The naive ansatz: $\hat{\Phi} = F\left(d_{\mathrm{CMB}}\right)$

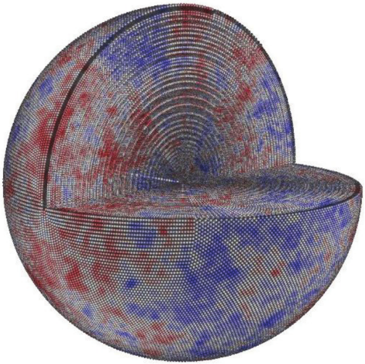
(F: Filter operation)

The problem :

$$N_{\mathrm{pixel}} \begin{cases} (d_{\mathrm{CMB}}) \propto \mathcal{O}(10^7) \\ (\Phi) \propto \mathcal{O}(10^7)^{3/2} \\ (R) \propto \mathcal{O}(10^7)^3 \end{cases}$$

Φ inference hardly numerically feasable & very expensive!

The solution: Reconstruct Φ slice by slice!



[Yadav & Wandelt et al. '05]

Data Model (T only)

$$d_{\ell m} = \underbrace{M_{\substack{\ell m \\ \ell' m'}} B_{\ell'} \frac{2}{\pi} \int dk\ k^2 \int dr\ r^2 \Phi_{\ell' m'}(r) g^T_{\ell'}(k) j_{\ell'}(kr)}_{(R\Phi)_{\ell m}} + n_{\ell m}.$$

Wiener Filter:

- Linear data model: $d = R\Phi + n$
- Bayes theorem: $P(\Phi|d) = \frac{P(d|\Phi)P(\Phi)}{P(d)}$
- Mean of Φ: $\hat{\Phi} = \langle\Phi\rangle_{(\Phi|d)} = \int \mathcal{D}\Phi \; \Phi \; \mathcal{P}(\Phi|d)$

Wiener Filter:

- Linear data model: $d = R\Phi + n$
- Bayes theorem: $P(\Phi|d) = \frac{P(d|\Phi)P(\Phi)}{P(d)}$
- Mean of Φ: $\hat{\Phi} = \langle\Phi\rangle_{(\Phi|d)} = \int \mathcal{D}\Phi \; \Phi \; \mathcal{P}(\Phi|d)$
- Statistics of Φ & n: Gaussian

$\Rightarrow$ $\hat{\Phi} = WF(d)$ WF: matrix/operator

Approach

Approach

- 1 slice = WF result $\hat{\Phi}$ projected onto a sphere:

$$\hat{\Phi}^{(2)} = T\left(\hat{\Phi}\right) = T\left\langle\Phi\right\rangle_{(\Phi|d)}$$

Approach

- 1 slice = WF result $\hat{\Phi}$ projected onto a sphere:

$$\hat{\Phi}^{(2)} = T\left(\hat{\Phi}\right) = T\left\langle\Phi\right\rangle_{(\Phi|d)}$$

- Resulting WF for 1 slice:

$$\hat{\Phi}^{(2)} = P_\ell^\Phi R^{(2)\dagger} C_d^{-1} d$$

P_ℓ^Φ: primo. power spectrum projected onto a sphere
C_d: data (CMB) covariance
$R^{(2)}$: 2d response, includes beam, mask, convolutions, physics

Including Polarization

$$d = \begin{pmatrix} d^T \\ d^Q \\ d^U \end{pmatrix} = R\Phi + \begin{pmatrix} n^T \\ n^Q \\ n^U \end{pmatrix}$$

Including Polarization

$$d = \begin{pmatrix} d^T \\ d^Q \\ d^U \end{pmatrix} = R\Phi + \begin{pmatrix} n^T \\ n^Q \\ n^U \end{pmatrix}$$

$$R = \underbrace{\begin{pmatrix} M_T B & 0 & 0 \\ 0 & M_P B & 0 \\ 0 & 0 & M_P B \end{pmatrix} W^{T,E}_{T,Q,U}}_{\equiv R^{T,E}_{T,Q,U}} \begin{pmatrix} R^T \\ R^E \\ 0 \end{pmatrix},$$

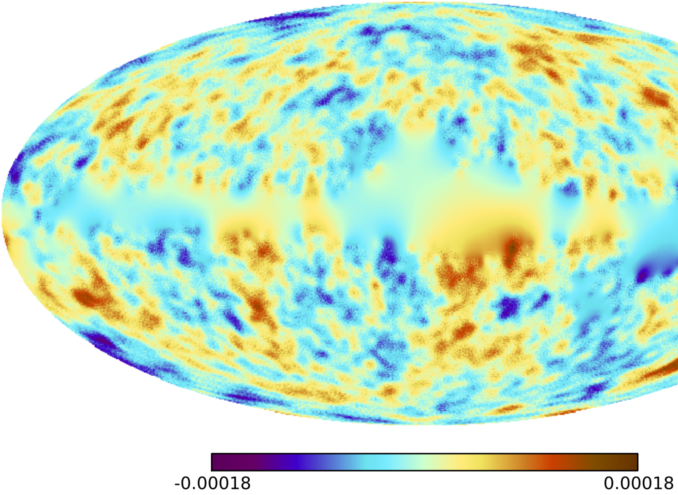
$$\left(R^{T,E}\Phi\right)_{\ell m} \equiv \frac{2}{\pi}\int dk\ k^2 \int dr\ r^2 \Phi_{\ell m}(r) g_\ell^{T,E}(k) j_\ell(kr).$$

Achievements

- Full parallelization of the 3d WF
- Fast & cheap reconstruction
- Inclusion of polarization data simple
- Uncertainty estimates (sampling) per slices affordable

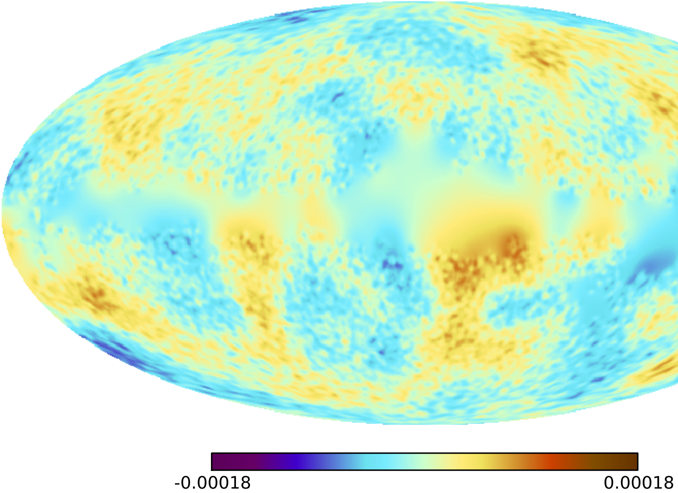
Reconstruction From Nine-Year WMAP T-Data (V-Band)

$r = 1.01 r_{\text{LSS}}$



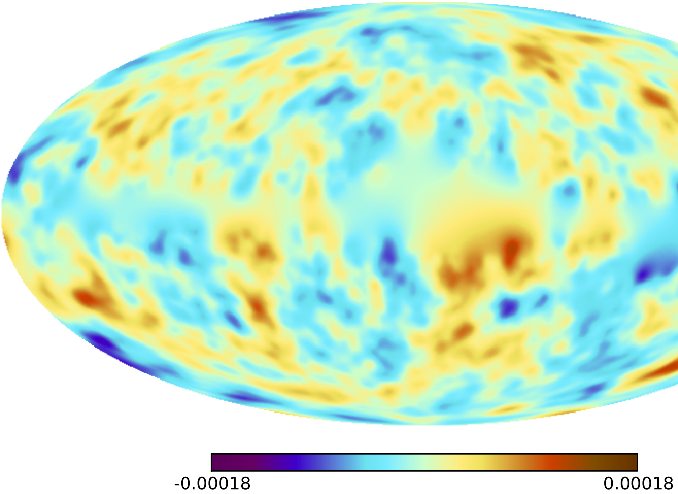
[SD et al. '14]

$r = 0.97 r_{\mathrm{LSS}}$



[SD et al. '14]

$r = 1.05 r_{\mathrm{LSS}}$



[SD et al. '14]

Uncertainty calculation:

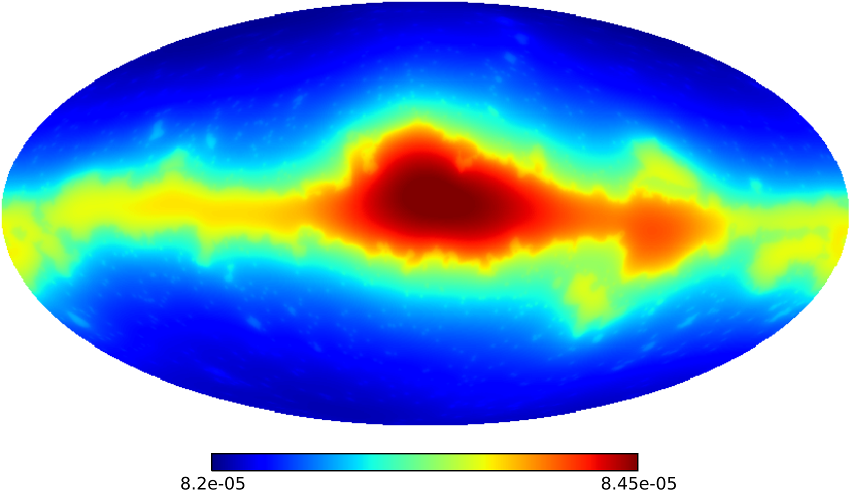
Within the Wiener filter theory,

$$\hat{\Phi} = WF(d) = DR^{\dagger}N^{-1}d,$$

the 1σ uncertainty is given by

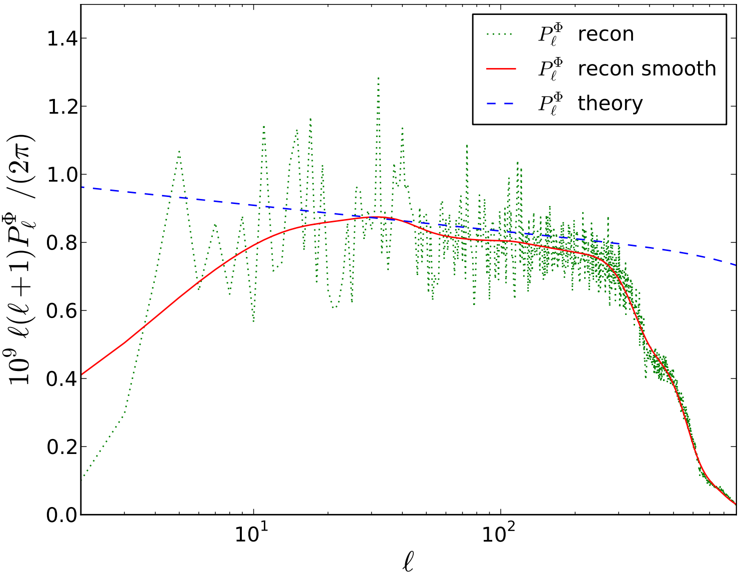
$$\sigma = \sqrt{\mathrm{diag}(D)}.$$

$r = 1.01 r_{\text{LSS}}$



[SD et al. '14]

Primordial Power Spectrum ($r = r_{LSS}$), T only [SD et al. ’14]



Next:

- Planck data including polarization
- Cross-checks with LSS reconstructions
- Inference of inflationary (recombinational) parameters
- Morphology/Symmetry investigations