Current Interactions from Nonlinear Higher-Spin Equations

M.A.Vasiliev

arXiv:1605.02662

Lebedev Institute, Moscow

Higher Spin Theories and Duality

MIAPP

Garching, May 25, 2016

Higher derivatives in HS interactions

HS interactions contain higher derivatives Bengtsson, Bengtsson, Brink (1983)

Nonanaliticity in Λ via dimensionless combination $\Lambda^{-\frac{1}{2}}\frac{\partial}{\partial x}$ (Fradkin, MV 1987)

By a seemingly local field redefinition it is possible to get rid of currents from HS field equations including the stress tensor (Prokushkin, MV 1998)

$$\phi \to \phi' = \phi + \sum_n a_{nm} (\rho D)^n \phi \, (\rho D)^m \phi + \dots ,$$

ρ is the AdS radius, D is the space-time covariant derivative.

The problem: find restrictions on a_{nm} distinguishing between truly non-local and generalized local field redefinitions containing an infinite number of terms but a_{nm} decreasing fast enough with n and m.

The problems in AdS_d and Minkowski space are essentially different

Locality versus Nonlocality

For a massive field equation

$$(\Box + m^2)\phi = 0$$

Greens function can be represented in the pseudolocal form

$$G = (\Box + m^2)^{-1} = m^{-2} \sum_{n=0}^{\infty} \left(-\frac{\Box}{m^2} \right)^n$$

Constant expansion coefficients imply nonlocality.

m^2 is a counterpart of Λ for massless particles in AdS

The problem is to look for a class of field redefinitions which

- are closed under multiple application: form an algebra
- rule out obviously nonlocal field redefinitions like those resulting from Greens functions

Nonlinear HS equations

$$\mathcal{W}(Z;Y;k,\bar{k}|x) = (\mathsf{d} + W) + S\,, \qquad W = dx^n W_n\,, \qquad S = \theta^\alpha S_\alpha + \bar{\theta}^{\dot\alpha}\bar{S}_{\dot\alpha}$$

$$\mathcal{W}\star\mathcal{W} = i(\theta^A\theta_A + \eta\theta^\alpha\theta_\alpha B\star k\star\kappa + \bar{\eta}\bar{\theta}^{\dot\alpha}\bar{\theta}_{\dot\alpha}B\star\bar{k}\star\bar{\kappa})$$

$$\mathcal{W}\star B = B\star\mathcal{W}\,, \qquad B = B(Z;Y;k,\bar{k}|x)$$

HS star product

$$(f\star g)(Z;Y) = \frac{1}{(2\pi)^4}\int d^4U\,d^4V\,\exp\left[iU_AV^A\right] f(Z+U;Y+U)g(Z-V;Y+V)$$

$$\kappa = \exp iz_\alpha y^\alpha\,, \qquad \bar{\kappa} = \exp i\bar{z}_{\dot\alpha}\bar{y}^{\dot\alpha}$$

Massless fields

$$\mathcal{W}(Z;Y;k,\bar{k}|x) = \mathcal{W}(Z;Y;-k,-\bar{k}|x),\ B(Z;Y;k,\bar{k}|x) = -B(Z;Y;-k,-\bar{k}|x)$$

Topological fields

$$\mathcal{W}(Z;Y;k,\bar{k}|x) = -\mathcal{W}(Z;Y;-k,-\bar{k}|x),\ B(Z;Y;k,\bar{k}|x) = B(Z;Y;-k,-\bar{k}|x)$$

Perturbative analysis

The standard vacuum solution is $B=0$ and

$$\mathcal{W}_0 = \mathsf{d}_x + Q + W_0(Y|x)\,, \qquad Q := \theta^A Z_A$$

The space-time one-form $W_0(Y|x)$ solves the flatness equation

$$AdS: \qquad \mathsf{d}_x W_0(Y|x) + W_0(Y|x) \star W_0(Y|x) = 0\,.$$

The star-commutator with Q yields de Rham derivative in Z^A

$$Q \star f(Z;Y) - (-1)^{deg_f} f(Z;Y) \star Q = -2i\mathsf{d}_Z f(Z;Y)\,, \qquad \mathsf{d}_Z = \theta^A \frac{\partial}{\partial Z^A}$$

Standard homotopy formula:

$$\mathsf{d}_Z g(\theta_Z;Z;Y) = f(\theta_Z;Z;Y) \quad \Longrightarrow \quad g(\theta_Z;Z;Y) = \partial_Z^* f + \mathsf{d}_Z \varepsilon + g(0;0;Y)$$

$\mathsf{d}_Z\varepsilon$: exact forms

$g(0;0;Y)$: de Rham cohomology

Dynamical fields in de Rham cohomology:

$$C(Y;k,\bar{k}|x) = B(0;Y;k,\bar{k}|x)\,, \qquad \omega(Y;k,\bar{k}|x) = W(0;Y;k,\bar{k}|x)$$

Central On-Shell Theorem

Unfolded equations for free massless fields of all spins (1989)

$$R_1(y,\overline{y}|x) = L(w,C) := i\Big(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\ \bar{C}(0,\overline{y}|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\ C(y,0|x)\Big)$$

$$\tilde{D}_0 C(y,\overline{y}|x) = 0$$

where

$$R_1(y,\bar{y}|x) := D_0^{ad}\omega(y,\bar{y}|x) := D_0^L\omega(y,\bar{y}|x) + \lambda h^{\alpha\dot\beta}\Big(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot\beta}} + \frac{\partial}{\partial y^\alpha}\bar{y}_{\dot\beta}\Big)\omega(y,\bar{y}|x)\,,$$

$$\tilde{D}_0 C(y,\bar{y}|x) := D_0^L C(y,\bar{y}|x) - i\lambda h^{\alpha\dot\beta}\Big(y_\alpha\bar{y}_{\dot\beta} - \frac{\partial^2}{\partial y^\alpha\partial\bar{y}^{\dot\beta}}\Big)C(y,\bar{y}|x)\,,$$

$$D_0^L f(y,\bar{y}|x) := \mathsf{d} f(y,\bar{y}|x) + \Big(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta} + \overline{\omega}^{\dot\alpha\dot\beta}\bar{y}_{\dot\alpha}\frac{\partial}{\partial\bar{y}^{\dot\beta}}\Big)f(y,\bar{y}|x)\,.$$

$$W_0 = (\omega_{\alpha\beta}, \overline{\omega}_{\dot\alpha\dot\beta}, h_{\alpha\dot\beta})$$

$$\mathsf{d} = dx^n\frac{\partial}{\partial x^n}\,,\qquad H^{\alpha\beta} := h^{\alpha\dot\alpha}h^{\beta}{}_{\dot\alpha}\,,\qquad \overline{H}^{\dot\alpha\dot\beta} := h^{\alpha\dot\alpha}h_{\alpha}{}^{\dot\beta}\,.$$

Fields and Currents

Spin s is described by the 1-forms $\omega(y,\bar{y}|x)$ and 0-form $C(y,\bar{y}|x)$ obeying

$$\omega(\mu y,\mu\bar{y}\mid x)=\mu^{2(s-1)}\omega(y,\bar{y}\mid x)\,,\qquad C(\mu y,\mu^{-1}\bar{y}\mid x)=\mu^{\pm 2s}C(y,\bar{y}\mid x)$$

Generalized Weyl tensors $C(y,0|x)$ and $C(0,\bar{y}|x)$ describe gauge invariant combinations of derivatives of the gauge fields of spins $s\geq 1$ and matter fields of spins $s=0,1/2$

$C(y,0|x)$ and $C(0,\bar{y}|x)$ are primaries of the Weyl module formed by $C(y,\bar{y}|x)$

Higher powers in y and $\bar{y}$ for a given spin contain higher derivatives

Conserved currents $J(Y_1,Y_2|x)$ are associated with the bilinears of $C(Y|x)$

$$J(Y_1,Y_2|x):=C(Y_1|x)\tilde{C}(Y_2|x)\,,\qquad \tilde{C}(y,\bar{y}|x)=C(-y,\bar{y}|x)\,.$$

As a consequence of the rank-one equation for $C(Y|x)$, the current $J(Y_1,Y_2|x)$ obeys the rank-two equation   Gelfond, MV (2003)

$$\tilde{D}_2J(Y_1,Y_2|x)=0\,,\qquad \tilde{D}_2:=D^L-i\lambda h^{\alpha\dot{\beta}}\Big(y_{1\alpha}\bar{y}_{1\dot{\beta}}-y_{2\alpha}\bar{y}_{2\dot{\beta}}-\frac{\partial^2}{\partial y_1^\alpha\partial\bar{y}_1{}^{\dot{\beta}}}+\frac{\partial^2}{\partial y_2^\alpha\partial\bar{y}_2{}^{\dot{\beta}}}\Big)$$

Current deformation

Current deformation can be formulated as a linear system

$$D\omega + L(w,C) + \Gamma_{cur}(w,J) = 0\,,$$

$$\tilde{D}C + \mathcal{H}_{cur}(w,J) = 0\,, \qquad \tilde{D}_2 J(Y_1,Y_2|x) = 0$$

Linear functionals Γ and $\mathcal{H}$ should obey the compatibility conditions

The freedom in $\Gamma_{cur}(w,J)$ and $\mathcal{H}_{cur}(w,J)$ results from field redefinitions

$$\omega \to \omega' = \omega + \Omega(w,J)\,, \qquad C \to C' = C + \Phi(J)\,.$$

Nontrivial $\Gamma_{cur}(w,J)$ and $\mathcal{H}_{cur}(w,J)$ cannot be removed by a field redefinition. Usual current interactions are nontrivial. Schematically,

$$J = J_0 + \Delta J\,,$$

where ΔJ is an improvement that can be removed by a field redefinition.

Concept of (non)triviality of the currents depends on the class of field redefinitions

Unfolded form of usual current interactions

For simplicity: 0-form sector Gelfond, MV (2010)

$$\mathcal{H}_{cur}(w,J)=\frac{1}{4}\int_0^1 d\tau \sum_{h_1,h_2,h_J} a(h_1,h_2,h_J)\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2}\exp i[\bar{s}_{\dot\beta}\bar{t}^{\dot\beta}]$$
$$h(y,\tau\bar{s}+(1-\tau)\bar{t})J_{h_1,h_2,h_J}(\tau y,-(1-\tau)y,\bar{y}+\bar{s},\bar{y}+\bar{t})+c.c.\,,$$

$$h(u,\bar{u})=h^{\alpha\dot\alpha}u_\alpha\bar{u}_{\dot\alpha}$$

J_{h_1,h_2,h_J} **is the projection of** J **to the helicities** h_1,h_2,h_J.

Coefficients $a(h_1,h_2,h_J)$ **remain undetermined at this level.**

$\mathcal{H}_{cur}(w,J)$ **is local, containing a finite number of terms for any** $h_1,h_2,\ h_J$.

$\mathcal{H}_{cur}(w,J)$ **properly reproduces usual current interactions** Gelfond, MV 2010

Locality in the twistor variables

Technically, locality is due to the absence of integration over s and t.

$$\int \frac{dsdt}{(2\pi)^2} \exp i[s_\beta t^\beta] f(y+s,\bar{y}) g(y+t,\bar{y}) = f(y,\bar{y}) \exp[-i \overleftarrow{\partial}_\alpha \overrightarrow{\partial}_\beta \epsilon^{\alpha\beta}] g(y,\bar{y})$$

$$\int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot\beta} \bar{t}^{\dot\beta}] f(y,\bar{y}+\bar{s}) g(y,\bar{y}+\bar{t}) = f(y,\bar{y}) \exp[-i \overleftarrow{\partial}_{\dot\alpha} \overrightarrow{\partial}_{\dot\beta} \epsilon^{\dot\alpha\dot\beta}] g(y,\bar{y})$$

For given helicities carried by g and f, only a single term in the sum contributes hence containing a finite number of derivatives.

When both integrations are present, the number of derivatives in y and $\bar{y}$ can be infinitely increased without affecting the helicities carried by g and f, implying appearance of infinite tails of derivatives and hence nonlocality.

Current deformation from nonlinear equations

In the 0-form sector the deformation is

$$D_0 C + [\omega\,, C]_* + \mathcal{H}(w, J) = 0\,,$$

$$J(y_1, y_2; \bar{y}_1, \bar{y}_2; K|x) = C(y_1, \bar{y}_1; K|x) C(y_2, \bar{y}_2; K|x)\,, \qquad K = k, \bar{k}$$

A simple computation using the new technique Didenko, Misuna, MV 2015

$$\mathcal{H}(w, J) = \mathcal{H}_\eta(w, J) + \mathcal{H}_{\bar{\eta}}(w, J)\,,$$

$$\mathcal{H}_\eta(w, J) = -\frac{i}{2}\eta \int \frac{dSdT}{(2\pi)^4} \exp iS_A T^A \int_0^1 d\tau$$
$$[h(s, \tau\bar{y} - (1-\tau)\bar{t}) J(\tau s, -(1-\tau)y + t; \bar{y} + \bar{s}, \bar{y} + \bar{t}; K)$$
$$-h(t, \tau\bar{y} - (1-\tau)\bar{s}) J((1-\tau)y + s, \tau t, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K)] * k$$

This deformation is not local, containing integrations over s, t and $\bar{s}, \bar{t}$.

Here h depends on $\bar{y}$ instead of y in the local current deformation.

The two terms result from those in the commutator $[\mathcal{W}, B]_*$

Options

This work has been triggered by the previous work Boulanger, Kessel, Skvortsov and Taronna (2015) leading to counterintuitive infinite results

A priori there are three options

i BKST analysis is correct &

nonlinear HS equations do not reproduce HS current interactions

ii BKST analysis is incorrect &

nonlinear HS equations do not reproduce HS current interactions

iii BKST analysis is incorrect &

nonlinear HS equations do reproduce HS current interactions

A weak point of the BKST analysis is the demand that the deformation resulting from nonlinear HS equations should admit the decomposition

$$\mathcal{H}^{nl} = \mathcal{H}^{cur} + \mathcal{H}^{impr}\,, \qquad \mathcal{H}^{impr} = \sum_n J_n^{impr}$$

J_n^{impr} are usual improvements containing a finite number of derivatives

Field redefinition

To show that the option iii is the right one it is useful first to solve the problem and then to analyse the result

To reproduce standard current interactions we have to find a field redefinition

$$C \to C'(Y;K|x) = C(Y;K|x) + \Phi(Y;K|x)$$

with Φ linear in J bringing $\mathcal{H}(w,J)$ to $\mathcal{H}_{cur}(w,J)$

First field redefinition

$$\Phi_{1\eta}(Y;K|x) = \quad \eta \int \frac{dSdT}{(2\pi)^4} \exp iS_A T^A \int d\tau_i \prod_{i=1}^{3} \theta(\tau_i)\delta\Big(1 - \sum_{i=1}^{3} \tau_i\Big) \frac{\partial}{\partial \tau_3} J(\tau_3 s + \tau_1 y, t - \tau_2 y; \bar{y} + \bar{s}, \bar{y} + \bar{t}; K) * k\,,$$

gives

$$D_0\Phi_{1\eta}(Y;K|x) = -\frac{i}{2}\eta\int\frac{dSdT}{(2\pi)^4}\exp i[S_A T^A]\int_0^1 d\tau$$
$$\Big[h(s,\tau\bar{y}-(1-\tau)\bar{t})J(\tau s,-(1-\tau)y+t,\bar{y}+\bar{s},\bar{y}+\bar{t})$$
$$-h(t,\tau\bar{y}-(1-\tau)\bar{s})J(s+(1-\tau)y,\tau t,\bar{y}+\bar{s},\bar{y}+\bar{t})$$
$$-ih(\partial_1-\partial_2,(1-\tau)\bar{t}+\tau\bar{s})J(\tau y,-(1-\tau)y;\bar{y}+\bar{s},\bar{y}+\bar{t};K)\Big]*k$$

The first two terms just give $\mathcal{H}_\eta(w,J)$

$$\mathcal{H}_\eta(w,J) = D_0\Phi_{1\eta}(J)+\mathcal{H}'_\eta(w,J)\,,$$

$$\mathcal{H}'_\eta(w,J) = \frac{\eta}{2}\int\frac{d\bar{s}d\bar{t}}{(2\pi)^2}\exp i[\bar{s}_{\dot\alpha}\bar{t}^{\dot\alpha}]\int_0^1 d\tau h(\partial_1-\partial_2,(1-\tau)\bar{t}+\tau\bar{s})$$
$$J(\tau y,-(1-\tau)y;\bar{y}+\bar{s},\bar{y}+\bar{t};K)*k\,.$$

Being free of the integration over s and t, $\mathcal{H}'_\eta(w,J)$ is a local functional of J. Depending on $\partial_1-\partial_2$ instead of y, $\mathcal{H}'_\eta(w,J)$ differs from $\mathcal{H}_{cur}$ containing one extra space-time derivative.

In Minkowski space this would imply that $\mathcal{H}'_\eta(w,J)$ is an improvement.

However this is not the case in AdS_4.

Reincarnation of y

For a local field redefinition

$$\Phi_{2\eta} = \frac{i}{2}\eta \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot{\beta}}\bar{t}^{\dot{\beta}}] \int_0^1 d\tau J(\tau y, -(1-\tau)y; \bar{y}+\bar{s}, \bar{y}+\bar{t}; K) * k$$

an elementary computation yields

$$D_0\Phi_{2\eta} = -\frac{\eta}{4} \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot{\beta}}\bar{t}^{\dot{\beta}}] \int_0^1 d\tau h(y + i(\partial_2 - \partial_1)), \tau\bar{s} + (1-\tau)\bar{t}) \\ J(\tau y, -(1-\tau)y; \bar{y}+\bar{s}, \bar{y}+\bar{t}; K) * k$$

Hence

$$\mathcal{H}_\eta(w,J) = \mathcal{H}_{\eta\, impr}(w,J) + \mathcal{H}_{\eta\, cur}(w,J)$$

$$\mathcal{H}_{\eta\, impr}(w,J) := D_0(\Phi_{1\eta} + \Phi_{2\eta})$$

$$\mathcal{H}_{\eta\, cur}(w,J) = \frac{\eta}{4} \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot{\beta}}\bar{t}^{\dot{\beta}}] \int_0^1 d\tau h(y, \tau\bar{s} + (1-\tau)\bar{t}) \\ J(\tau y, (\tau-1)y; \bar{y}+\bar{s}, \bar{y}+\bar{t}; K) * k\,.$$

The coefficients in the current deformation are determined as

$$a(h_1, h_2, h_J) = \eta\,, \qquad \bar{a}(h_1, h_2, h_J) = \bar{\eta}$$

The phase (in)dependence

An important consequence of the flip of chirality $\bar{y} \to y$ is that the current contributions are proportional to $\eta\bar{\eta}$.

Spin-one example

$$R_1(y,\overline{y}|x) = i\Big(\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\overline{y}^{\dot{\alpha}}\partial\overline{y}^{\dot{\beta}}}\ \bar{C}(0,\overline{y}|x) + \bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\ C(y,0|x)\Big)$$

shift of the first and second terms are proportional to $\bar{\eta}$ **and** η, **respectively**.

Contribution to *r.h.s.* **of the Maxwell equations is proportional to** $\eta\bar{\eta}$.

Locality

For functions

$$f(y,\bar{y}|x) = \frac{1}{2i}\sum_{n,m=0}^{\infty}\frac{1}{n!m!}y_{\alpha_1}\dots y_{\alpha_n}\bar{y}_{\dot\beta_1}\dots\bar{y}_{\dot\beta_m}f^{\alpha_1\dots\alpha_n}{}_{,}{}^{\dot\beta_1\dots\dot\beta_m}(x)$$

Star product is

$$(f*g)f_{\alpha(n)\,,\dot\alpha(m)} \sim \sum_{p,q,k,l,s,r=0}^{\infty}\delta^{n}_{p+q}\delta^{m}_{k+l}a(p,q,k,l,s,t)f_{\alpha(p)\gamma(s)\,,\dot\alpha(l)\dot\gamma(t)}g_{\alpha(q)}{}^{\gamma(s)}{}_{,\dot\alpha(k)}{}^{\dot\gamma(t)}\,.$$

$$a(p,q,k,l,s,t) = \frac{1}{p!q!k!l!s!t!}$$

To restrict expansion coefficients in space-time derivatives, it suffices to restrict the behavior in the number of either undotted or dotted contracted indices as

$$\lim_{p_i,q_j\to\infty} p_k^{\varepsilon}\left(\prod_{j=1}^{3}p_j!q_j!\right)a(p,q)<\infty\,,\qquad k=1,2,3\,,\qquad \varepsilon>0$$

or/and

$$\lim_{p_i,q_j\to\infty} q_k^{\varepsilon}\left(\prod_{j=1}^{3}p_j!q_j!\right)a(p,q)<\infty\,,\qquad k=1,2,3\,,\qquad \varepsilon>0\,.$$

Composition property

The condition has to be symmetric with respect of free and contracted indices to guarantee that a composition of generalized local transformations is again a generalized local transformation dominated by a multiple star product.

For the field redefinition induced by Φ_1

$$\Phi_{1\eta}(Y;K|x) = \int \delta(1-\sum_{j=1}^{3}\tau_i)\prod_{i=1}^{3} d\tau_i\theta(\tau_i)\delta(1-\sum_{j=1}^{3}\tau_i)$$
$$\frac{\partial}{\partial\tau_3}J(\tau_3 s+\tau_1 y, t-\tau_2 y;\bar{y}+\bar{s},\bar{y}+\bar{t};K)*k$$

Integration over homotopy parameters τ_i over a triangle $\sum_{j=1}^{3}\tau_i = 1$ softens the coefficients in $\Phi_{1\eta}(Y;K|x)$ compared to the star product

Exact estimate by

$$\int_0^1 dt_1 t_1^{a_1}\dots\int_0^1 dt_p t_p^{a_p}\delta(\sum_i t_i - 1) = \frac{\prod_{i=1}^{p} a_i!}{(\sum_{i=1}^{p} a_i + p - 1)!}$$

implies $\varepsilon = 1$.

HS holography

The Aharony - Gur-Ari - Yacoby (2011) generalization of the Klebanov-Polyakov conjecture (2002) is that the phase φ of η

$$\eta = |\eta| \exp i\varphi$$

should be related to the Chern-Simons coupling of the boundary vector model

$$\varphi = \frac{\pi}{2}\lambda_B\,, \qquad \varphi = \frac{\pi}{2}(1-\lambda_F) \qquad \lambda := \frac{N}{k}\,.$$

Does this fit the conclusion that the HS cubic vertex is φ-independent?

Yes: the dependence on φ results from the boundary conditions

Boundary behaviour

$$C^{j\,1-j}(y,\bar{y}|\mathbf{x},\mathbf{z}) = \mathbf{z}\exp(y_\alpha \bar{y}^\alpha) T^{j\,1-j}(w,\bar{w}|\mathbf{x},\mathbf{z})\,, \qquad w^\alpha = \mathbf{z}^{1/2} y^\alpha \quad \bar{w}^\alpha = \mathbf{z}^{1/2}\bar{y}^\alpha$$

where $\mathbf{z}$ is the Poincaré coordinate while the 0-forms $T^{j\,1-j}$ are associated with the boundary currents.

Phase dependence via boundary conditions

The contribution of HS connections at the boundary cannot be neglected except for the boundary conditions MV 2012

$$\bar{\eta} T_{+}^{j\,1-j}(y,\bar{y}|\mathbf{x},0) - \eta T_{-}^{1-j\,j}(i\bar{y},iy|\mathbf{x},0) = 0\,,$$

where T_{+} and T_{-} are the positive and negative helicity parts of $T(y,\bar{y}|x)$.

Upon imposing boundary conditions, remaining real boundary fields are

$$j^{j}(y,\bar{y}|\mathbf{x}) := \frac{1}{2}\Big(\bar{\eta} T_{+}^{j\,1-j}(y,\bar{y}|\mathbf{x},0) + \eta T_{-}^{1-j\,j}(i\bar{y},iy|\mathbf{x},0)\Big) = \bar{\eta} T_{+}^{j\,1-j}(y,\bar{y}|\mathbf{x},0)\,.$$

Independence of the bulk HS vertex on φ gives the following formula matching the form of the deformation of the HS current algebra found by Maldacena and Zhiboedov

$$V = cos^{2}(\varphi)V_{b} + sin^{2}(\varphi)V_{f} + \frac{1}{2}sin(2\varphi)V_{o}\,,$$

Conclusion

Nonlinear HS equations properly reproduce the HS current interactions with the φ-independent coupling constant.

Explicit form of the appropriate field redefinition suggests a proper form of generalized local field redefinitions

Proper dependence on the phase parameter in the holographic duals of the AdS_4 HS theory is reproduced by the phase-independent vertex of the bulk theory HS theory via phase-dependent boundary conditions.

Green light for the analysis of HS field equations

Invariant functionals

String-like HS theory

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