Twisted Self-Duality and $SO(2)$ Electric-Magnetic Duality for Gravity and Higher Spin Gauge Fields

Marc Henneaux

Munich – May 2016
MIAPP's topical workshop "Aspects of Higher Spin Theory"

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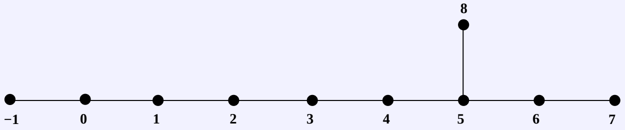

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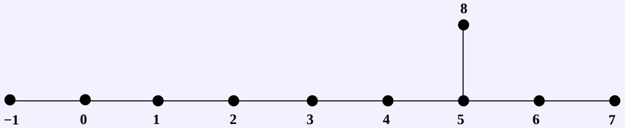

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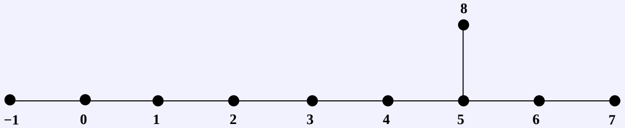

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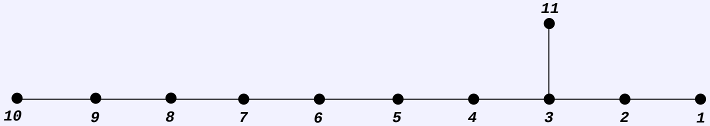

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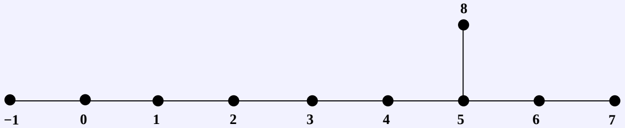

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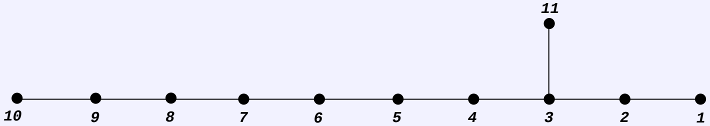

might be a “hidden symmetry" of maximal supergravity or of an appropriate extension of it.

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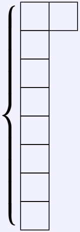

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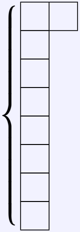

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We will see that this necessitates introducing "prepotentials" which have the somewhat unexpected and intriguing gauge symmetries characteristic of higher spin conformal geometry.

Tools for handling higher spin conformal geometry will also be discussed.

Twisted self-duality for linearized gravity - Dual Riemann tensor

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and conversely.

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Conversely, if one considers two spin two fields $h_{\mu\nu}$ and $f_{\mu\nu}$,

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then both $h_{\mu\nu}$ and $f_{\mu\nu}$ fulfill the Einstein equations.

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In this formulation, the graviton and its dual appear on an equal footing.

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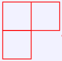
but then, the graviton and its dual are not of same Young symmetry types. [In 5D for instance, the dual graviton is described by a tensor of Young symmetry type .]

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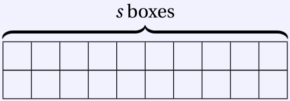
The “curvature" tensor $R_{\lambda_1 \mu_1 \lambda_2 \mu_2 \cdots \lambda_s \mu_s}$ has Young symmetry type



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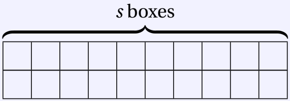

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The key feature that allows the generalization is the result by X. Bekaert and N. Boulanger that states that the second-order Fronsdal equations (with “restricted" gauge invariance) are equivalent to the local equations “Ricci = 0" of order s (with “unrestricted" gauge invariance).

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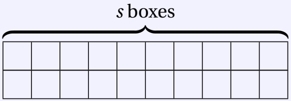

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One can then proceed exactly as for spin-2.

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$$\mathscr{F} = \begin{pmatrix} R[h] \\ S[f] \end{pmatrix}, \quad {}^*\mathscr{F} = \begin{pmatrix} {}^*R[h] \\ {}^*S[f] \end{pmatrix}, \quad \mathscr{S} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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One gets the same twisted self-duality formulation of the equations of motion in terms of the spin s-field and its dual,

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In this formulation, the spin-s field and its dual appear on an equal footing.

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In this formulation, the spin-s field and its dual appear on an equal footing.

The twisted self-duality conditions are highly redundant. In fact, the twisted self-duality conditions with spatial indices imply all the other ones. These equations contain at most one derivative with respect to time.

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in which both the spin-s field and its dual are also treated on *exactly* the same footing (without doubling the number of degrees of freedom) ?

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The answer is yes, but the formulation has some non-usual features :

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The answer is yes, but the formulation has some non-usual features :

First-order action, lack of manifest spacetime covariance, need to introduce “prepotentials" (in the minimal formulation without auxiliary fieds).

Duality invariance of the Einstein equations

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We shall focus for definitess on the $D = 4$ case and on gravity

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Exhibiting twisted self-duality proceeds in the same way independently of the spacetime dimension.

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Because the graviton and its dual are tensors of the same type in $D = 4$, so are the corresponding curvature tensors.

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It follows that the Einstein equations are invariant under the duality rotations

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\begin{aligned}
R &\rightarrow \cos\alpha\, R - \sin\alpha\, {}^*R \\
{}^*R &\rightarrow \sin\alpha\, R + \cos\alpha\, {}^*R,
\end{aligned}
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where $\mathcal{E}^{ij}$ and $\mathcal{B}^{ij}$ are the electric and magnetic components of the Riemann tensor, respectively.

This transformation rotates the Schwarschild mass into the Taub-NUT parameter N.

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It turns out that this is also a symmetry of the Pauli-Fierz action

$$S[h_{\mu\nu}] = -\frac{1}{4}\int d^4x \left[\partial^\rho h^{\mu\nu}\partial_\rho h_{\mu\nu} - 2\partial_\mu h^{\mu\nu}\partial_\rho h^\rho_{\ \nu} + 2\partial^\mu h \partial^\nu h_{\mu\nu} - \partial^\mu h \partial_\mu h\right].$$

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That this is true will come out automatically when we shall reformulate the theory

in a manner appropriate to twisted self-duality.

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That this is true will come out automatically when we shall reformulate the theory

in a manner appropriate to twisted self-duality.

The fastest way to get the action for twisted self-duality is to go to the first-order (Hamiltonian) action and solve the constraints.

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The temporal components $h_{0\mu}$ are Lagrange multipliers for the constraints :

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which expresses that π^{ij} is transverse.

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The explicit resolution of the constraints introduces two “prepotentials", one for the metric and one for its conjugate momentum.

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For instance, the momentum constraint $\partial_i \pi^{ij} = 0$ is solved by

$$\pi^{ij} = \epsilon^{ipq} \epsilon^{jrs} \partial_p \partial_r Z^1_{qs}.$$

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This is because any transverse symmetric tensor can be written as the Einstein tensor $G^{ij}[Z^1]$ for some symmetric prepotential Z^1_{ij} (Young symmetry type $\square\square$).

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$$\delta Z^1_{ij} = \partial_i \xi^1{}_j + \partial_j \xi^1{}_i + 2\epsilon^1 \delta_{ij}$$

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... of a massless, conformal spin-2 field !

Duality invariance of the Pauli-Fierz action

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Indeed, the Hamiltonian constraint states that the Einstein tensor, which is transverse, should also be traceless $G^{ij}[h]\delta_{ij} = 0$. This implies that $G^{ij}[h]$ is equal to the Cotton tensor $D^{ij}[Z^2]$ of some symmetric field Z^2_{ij}, which is the second prepotential.

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The Cotton tensor involves three derivatives of Z^2_{ij}, is symmetric, transverse and traceless,

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The Cotton tensor involves three derivatives of Z^2_{ij}, is symmetric, transverse and traceless,

and also invariant under diffeomorphisms and Weyl rescalings

$$\delta Z^2_{ij} = \partial_i \xi^2_j + \partial_j \xi^2_i + 2\epsilon^2 \delta_{ij}$$

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Duality conjugate and canonically conjugate are thus equivalent.

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This turns out to be the case.

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$$S[Z^a_{mn}] = \int dt \left[\frac{1}{2} \int d^3x \epsilon^{ab} D^{ij}_a \dot{Z}_{bij} - H \right]$$

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where $D^{ij}_a \equiv D^{ij}[Z_a]$ is the Cotton tensor of the prepotential Z_{aij},

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To that end, one needs the appropriate geometrical concepts associated with higher spin conformal geometry.

Cotton tensor

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The Cotton tensor $B^{i_1 \cdots i_s}$ describes the higher spin conformal geometry in three dimensions, where the Weyl tensor identically vanishes.

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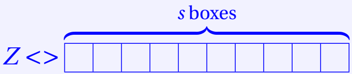
We consider a spin-s gauge field $Z_{i_1 \cdots i_s}$ in three dimensions,



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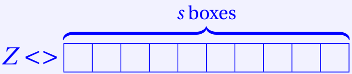

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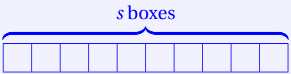
$Z <>$ 

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$$\delta Z_{i_1 \cdots i_s} = s \partial_{(i_1} \xi_{i_2 \cdots i_s)} + \frac{s(s-1)}{2} \delta_{(i_1 i_2} \lambda_{i_3 \cdots i_s)}$$

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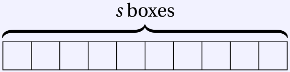
$B <>$ 

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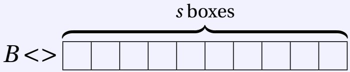

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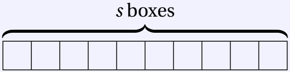
$B <>$ 

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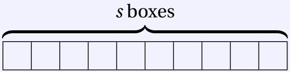
$B <>$ 

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A necessary and sufficient condition for a spin-s field $Z_{i_1 \dots i_s}$ to be pure gauge is that its Cotton tensor vanishes,

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with $1 \leq k < m \leq s$ and $1 \leq p \leq s$.

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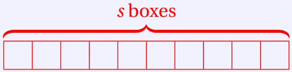
Conversely, any symmetric tensor B that is both transverse and traceless can be written as the Cotton tensor of some symmetric tensor Z,

Z <> 

$B = B[Z]$.

From $d^2 = 0$ (spin 1) to $d^{s+1} = 0$ (spin s)

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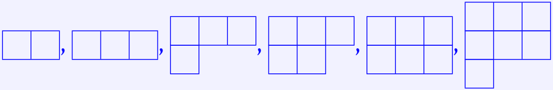

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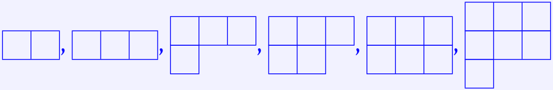
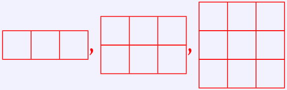
For instance, for $s = 3$, the tensors with Young symmetry type ,  etc are all in the complex.

The tensors  etc correspond to well-filled diagrams.

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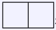
- $T <>$ , $dT <>$ , $(dT)_{\alpha_1 \beta_1} \sim \partial_{(\alpha_1} T_{\beta_1)}$

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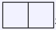
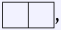
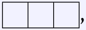
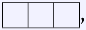
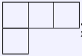
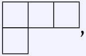
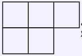
- $T <>$ , $dT <>$ , $(dT)_{\alpha_1 \beta_1} \sim \partial_{(\alpha_1} T_{\beta_1)}$
- $T <>$ , $dT <>$ , $(dT)_{\alpha_1 \beta_1 \gamma_1} \sim \partial_{(\alpha_1} T_{\beta_1 \gamma_1)}$
- $T <>$ , $dT <>$ ; $(dT)_{\alpha_1 \alpha_2 \beta_1 \gamma_1} \sim \partial_{[\alpha_2} T_{\alpha_1] \beta_1 \gamma_1}$
- $T <>$ , $dT <>$ ; $(dT)_{\alpha_1 \alpha_2 \beta_1 \beta_2 \gamma_1} \sim P\Big(\partial_{\beta_2} T_{\alpha_1 \alpha_2 \beta_1 \gamma_1}\Big)$

One has $d^{s+1} = 0$ ($d^4 = 0$ for spin 3) because two partial derivatives operators will inevitably occur in the same column in $d^{s+1} T$.

From $d^2 = 0$ (spin 1) to $d^{s+1} = 0$ (spin s)

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If T is in the complex, dT has one more box and is obtained by projecting ∂T on the corresponding symmetry type.

For instance for $s = 3$:

- $T <> \ydiagram{1}$, $dT <> \ydiagram{2}$, $(dT)_{\alpha_1\beta_1} \sim \partial_{(\alpha_1} T_{\beta_1)}$
- $T <> \ydiagram{2}$, $dT <> \ydiagram{3}$, $(dT)_{\alpha_1\beta_1\gamma_1} \sim \partial_{(\alpha_1} T_{\beta_1\gamma_1)}$
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For instance, $(d^4 f)_{\alpha_1\alpha_2\beta_1\gamma_1} = \partial_{[\alpha_2}\partial_{\alpha_1]}\partial_{\beta_1}\partial_{\gamma_1} f = 0$.

$d^{s+1} = 0$ – Generalized Poincaré lemma

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acting on antisymmetric tensors (rows of length one),

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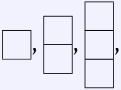
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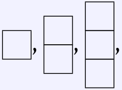

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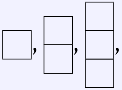

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One has similarly, *for tensors described by well-filled diagrams*, $d^k T = 0 \rightarrow T = d^{s+1-k} S$ for some S, with $0 < k \leq s$.

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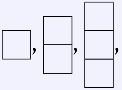
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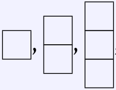

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Related construction by Olver, and Bekaert & Boulanger.

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THANK YOU !